SAMPLING THEORY :-

1. It is a process to convert continuous time signals in to Discrete signal.
2. Sufficient number of samples must be taken, so that the original signal is reconstructed properly:
3. Number of samples to be taken depends on Maximum signal Frequency present in the signal.
4. Different Types of sampling are:
   (a) Ideal samples, (b) Natural samples, (c) Flat Top samples.

Statement of Sampling Theorem :-

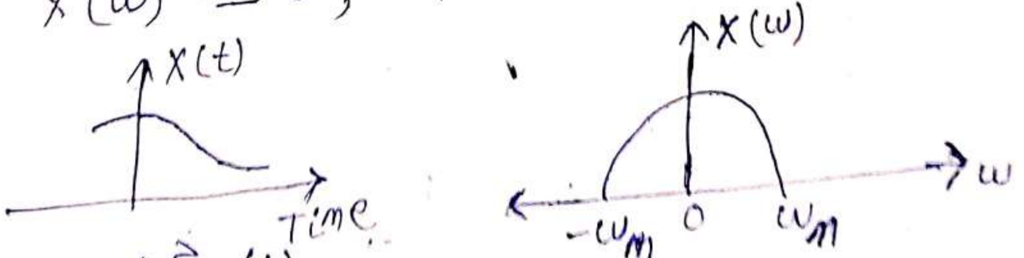
(i) A band limited signal of Finite energy, which has no Frequency component higher than f_m (HZ), is completely described by its sample values at uniform intervals less than (or) equal to $\frac{1}{2f_m}$.

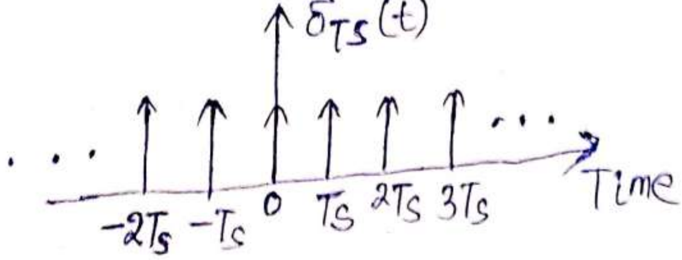
$$\boxed{T_s \leq \frac{1}{2f_m}}$$

(ii) A Band limited signal of finite energy, which has no Frequency components higher than f_m (HZ), may be completely recovered from the knowledge of its samples taken at the rate of $2f_m$ samples per second.

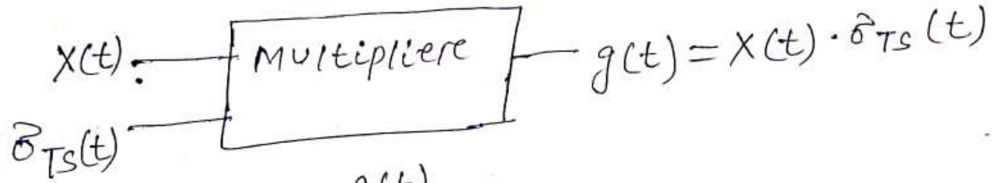
$$\boxed{F_s \geq 2f_m}$$

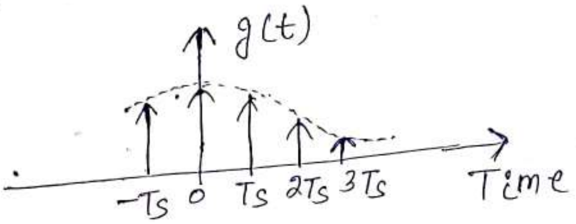
↳ If the signal is band limited to f_m then
$X(\omega) = 0$, for $\omega > \omega_m$



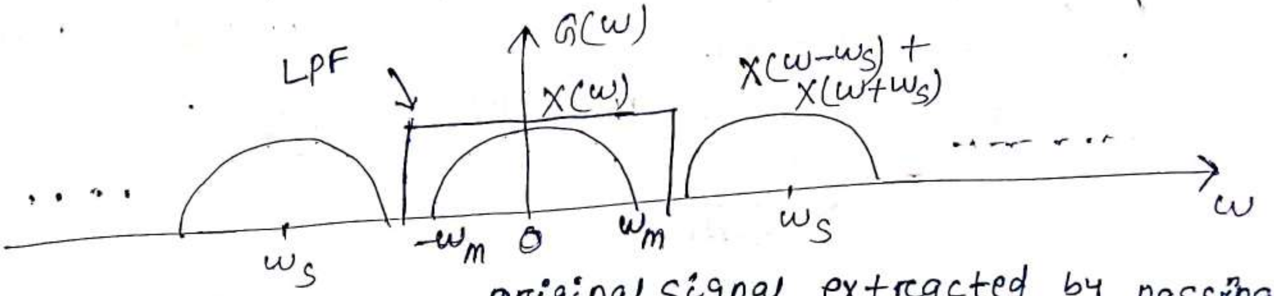


$$\delta_{T_s}(t) = \frac{1}{T_s}\left[1 + 2\cdot\cos\omega_s t + 2\cdot\cos 2\omega_s t + 2\cdot\cos 3\omega_s t + \cdots\right]$$





↳ In Frequency Domain, $G(\omega) = \frac{1}{T_s}\left[X(\omega) + X(\omega-\omega_s) + X(\omega+\omega_s) + X(\omega-2\omega_s) + X(\omega+2\omega_s) + \cdots\right]$



original signal extracted by passing through LPF.

$\boxed{\omega_s > 2\omega_m}$

↳ As long as $f_s > 2f_m$, $G(\omega)$ will repeat periodically without overlapping.

↳ Spectrum $G(\omega)$ extends upto ∞ (infinite) Frequency but our purpose is to extract original spectrum $X(\omega)$ out of the spectrum $G(\omega)$.

↳ At receiver we place LPF of Frequency ω_m. So we can extract original information.

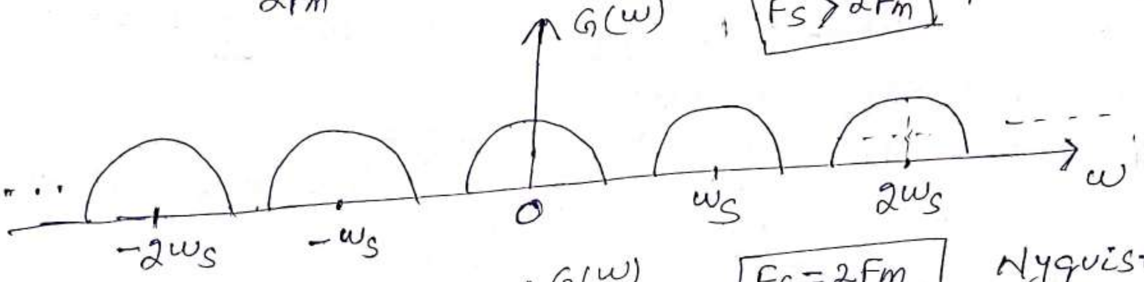
↳ $f_s > 2f_m$, To avoid successive cycles not to overlape.
$f_s = 2f_m$, successive cycles just touch each other.
$f_s < 2f_m$, successive cycles overlape each other.

↳ Hence, for reconstruction without distortion

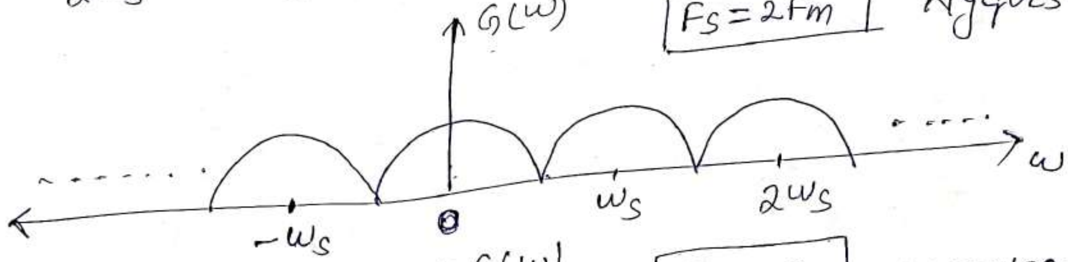
$\boxed{f_s \geq 2f_m}$

↳ $F_s = 2f_m$, Hence F_s is refererred as Nyquist Rate.
$T_s = \frac{1}{2f_m}$, T_s is Nyquist Interval

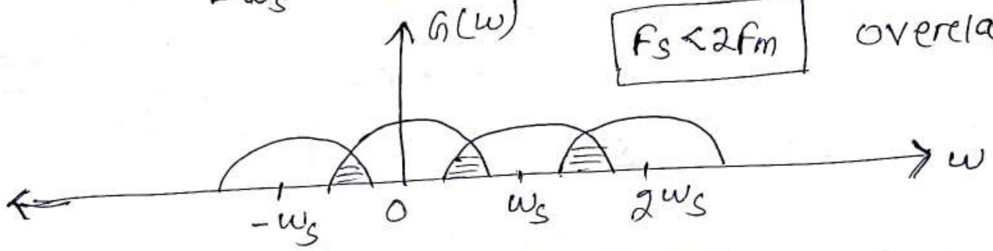
$\boxed{F_s > 2F_m}$ No overlapping



$\boxed{F_s = 2F_m}$ Nyquist sampling



$\boxed{F_s < 2F_m}$ overelape



↳ If $F_s < 2f_m$, then successive samples cycles of G(ω) will overelape each other.
↳ Due to Aliasing effect, it is not possible to recover original signal x(t) by LPF.
↳ Hence due to overelape of one region to other region, signal x(t) is distorted.
↳ So, before we go for sampling, we pass original signal through LPF. This is even refererred as pre-alias filter, other name is Band limit Filter.
↳ In short, to avoid aliasing:

① pre aliasing filter can be used.

② $F_s \geq 2F_m$

— x —

Examples based on sampling and Nyquist Rate :-

Que:- $x(t) = 3\cdot\cos(50\pi t) + 10\cdot\sin(300\pi t) - \cos(100\pi t)$
Calculate the Nyquist Rate for this signal.

Sol^n:- $f_1 = \frac{\omega_1}{2\pi} = 25\,Hz$, $f_2 = \frac{\omega_2}{2\pi} = 150\,Hz$, $f_3 = \frac{\omega_3}{2\pi} = 50\,Hz$

Maximum Frequency $f_m = 150\,Hz$

Nyquist Rate $F_s = 2f_m = 2\times150 = 300\,Hz$

—— xx —— xx —— x ——

Que:- Find the Nyquist Rate and Nyquist Interval for the signal $x(t) = \frac{1}{2\pi}\cdot\cos(4000\pi t)\cdot\cos(1000\pi t)$

Sol^n:- $x(t) = \frac{1}{4\pi}\left[2\cdot\cos(4000\pi t)\cdot\cos(1000\pi t)\right]$

$$= \frac{1}{4\pi}\left[\cos(3000\pi t) + \cos(5000\pi t)\right]$$

$f_1 = \frac{\omega_1}{2\pi} = 1500\,Hz$, $f_2 = \frac{\omega_2}{2\pi} = 2500\,Hz$, Maximum Frequency $f_m = 2500\,Hz$, Nyquist Rate $F_s = 2f_m = 2\times2500 = 5000\,Hz$,

Nyquist Interval $T_s = \frac{1}{F_s} = \frac{1}{5000} = \boxed{0.2\,msec}$

—— x —— x ——

Que:- Determine the Nyquist rate for a continuous time signal $x(t) = 6\cdot\cos 50\pi t + 20\cdot\sin 300\pi t - 10\cdot\cos 100\pi t$.

Sol^n:- $f_1 = \frac{\omega_1}{2\pi} = 25\,Hz$, $f_2 = \frac{\omega_2}{2\pi} = 150\,Hz$, $f_3 = \frac{\omega_3}{2\pi} = 50\,Hz$

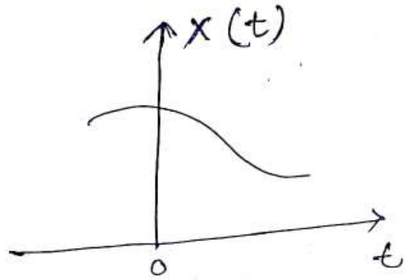
Maximum Frequency $f_m = 150\,Hz$

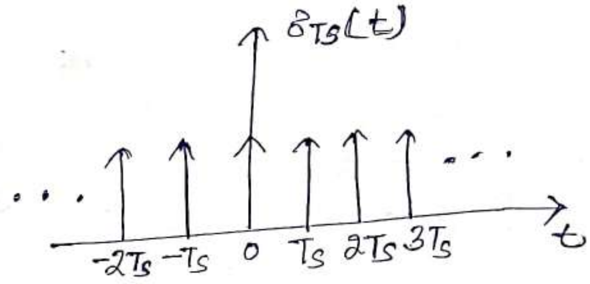
Nyquist Rate $F_s = 2f_m = 300\,Hz$

—— x ——

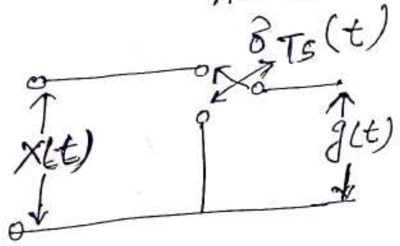
Instantaneous sampling (or) Impulse sampling (or) Ideal sampling :-

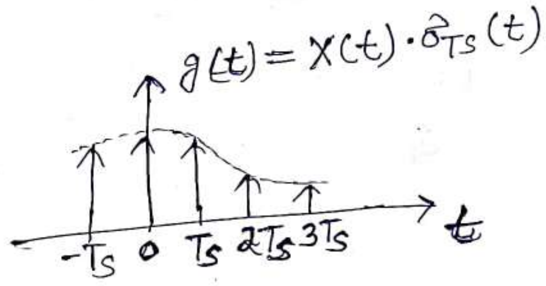
- It uses principle of multiplication.





$$\delta_{Ts}(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT_s)$$

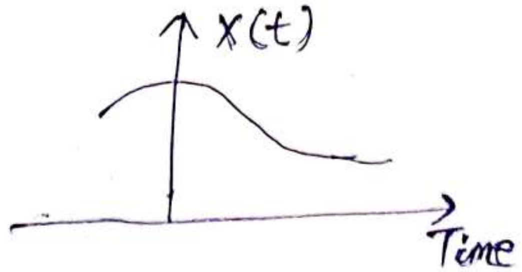


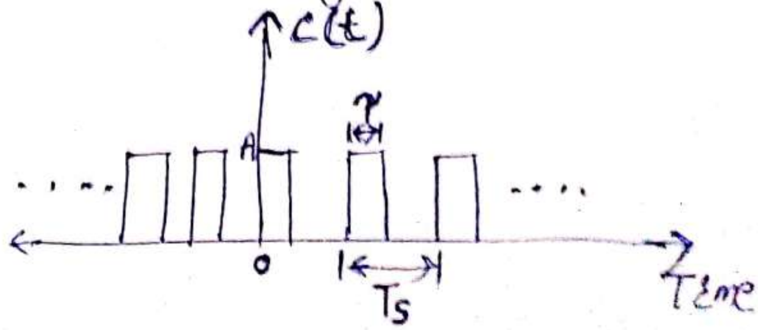


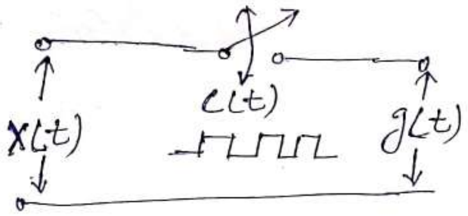
- To generate ideal samples train, we use switching sampler.
- If we assume, closing time $t \to 0$, then it has to be consider ideal impulse train.
- Impulse Train $\delta_{Ts}(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT_s)$
- output $g(t) = x(t)\cdot\delta_{Ts}(t) = x(t)\sum_{n=-\infty}^{\infty}\delta(t-nT_s)$
- In Frequency Domain, $G(\omega) = F_s \cdot \sum_{n=-\infty}^{\infty} X(F-nF_s)$
- practically, This method is not possible. High noise intercference is available. signal Energy is very Low.

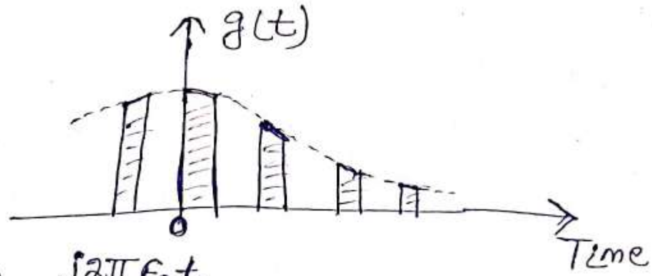
NATURAL SAMPLING :-

① It uses chopping principle.









$$c(t) = \frac{\tau \cdot A}{T_s} \cdot \sum_{n=-\infty}^{\infty} \operatorname{sinc}(f_n \tau) \cdot e^{j2\pi f_s t}$$

↳ $g(t) = x(t)$, for $c(t) = A$

$g(t) = 0$, for $c(t) = 0$

↳ So, Mathematically $g(t) = x(t) \cdot c(t)$

$$= \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} x(t) \cdot \operatorname{sinc}(f_n \cdot \tau) \cdot e^{j2\pi f_s t}$$

↳ Frequency Domain

$$G(\omega) = \frac{\tau \cdot A}{T_s} \sum_{n=-\infty}^{\infty} \sin(n f_s \tau) \cdot X(f - n f_s)$$

↳ This method is used practically. Noise interference is less. Because g(t) sampled output impulses have finite pulse duration (τ) and finite Energy.

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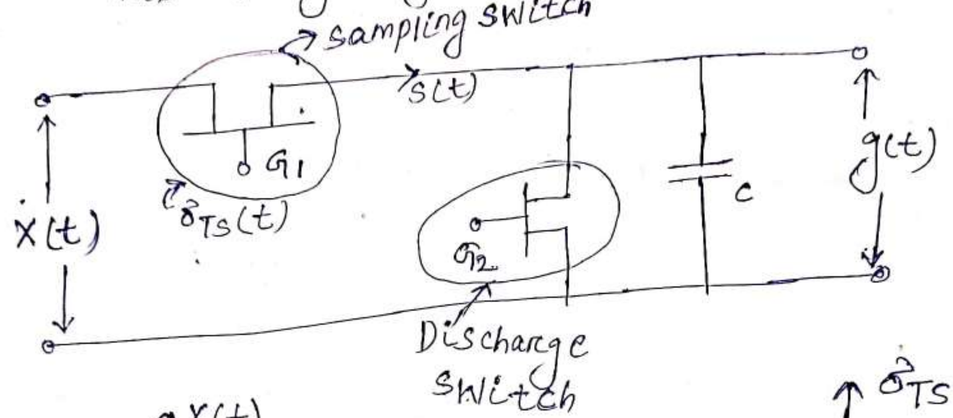
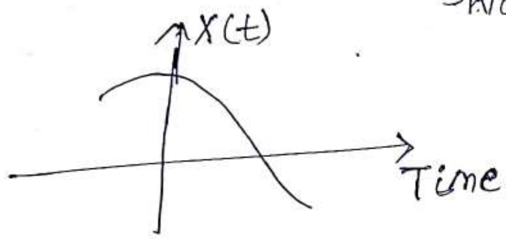
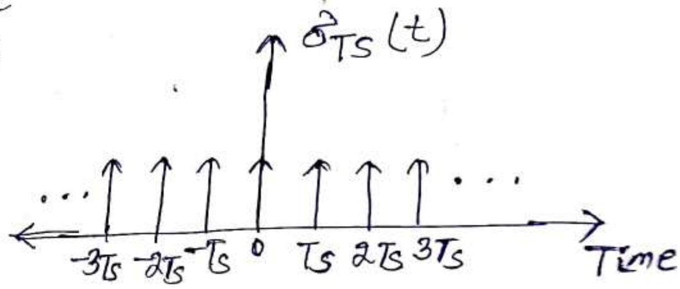
FLAT-TOP SAMPLING (PAM) :-

- It uses sample and hold circuit.
- It is practically possible like natural sampling but Flat top sampling is easier compared to natural sampling.
- It has very high noise interference.

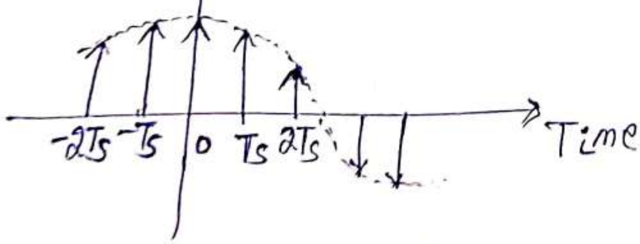


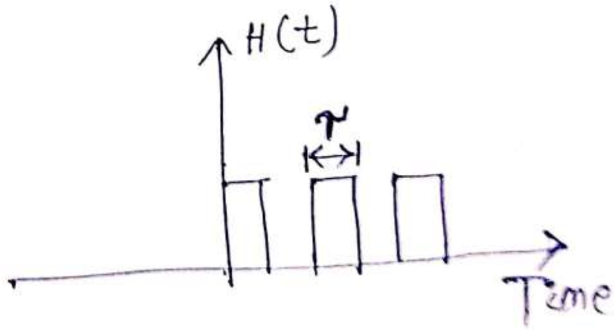




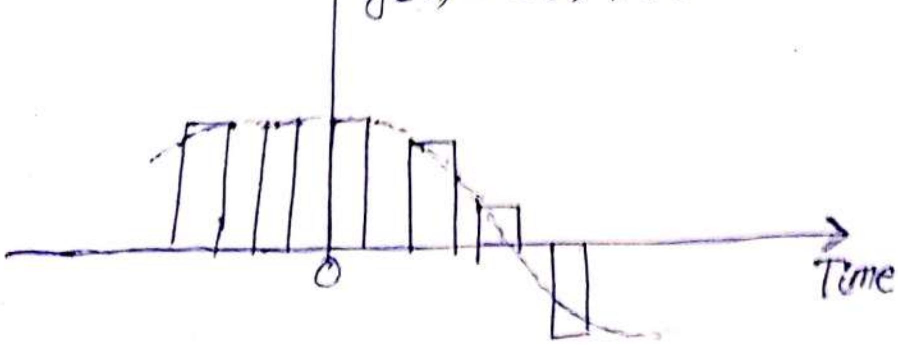
$$\delta_{Ts}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$s(t) = x(t) \cdot \delta_{Ts}(t)$$





$$g(t) = s(t) \cdot H(t)$$



τ = Time period that capacitor holds the output.

$H(t)$ = Function of Discharge switch

$s(t)$ signal is holded up to time period τ.

↳ $g(t) = s(t) \cdot H(t)$

$$= \sum_{n=-\infty}^{\infty} x(t) \cdot h(t - nT_s)$$

In Frequency Domain $G(\omega) = f_s \cdot \sum_{n=-\infty}^{\infty} X(F - nF_s) \cdot H(F)$

↳ By sampling switch, sampling can be done. Discharge switch will define the time period up to which capacitor will charge. By pressing on Discharge switch, capacitor will discharge. By pressing on to sampling switch, then sampled output obtained that is constant voltage which is similar across capacitor. So, sampling switch is identical to impulse train switch.

Sampling switch on → capacitor will charge

Discharge switch on → capacitor will discharge, and output will zero.

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Performance comparison of sampling Techniques :-

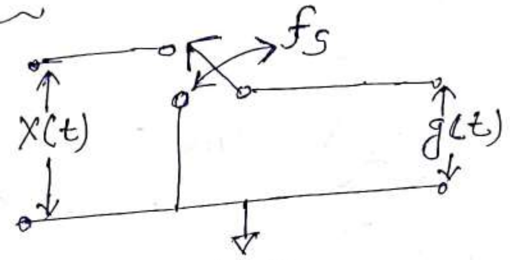
(a) performance parameters :

(1) sampling principle :-

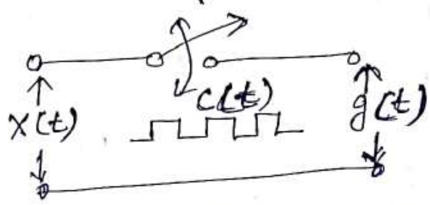
- ↳ In Ideal sampling, Multiplication is done.
- ↳ In Natural sampling, chopping is done.
- ↳ In Flat Top sampling, sample and Hold circuit is used.

(2) Generation circuit :-

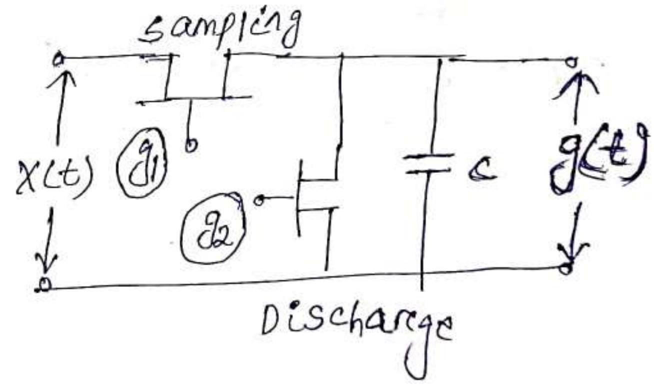
↳ In Ideal sampling,



↳ In Natural sampling,



↳ In Flat top sampling,



(3) Waveforms :-

(4) Feasibility :-

- ↳ Ideally Sampling practically not possible.
- ↳ Natural sampling practically used.
- ↳ Flat Top sampling practically used.

④ **Noise Interference :-**

↳ In Ideal sampling Noise interference is very high, In Natural sampling it is less, and in Flat Top sampling Noise Interference is high.

⑤ **Time Domain Representation :-**

$$g(t) = \sum_{n=-\infty}^{\infty} x(t) \cdot \delta(t - nT_s)$$ [For Ideal sampling]

$$g(t) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} x(t) \cdot \text{sinc}(nf_s\tau) \cdot e^{j2\pi f_s t}$$ [For Natural sampling]

$$g(t) = \sum_{n=-\infty}^{\infty} x(t) \cdot h(t - nT_s)$$ [For Flat Top sampling.]

⑥ **Frequency Domain Representation :-**

$$G(f) = f_s \cdot \sum_{n=-\infty}^{\infty} x(f - nf_s)$$ [For Ideal sampling.]

$$G(f) = \frac{\tau A}{T_s} \cdot \sum_{n=-\infty}^{\infty} \text{sinc}(nf_s\tau) \cdot X(f - nf_s)$$ [For Natural sampling.]

$$G(f) = f_s \cdot \sum_{n=-\infty}^{\infty} X(f - nf_s) \cdot H(f)$$ [For Flat Top sampling.]

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BIT RATE AND BAUD RATE :-

↳ Bit Rate : It is number of Bits per second (bits/sec)
(R_b) or R

↳ Baud Rate : It is number of symbols per second or Elements per second. [symbols/second / elements/second]
(r) ~ notation of Baud rate

↳ If n = number of bits/symbol (or) bits/element

Then $\boxed{r = \frac{R}{n}}$

↳ Baud rate always ~~greater~~ Less than Bit Rate.

$\boxed{r < R}$

↳ Total number of symbols [elements] $= L = 2^n$

Example :- An Analog signal carries 4 bits/signal elements. If 1000 signal elements are sent per second. Then Find the BitRate.

sol^n :- $n = 4$ bits/~~second~~ elements , Baud rate $r = 1000$ baud / elements/second / symbols/second

$R = n \cdot r = 4 \times 1000 = 4000$ bits/sec
$= \boxed{4 \text{ Kbps}}$

Total number of elements or symbols $L = 2^n = 2^4 = \boxed{16}$

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Que-2 :- An Analog signal has a bit rate of 8000 bps and a baud rate of 1000 baud. How many data elements are carried by each signal element? How many signal elements do we need?

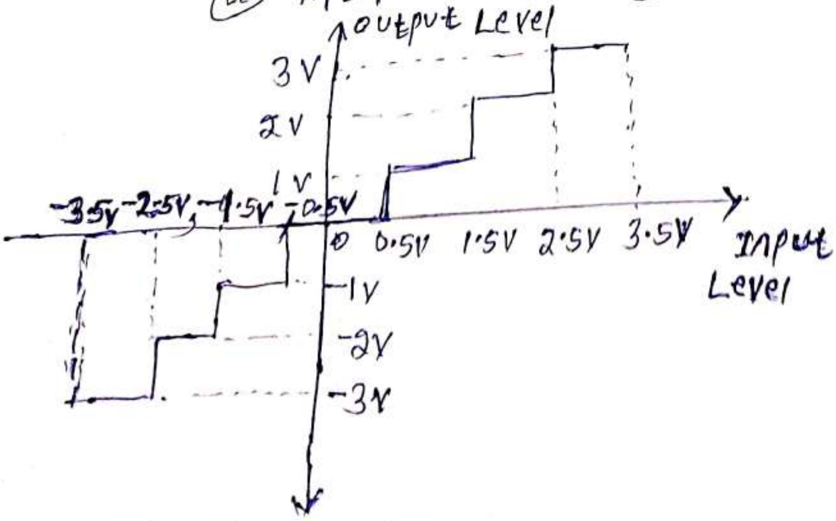
sol^n :- $R = 8000$ bps , $r = 1000$ baud ; $n = ?$, $L = ?$

$n = \frac{R}{r} = \frac{8000}{1000} = 8$ bits/element

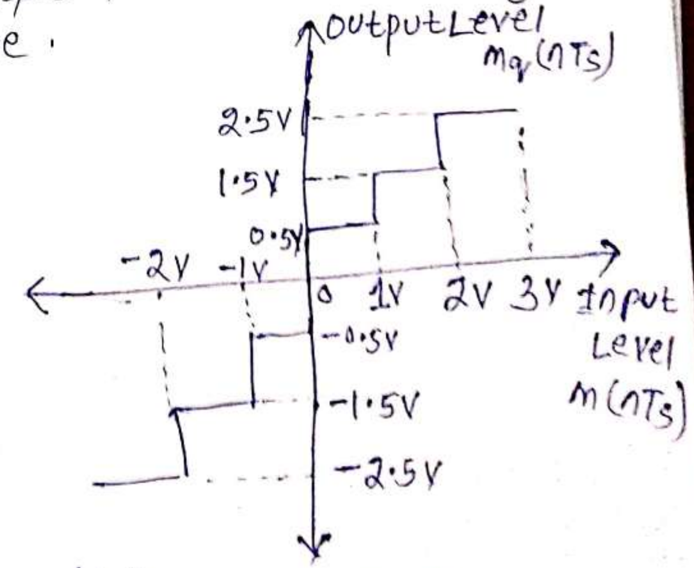
Total number elements $L = 2^n = 2^8 = 256$

QUANTIZATION :-

- A continuous signal such as voice has a continuous range of amplitudes therefore its samples have a continuous amplitude range.
- In otherwords we can say, within the finite amplitude range of signal we can find an infinite number of amplitude levels.
- It is not necessary to transmit the exact amplitude of the samples because any human sense (the ear or the eye) works as an ultimate receiver that can detect finite intensity differences.
- This means that the original continuous signal may be approximated by a signal constructed of discrete amplitudes selected on a minimum error basis from an available set.
- Amplitude Quantization is defined as the process of transforming the sample amplitude $m(nT_s)$ of a message signal $m(t)$ at time $t = nT_s$ in to a discrete amplitude $m_q(nT_s)$ taken from a finite set of possible amplitudes.
- Quantizer can be of a uniform or nonuniform type.
- In a uniform quantizer, the representation levels are uniformly spaced otherwise the quantizer is non uniform.
- The quantizer characteristics can be of two types
  (a) Mid tread (b) Midrise.



(a) Midtread Type



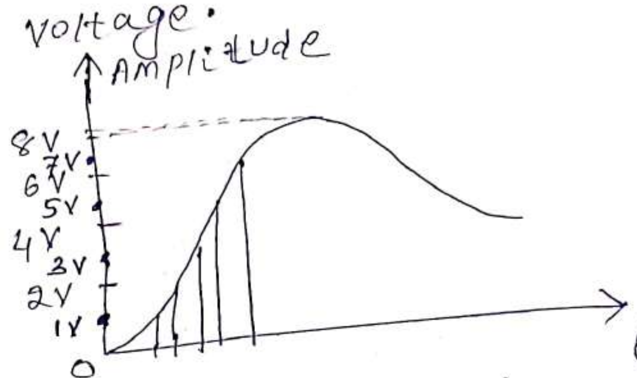
(b) Midrise Type

QUANTIZATION PROCESS :-

↳ Total dynamic range of the signal is divided in to L equal number of steps.

↳ Middle of each step will be selected as quantization voltage corresponding to a step voltage.

↳ Each of voltage will be rounded off to middle of the step (or) each of the sample will be rounded off to one of the nearest quantization voltage.



TAKE $L=4$

1 V → 00
3 V → 01
5 V → 10
7 V → 11

[Encoded the quantization voltage.]

Sample voltage	Quantization voltage	Encoded output	Quantization Error Qe (Sample voltage − quantized voltage)
0.6 volt	1 Volt	00	−0.4 V
1.7 Volt	1 Volt	00	0.7 V
2.2 V	3 V	01	−0.8 V
0 V	1 V	00	−1 V
8 V	7 V	11	1 V

$$\text{step size } \Delta = \frac{V_{max} - V_{min}}{\text{Number of Levels}} = \frac{8-0}{4} = 2$$

$Q_e(max)$ = Maximum Quantization Error $= \pm \frac{\Delta}{2}$

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NONUNIFORM QUANTIZATION :-

↳ If quantization characteristics is nonlinear then step-size is not constant, it means quantization is non-uniform quantization.

↳ In non uniform quantization, stepsize reduce with respect to reduction in signal so quantization noise decreases. ↳ By companding we can achieve it. ↳ Non uniform quantization is generally used for speech and music signals.

↳ crest factor = $\frac{\text{peak value of signal}}{\text{RMS value of signal}} = \frac{X_{max}}{X_{rms}}$

↳ crest factor usually very high for speech and music signals.

↳ signal power $p = \frac{x^2(t)}{R}$, where $x(t)$ = Mean value of signal

$R = 1$ for normalized power

so, power $\boxed{p = x^2(t)}$

↳ crest factor $C.F. = \frac{X_{max}}{X_{rms}} = \frac{X_{max}}{\sqrt{x^2(t)}} = \frac{X_{max}}{\sqrt{p}}$

↳ For normalized signal $X_{max} = 1$

$C.F. = \frac{1}{\sqrt{p}} \Rightarrow p = \frac{1}{(C.F.)^2}$

↳ For Non sinusoidal signal, signal to Noise Ratio

$SNR = 3 \times 2^N \times p$ where p = power

↳ For voice & speech signal $CF \gg 1$, so $p \ll 1$

Hence SNR is poor

↳ By using Non uniform quantization, we can change the step size with respect to signal. For weak signal we decrease the step size and for strong signal we increase the step size. That will improve SNR.

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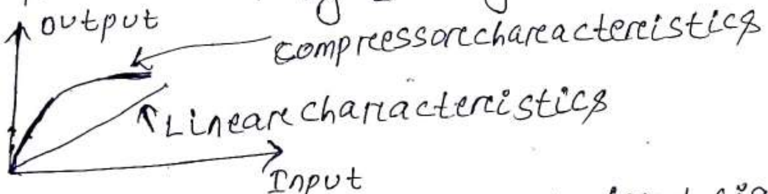
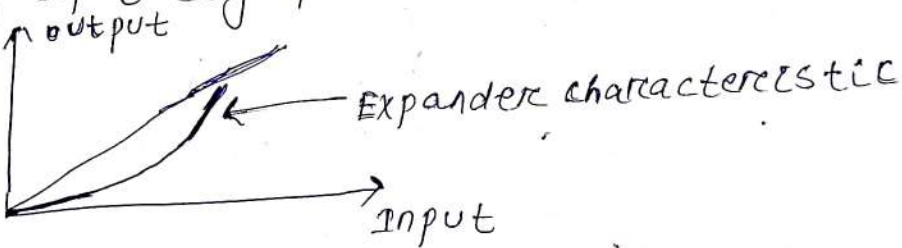
COMPANDING :- (1) companding is Nonuniform quantization. (2) It is required to be implemented to improve SNR of weak signal. (3) Quantization Noise is given by $\boxed{N_q = \frac{\Delta^2}{12}}$ for uniform quantization. and N_q is very high of weak signals in uniform quantization. In uniform quantization step size Δ is constant. (4) For weak signal noise is constant. (5) companding is derived from two word: (a) compression, (b) Expansion.

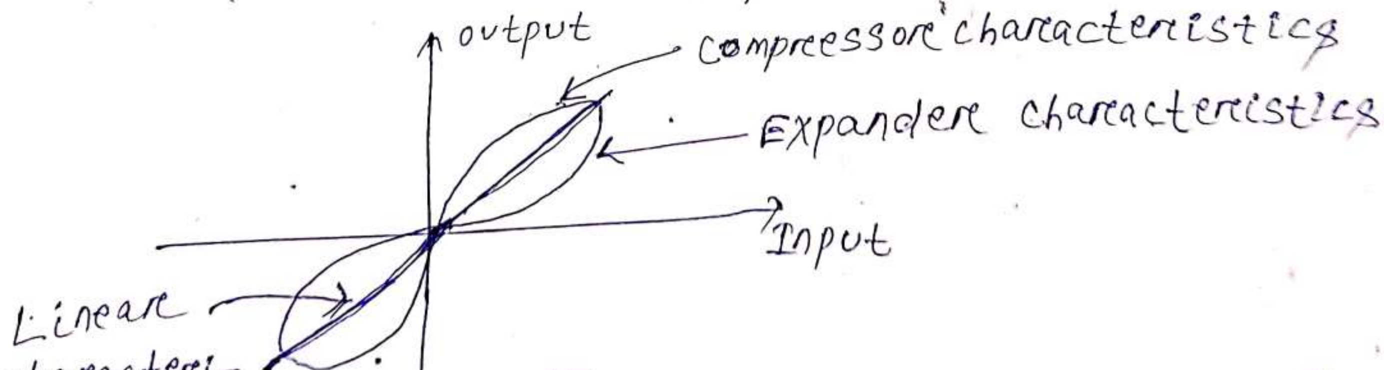
Input → Compressor → Uniform Quantizer → Expander → output

↳ compressor amplify low signal and attenuates strong signal.



↳ Expander attenuates weak input signal. and amplify the strong input signal





[COMPANDING CHARACTERISTICS]

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M-LAW COMPANDING FOR Non UNIFORM QUANTIZATION :-

↳ It is very populare in USA and Japan. ↳ Input, output relationship is given by $\frac{|y|}{X_{max}} = \frac{\ln\left[1+M\cdot\frac{|x|}{X_{max}}\right]}{\ln[1+M]}$

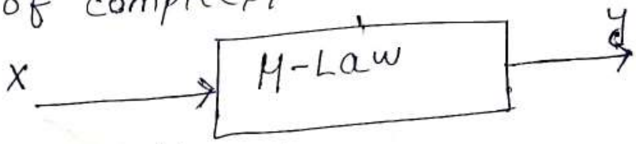
where,

x = Amplitude of Input signal at a particulare Instant.

y = compressed output signal.

X_{max} = Maximum Amplitude of Input signal

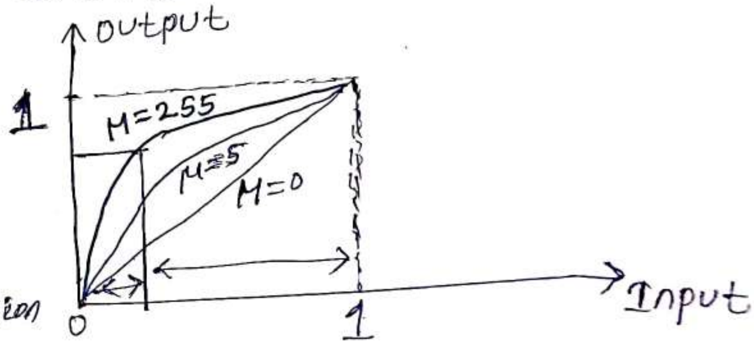
M = unitless parameter used to define the amount of compression.



$\left(\frac{|X|}{X_{max}}\right)$ → Range is (0 to 1)

↳ For $M=0$, $\frac{|y|}{X_{max}} = \frac{\ln[1+0]}{\ln[1+0]} = 1$, so There is no compression.

↳ Largere the value of M, results in to largere compression of output to input with highere amplitude.



↳ Recently in digital Transmission we use 8 bit pcm with $M = 255$.

x x — x — x —

A-Law companding fore Non uniform quantization :-

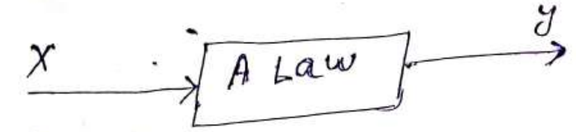
↳ It is very populare in India and in many European countries.

↳ It has slightly Flatter output characteristics than M-Law.

↳ Input to output relationship is given by

$$\frac{|y|}{X_{max}} = \begin{cases} \dfrac{\frac{A|x|}{X_{max}}}{1+\ln A} ; & 0 \le \dfrac{|x|}{X_{max}} \le \dfrac{1}{A} \\[2ex] \dfrac{1+\ln\left[\frac{A|x|}{X_{max}}\right]}{1+\ln A} ; & \dfrac{1}{A} \le \dfrac{|x|}{X_{max}} \le 1 \end{cases}$$

Scanned by CamScanner

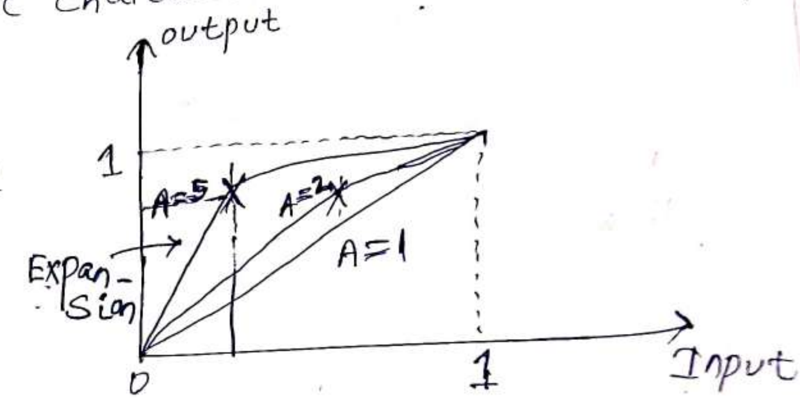


$\left(\frac{|x|}{X_{max}}\right) \rightarrow$ Range is 0 to 1.

↳ If $A=1$, $\frac{|y|}{X_{max}} = \frac{A|x|/X_{max}}{1+\ln A} = \frac{|x|}{X_{max}}$, That means

output = Input. So there is no compression.

↳ Larger the value of A, Larger the compression. Larger the value of A, linear characteristics shift towards Left.



↳ In India for digital Telephone system we use $A = 87.6$

——— x ——— x ———