Discrete Fourier Transform [D.F.T.] :- ①

$$x[n] \xleftrightarrow[\text{DFT}]{N\text{-point}} X(K) \qquad \text{— ①}$$

$$X(K) = \sum_{n=0}^{N-1} x[n] \cdot e^{\frac{-j2\pi Kn}{N}}, \quad K = 0, 1, 2, \cdots N-1$$

— ②

$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K) \cdot e^{\frac{j2\pi Kn}{N}}, \quad n = 0, 1, 2, \cdots N-1$$

$$w_N \equiv e^{-j2\pi/N} = \text{phase factor, Twiddle Factor}$$

$$X(K) = \sum_{n=0}^{N-1} x(n) \cdot w_N^{Kn}, \quad x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K) \cdot w_N^{-Kn}$$

→ In Discrete Time Fourier Transform (D.T.F.T.)

$$x[n] \xleftrightarrow{\text{D.T.F.T.}} X(e^{jw})$$

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-jwn}$$

continuous Frequency, Discrete Frequency

$$w_d = \frac{\text{Analog Frequency } (w_a)}{T_s}$$

where, T_s = Sampling Interval

By putting $K = 0$ in eqn-①, $X(0) = \sum_{n=0}^{N-1} x[n]$

$$\Rightarrow X(0) = x[0] + x[1] + x[2] + \cdots + x[N-1] \qquad \text{— ③}$$

By putting $K = N/2$ if N is even number in eqn ①

$$X\left(\frac{N}{2}\right) = \sum_{n=0}^{N-1} x(n) \cdot (-1)^n \qquad \text{— ④}$$

$$= x(0) - x[1] + x[2] - x[3] + x[4] \cdots$$

$$e^{-j\frac{2\pi}{N} \cdot \frac{N}{2} \cdot n} = e^{-j\pi n} = (-1)^n, \quad e^{-j\pi} = -1$$

By Adding Equation-③ and Equation-④

$$X(0) + X\left(\frac{N}{2}\right) = 2\left[x(0) + x(2) + x(4) + \cdots\right]$$

By subtracting Equation-(4) from Equation-(3)

$$X(0) - X\left(\frac{N}{2}\right) = 2\left[x(1) + x(3) + x(5) + \cdots\cdots\right]$$

$$X(K) = \begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ \vdots \\ X(N-1) \end{bmatrix} = X_N$$

$$\begin{array}{c} \\ [W_N] = \end{array} \begin{array}{c|ccccc} & n=0 & n=1 & n=2 & \cdots & n=N-1 \\ K=0 & W_N^0 & W_N^0 & W_N^0 & \cdots & W_N^0 \\ K=1 & W_N^0 & W_N^1 & W_N^2 & \cdots & W_N^{N-1} \\ K=2 & W_N^0 & W_N^2 & W_N^4 & \cdots & W_N^{2(N-1)} \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ K=N-1 & W_N^0 & W_N^{N-1} & W_N^{2(N-1)} & \cdots & W_N^{(N-1)^2} \end{array}$$

$$x(n) = \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ \vdots \\ x(N-1) \end{bmatrix} = x_N$$

$$= \left[W_N^{Kn}\right] \text{ in Matrix format}$$

In vector form we can write

$$X_K = [W_N] \cdot x_N \quad \text{and} \quad x_N = \frac{1}{N} \cdot [W_N]^* \cdot X_K$$

$$X[K] = \sum_{n=0}^{N-1} x[n] \cdot W_N^{Kn}, \quad x[n] = \frac{1}{N} \sum_{K=0}^{N-1} X(K) \cdot \underbrace{W_N^{-Kn}}_{(W_N^*)^{Kn}}$$

For any 1 value of K number of computational multiplications $= N$, and number of Additions $= (N-1)$, so N numbers of values for 'K' the Total Number of Multiplications require $= N^2 (N \cdot N)$ and Total Number of Additions require $= (N-1) \cdot N = N^2 - N$

↳ $x(t) \xleftrightarrow{\text{Fourier Transform}} X(\omega)$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega_a t} \cdot dt \quad \text{where } \omega_a = \text{Analog Frequency}$$

$$t \xrightarrow{\text{By sampling}} n \cdot T_s$$

$$x(t) \longrightarrow x[n]$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-j\omega_a \cdot n \cdot T_s}$$

$$= \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-j\omega_d \cdot n}$$

$$\boxed{\omega_d = \omega_a \cdot T_s} \quad , \quad 2\pi f_d = 2\pi f_a \cdot \frac{1}{f_s}$$

$$\Rightarrow \boxed{f_d = \frac{f_a}{f_s}}$$

f_d = Digital Frequency
f_a = Analog Frequency,
f_s = sampling Frequency.

properties of Twiddle Factor :-

$$\omega_N = e^{-j2\pi/N}$$

(1) $\omega_N^{K+N} = \omega_N^{K}$,

(2) $\omega_N^{K+\frac{N}{2}} = -\omega_N^{K}$

(3) $W_{N/2} = W_N^2$

PROPERTIES OF DFT :-

$$X(K) = \sum_{n=0}^{N-1} x[n] \cdot W_N^{Kn}, \quad K = 0, 1, 2, \cdots N-1$$

$$x[n] = \frac{1}{N} \sum_{K=0}^{N-1} X(K) \cdot W_N^{-Kn}, \quad n = 0, 1, 2, \cdots N-1$$

(1) periodicity property :-

If $x(n+N) = x(n)$ Then $X(K+N) = X(K)$

(2) Linearity property :-

If $x_1[n] \xleftrightarrow{DFT} X_1(K)$

$x_2[n] \xleftrightarrow{DFT} X_2(K)$

$$\alpha \cdot x_1[n] + \beta \cdot x_2[n] \xleftrightarrow{DFT} \alpha \cdot X_1(K) + \beta \cdot X_2(K)$$

③ Time shifting property :-

If $x[n] \xleftrightarrow{DFT} X(K)$

Then, $x((n-n_0))_N \xleftrightarrow{DFT} X(K) \cdot e^{-j\frac{2\pi}{N}Kn_0}$

$X(K) \cdot \omega_N^{Kn_0}$

$x((n+n_0))_N \xleftrightarrow{DFT} X(K) \cdot e^{j\frac{2\pi}{N}Kn_0}$

$\approx X(K) \cdot W_N^{-Kn_0}$

④ Frequency shifting property :-

If $x[n] \xleftrightarrow{DFT} X(K)$ Then

$x[n] \cdot e^{j\frac{2\pi \cdot l}{N} \cdot n} \xleftrightarrow{DFT} X((K-l))_N$

⑤ Expansion in Time property :-

In DTFT, $x[n] \xleftrightarrow{DTFT} X(e^{jw})$, periodic with 2π

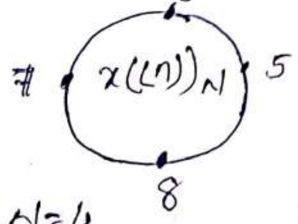
$x[\frac{n}{K}] \xleftrightarrow{DTFT} X(e^{jwK})$, periodic with $\frac{2\pi}{K}$

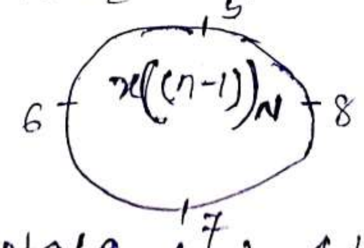
In DFT, $x[n] \longleftrightarrow X(K)$

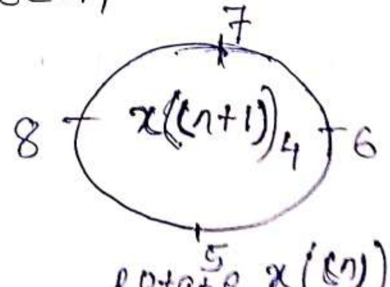
$\{x(0), x(1), x(2), \dots x(N-1)\} \xleftrightarrow{DFT} \{X(0), X(1), X(2), \dots X(N-1)\}$

$x[\frac{n}{K}] \xleftrightarrow{DFT} \{\underbrace{X(0), X(1), X(2) \dots X(N-1)}_{1st\ Repetition}, \underbrace{X(0), X(1), \dots X(N-1)}_{2nd\ Repetition}, \underbrace{X(0), X(1), X(2), \dots X(N-1)}_{Kth\ Repetition}\}$

$x(n) = \{5, 6, 7, 8\}$



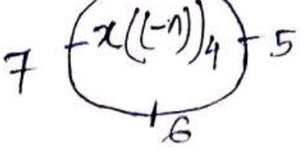




$N=4$

Rotate $x((n))_N$ 1 time in Anticlockwise Direction

Rotate $x((n))_4$ clockwise by 1 unit.



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(6) circular convolution :-

$x_1[n] \xleftrightarrow{DFT} X_1(K)$, $x_2[n] \xleftrightarrow{DFT} X_2(K)$

$$w(n) = x_1(n) \; (N) \; x_2(n) = \sum_{m=0}^{N-1} x_1(m) \cdot x_2((n-m))_N$$

↑ circular convolution

$x_1(n)$, $x_2(n)$ sequence having equal length l. otherwise zeropadding will occur in the sequence, which has less number of samples.

$$w(n) = x_1(n) \; (N) \; x_2(n) \xleftrightarrow{DFT} X_1(K) \cdot X_2(K)$$

(7) Multiplication property :-

$$x_1[n] \cdot x_2[n] \xleftrightarrow{DFT} \frac{1}{N} \cdot \left[X_1(K) \; (N) \; X_2(K)\right]$$

Que:- Find circular convolution between Two sequence $x_1(n) = \{2, 3, 4, 5\}$, $x_2(n) = \{5, 6, 2, 1\}$

Sol^n:- $y(n) = x_1(n) \; (N) \; x_2(n)$

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} = \begin{bmatrix} 2 & 5 & 4 & 3 \\ 3 & 2 & 5 & 4 \\ 4 & 3 & 2 & 5 \\ 5 & 4 & 3 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 10+30+8+3 \\ 15+12+10+4 \\ 20+18+4+5 \\ 25+24+6+2 \end{bmatrix} = \begin{bmatrix} 51 \\ 41 \\ 47 \\ 57 \end{bmatrix}$$

— x — x —

Que:- N-point DFT of $x[n] = a^n$, $0 \le n \le N-1$ is $X(K)$. Then the value of $X(K)$ for $K=2$, $a = 0.5$ and $N=4$ is ______ .

Sol^n:- $$X(K) = \sum_{n=0}^{N-1} x[n] \cdot e^{\frac{-j2\pi Kn}{N}} = \sum_{n=0}^{N-1} a^n \cdot \left(e^{\frac{-j2\pi K}{N}}\right)^n$$

$$= \sum_{n=0}^{N-1} \left(a \cdot e^{\frac{-j2\pi K}{N}}\right)^n$$

we know, $\sum_{n=0}^{N} a^n = \frac{1-a^{N+1}}{1-a}$

By applying this to above Equation, we will get

$$= \frac{1-\left(a \cdot e^{-j2\pi K/N}\right)^N}{1-a \cdot e^{-j2\pi K/N}}$$

$$\Rightarrow X(K) = \frac{1-(a)^N}{1-a \cdot e^{-j2\pi K/N}}$$

putting $K=2$,

$$\Rightarrow X(2) = \frac{1-(0.5)^4}{1-0.5\, e^{-j2\pi \times 2/4}}$$

$$\Rightarrow X(2) = \frac{1-(0.5)^4}{1-0.5\, e^{-j\pi}} \quad , \quad \boxed{e^{-j\pi} = -1}$$

$$= \frac{1-0.0625}{1-0.5 \times (-1)} = 0.625$$

x —— x —— x —— x ——

(8). CIRCULAR REVERSAL :-

If $x[n] \xleftrightarrow{DFT} X(K)$ Then

$$\boxed{x((-n))_N \xleftrightarrow{DFT} X((-K))_N}$$

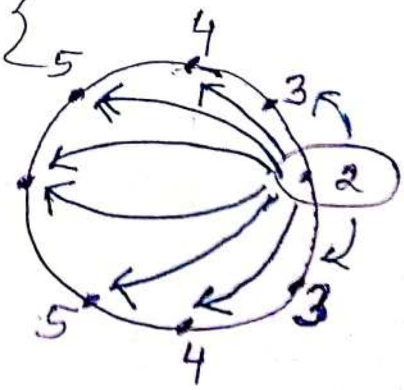
$$\boxed{x((-n))_N = x(N-n)}$$

(9.) circularly Even sequence :-

$x(N-n) = x(n)$, $1 \leq n \leq N-1$

The sequence will be symmetrical about origin point. $x[n] = \{2, 3, 4, 5, 6, 5, 4, 3\}$

↳ It is circularly even sequence



(10) circularly odd sequence :-

$x((-n))_N = x(N-n) = -x(n)$; $1 \leq n \leq N-1$

↳ The sequence will be antisymmetric about the origin.

$x[n] = \{2, 3, 4, 5, 6, -5, -4, -3\}$

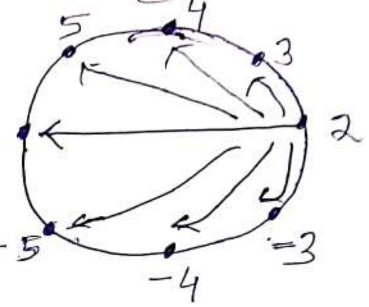
↑

Not circularly odd sequence.

To be circularly odd sequence insted of $\underset{\sim}{6}$, there should be $\underset{\sim}{0}$.

$\underline{\underline{0}}$

$6+(-6)$
$= \underline{\underline{0}}$



(11) conjugate property :-

If $x[n] \xleftrightarrow{DFT} X(K)$ Then

$x^*[n] \xleftrightarrow{DFT} X^*((-K))_N$

(12) parseval's Relation :-

$$\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \cdot \sum_{K=0}^{N-1} |X(K)|^2$$

(13) symmetry property of a 'real valued sequence :-

$x[n] \xleftrightarrow{DFT} X(K)$, Then

$$\boxed{X(N-K) = X^*(K)}$$

$$\boxed{\Rightarrow X(K) = X^*(N-K)}$$

Que:- If $X(K) = \{\underset{X(0)}{5}, \underset{X(1)}{2+j}, \underset{X(2)}{0}, \underset{X(3)}{0}, \underset{X(4)}{3+j}, -, -, -\}$

Then find the value of $x(0)$ is ——.

Soln :- $x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X[K] \cdot W_N^{-Kn}$, $x(0) = \frac{1}{N} \sum_{K=0}^{N-1} X(K)$

$X(5) = X^*(8-5) = X^*(3) = \boxed{0}$, $X(6) = X^*(8-6) = X^*(2) = \boxed{0}$,

$X(7) = X^*(8-7) = X^*(1) = \boxed{2-j}$

$$x(0) = \frac{1}{8}\left[X(0)+X(1)+X(2)+X(3)+X(4)+X(5)+X(6)+X(7)\right]$$

By putting all these values we will get $x(0)$

Que:- Two sequence $[\overline{a,b,c}]$ and $[\overline{A,B,C}]$ are related as

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & W_3^{-1} & W_3^{-2} \\ 1 & W_3^{-2} & W_3^{-4} \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}, \text{ where } W_3 = e^{j\frac{2\pi}{3}}$$

If another sequence $[p, q, r]$ is derived as

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & W_3^1 & W_3^2 \\ 1 & W_3^2 & W_3^4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & W_3^2 & 0 \\ 0 & 0 & W_3^4 \end{bmatrix} \begin{bmatrix} A/3 \\ B/3 \\ C/3 \end{bmatrix}$$

Then the relation between sequences $[p, q, r]$ and $[a, b, c]$ is

(A) $[p, q, r] = [b, a, c]$, (B) $[p, q, r] = [b, c, a]$

✓(C) $[p, q, r] = [c, a, b]$, (D) $[p, q, r] = [c, b, a]$

Sol^n:- $\displaystyle\sum_{n=-\infty}^{-1} a^{-n} \cdot e^{\frac{-j2\pi kn}{N}}$, $\displaystyle S = \sum_{n=-\infty}^{-1} \left(a \cdot e^{\frac{j2\pi k}{N}}\right)^{-n}$

Let $-n = m$, when $n \to -\infty$, $m \to +\infty$

$n \to -1$, $m \to +1$

$$S = \sum_{m=1}^{\infty} \Big(\underbrace{a \cdot e^{\frac{j2\pi k}{N}}}_{\text{Take } A}\Big)^m = A + A^2 + A^3 + \cdots$$

$$= A[1 + A + A^2 + A^3 + \cdots]$$

$$= A \cdot \sum_{n=0}^{\infty} (A)^n = A \cdot \frac{1}{1-A}$$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & W_N^1 & W_N^2 \\ 1 & W_N^2 & W_N^4 \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \end{bmatrix}, \text{ if } W_N = e^{-j2\pi/N}$$

for $N = 3$ here

$$\underbrace{\begin{bmatrix} p \\ q \\ r \end{bmatrix}}_{x'(n)} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & W_3^1 & W_3^2 \\ 1 & W_3^2 & W_3^4 \end{bmatrix} \underbrace{\begin{bmatrix} A \\ W_3^2 \cdot B \\ W_3^4 \cdot C \end{bmatrix}}_{X'(K)}$$

$x[n] = [a, b, c]$

$\downarrow$

$X(K) = [A, B, C]$

(5)

$x'[n] = [p, q, r]$

$X'[K] = [A, W_3^2 \cdot B, W_3^4 \cdot C]$

$\downarrow W_3^0 \cdot A$ we can consider

$X[K] = [A, B, C]$

$\downarrow X(0), X(1), X(2)$

we can write, $X'[K] = W_N^{2K} \cdot X(K)$

$N=3$ it is given $\longrightarrow 2 = l$

$X'[K] = e^{j\frac{2\pi}{3} 2K} \cdot X(K)$

we know, if $x[n] \xleftrightarrow{DFT} X(K)$ Then,

$x((n+l))_N \xleftrightarrow{DFT} e^{j\frac{2\pi K}{N} \cdot l} \cdot X(K)$

By comparing $l=2$ we will get.

$x[n] = [a, b, c] \longleftrightarrow X(K)$

$x((n+2))_3 \longleftrightarrow e^{j\frac{2\pi K}{N} \cdot 2} \cdot X(K) = X'[K]$

$\approx x'[n]$

Therefore $\boxed{x'[n] = x((n+2))_3}$

$[p, q, r] = [c, a, b]$

x — x — x — x

FAST FOURIER TRANSFORM (FFT) :-

For calculating N-point DFT of $x(n)$,

$$X(K) = \sum_{n=0}^{N-1} x[n] \cdot W_N^{Kn} \quad , \text{ where } W_N = \text{Twiddle Factor} = e^{\frac{-j2\pi}{N}}$$

$$X[K] = x[0] \cdot W_N^{0 \cdot K} + x[1] \cdot W_N^{1 \cdot K} + x[2] \cdot W_N^{2 \cdot K} + \cdots + x[N-1] \cdot W_N^{(N-1)K}$$

$K = 0, 1, 2, \text{-----} N-1$

To calculate N-point DFT we require

(i) Total Number of complex Multiplications $= N \times N = \boxed{N^2}$

(ii) Total Number of complex Additions $= N \cdot (N-1) = N^2 - N$

RELATION BETWEEN DFT SEQUENCE AND FOURIER SERIES COEFFICIENTS :-

The periodic sequence $x_p(n)$ with period N is

$$x_p(n) = \sum_{K=0}^{N-1} c_K \cdot e^{j2\pi nK/N}, \text{ where } -\infty < n < \infty$$

where c_K is Fourier series coefficients.

$$c_K = \frac{1}{N} \cdot \sum_{n=0}^{N-1} x_p(n) \cdot e^{-j2\pi Kn/N}, \text{ where } K = 0, 1, \cdots N-1 \quad \text{——— (1)}$$

If $x(n) \xleftrightarrow{DFT} X(K)$, By making Inverse DFT

$$x(n) = \frac{1}{N} \cdot \sum_{K=0}^{N-1} X(K) \cdot e^{j2\pi nK/N} \quad \text{——— (2)}$$

where $x(n)$ is Aperiodic sequence.

$x(n) = x_p(n)$ in $0 \leq n \leq N-1$

By comparing Equation-(1) and Equation-(2) we will get, $\boxed{c_K = \frac{X(K)}{N}}$

(or) $\boxed{X(K) = N \cdot c_K}$

——— x ——— x ———

RELATION OF DFT WITH Z-TRANSFORM :-

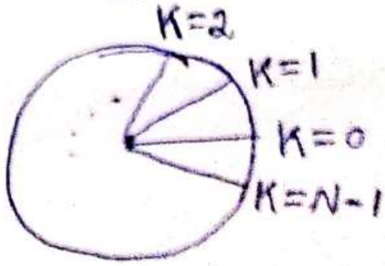
(1) DFT From Z-Transform

If $x(n)$ is sequence Then it's Z-Transform is

$$Z[x(n)] = X(Z) = \sum_{n=-\infty}^{\infty} x(n) \cdot Z^{-n}$$

If $X(Z)$ is sampled on the unit circle and the sampling rate is uniform and the number of samples $= N$

Here radius $r = 1$



$$Z_K = e^{\frac{j2\pi K}{N}}$$

$$* \text{ DFT } X(K) = X(Z)\Big|_{\text{at } Z = e^{\frac{j2\pi K}{N}}}$$

$$X(K) = \sum_{n=-\infty}^{\infty} x(n) \cdot e^{\frac{j2\pi K n}{N}}$$

$x(n)$ is limited to $n = 0, 1, 2, \dots N$

$$\therefore X(K) = \sum_{n=0}^{N-1} x(n) \cdot e^{\frac{-j2\pi n K}{N}}$$

$X(Z)$ to DFT, $X(Z) = \sum_{n=0}^{N-1} x(n) \cdot Z^{-n}$, where $n = 0, 1, \dots N-1$

$$= \sum_{n=0}^{N-1} \frac{1}{N} \sum_{K=0}^{N-1} X(K) \cdot e^{j2\pi Kn/N} \cdot Z^{-n}$$

$$= \frac{1}{N} \sum_{K=0}^{N-1} X(K) \cdot \sum_{n=0}^{N-1} e^{j2\pi Kn/N} \cdot Z^{-n}$$

$$= \frac{1}{N} \sum_{K=0}^{N-1} X(K) \cdot \sum_{n=0}^{N-1} \left(e^{\frac{j2\pi K}{N}} \cdot Z^{-1}\right)^{n}$$

$$= \frac{1}{N} \cdot \sum_{K=0}^{N-1} X(K) \cdot \frac{1 - \left(e^{j2\pi K/N} \cdot Z^{-1}\right)^{N}}{1 - e^{j2\pi K/N} \cdot Z^{-1}} \qquad \left[\because \sum_{n=0}^{N} a^{n} = \frac{1 - a^{N+1}}{1 - a}\right]$$

$$\left(e^{j2\pi K/N} \cdot Z^{-1}\right)^{N} = e^{\frac{j2\pi KN}{N}} \cdot Z^{-N} = e^{j2\pi K} \cdot Z^{-N}, \quad e^{j2\pi K} = 1$$

$$\boxed{X(Z) = \frac{1 - Z^{-N}}{N} \cdot \sum_{K=0}^{N-1} \frac{X(K)}{1 - e^{j2\pi K/N} \cdot Z^{-1}}}$$

This is the relationship between DFT and Z-Transform.

— x — x —

FILTERING OF LONG DATA SEQUENCES :- (7)

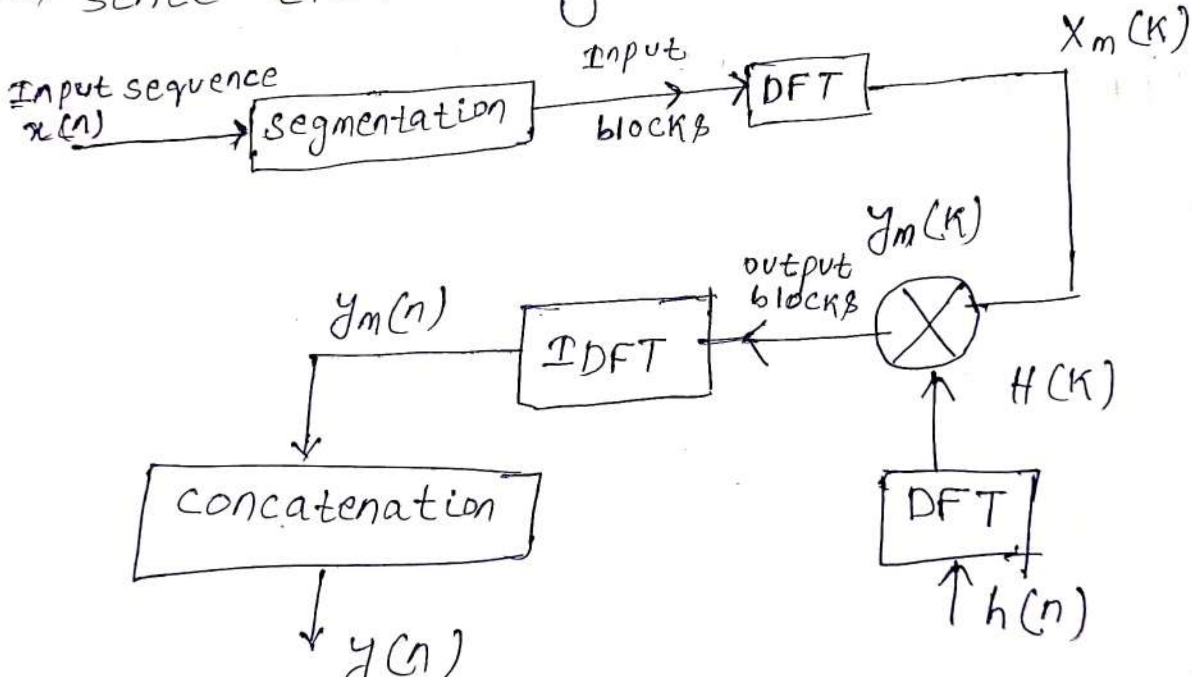
It can be very long

example :- Real Time signal processing

↳ Linear filtering using DFT must be done on a block of input data.

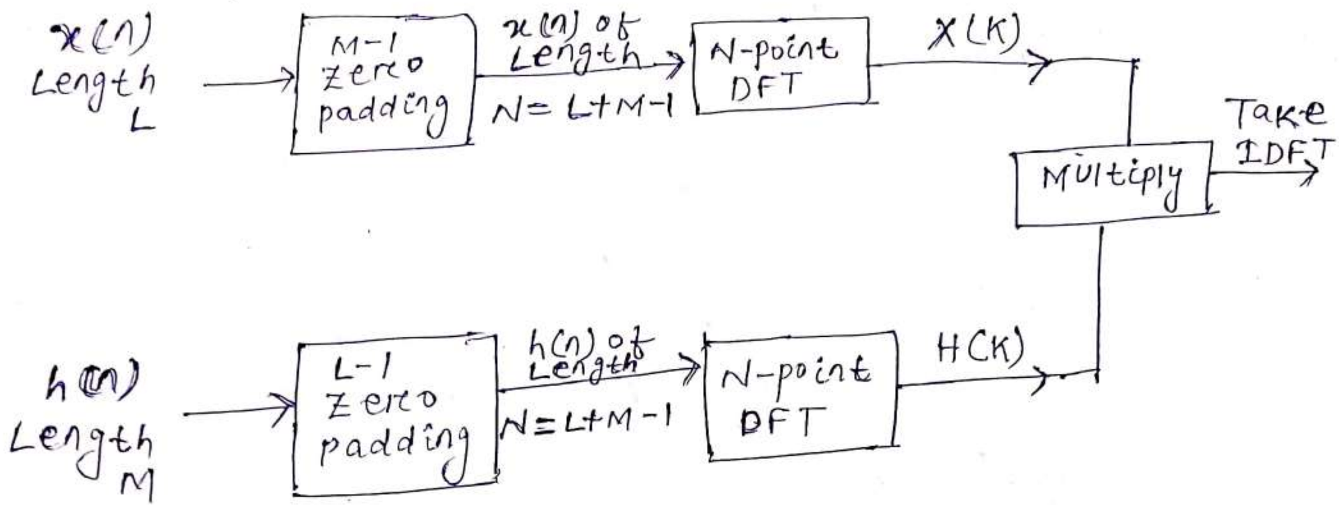
↳ At first Long data is segmented into Fixed size blocks.

↳ since the Filtering is a Linear process



↳ For segmentation, concatenation there are Two Methods available:
① overlap save method, ② overlap Add Method.

DFT for Linear Filtering :-



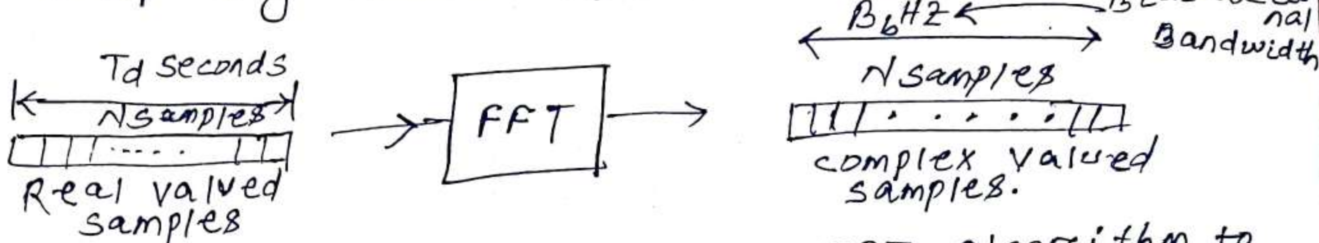
$$y(n) = x(n) * h(n)$$

IDFT → Inverse Discrete Fourier Transform

— x — x — x — x — x —

FAST FOURIER TRANSFORM (FFT) BASICS :-

- It is fast computation algorithm for Discrete Fourier Transform (DFT).
- In FFT we are taking array of Time Domain waveform samples and producing array of Frequency domain spectrum samples.



- N must be a power of 2 for FFT algorithm to be truly "fast".
- In Input side Real valued samples we are giving, in output side complex valued samples we will get. Typically work with magnitude and phase representation of the complex values.
- Sampling Interval $\Delta t = \frac{T_d}{N}$, sampling frequency $\frac{1}{\Delta t} = \frac{N}{T_d} = f_s (Hz)$
  In time domain

  similarly in frequency Domain, $\Delta f = \frac{B_b}{N} = \frac{f_s}{N}$ = frequency spacing

  $f_{max} = \frac{B_b}{2} = \frac{f_s}{2}$ ⇒ Typically display only Lower Half of the output Array.

  Nyquist frequency
- FFT is an efficient way (or) Algorithm to compute DFT with reduced computations. It is not a Transform. It is an algorithm.

Radix-2 FFT Algorithms :-

DIT (Decimation in Time)

DIF (Decimation in Frequency)

Both Algorithms use Divide and conquer Approach.

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↳ we have to choose the signal length 'N' such that it can be factored like

$$N = r_1 \cdot r_2 \cdot r_3 \cdot \cdots \cdot r_m$$

If $r_1 = r_2 = \cdots = r_m = r$

Then $\boxed{N = r^m}$

where $r \rightarrow$ Represents Radix.

For Radix-2, $r = 2$, $N = 2^m$

N-point DFT

Divides into

$\frac{N}{2}$ point DFT — $\frac{N}{2}$ point DFT

$\frac{N}{4}$ point DFTs — $\frac{N}{4}$ point DFTs — $\frac{N}{4}$ point DFTs — $\frac{N}{4}$ point DFTs

It will continue the division array like this until 2-point DFTs we obtaine.

① symmetry property: $W_N^{K+\frac{N}{2}} = -W_N^K$

proof:- $W_N^{K+\frac{N}{2}} = e^{-j\frac{2\pi}{N}\left(K+\frac{N}{2}\right)} = e^{-j\frac{2\pi}{N}\cdot K} \cdot e^{-j\frac{2\pi}{N}\cdot\frac{N}{2}}$

$$= e^{-j\frac{2\pi}{N}\cdot K} \cdot e^{-j\pi} = \boxed{-W_N^K}$$

② periodicity:- $W_N^{K+N} = W_N^K$

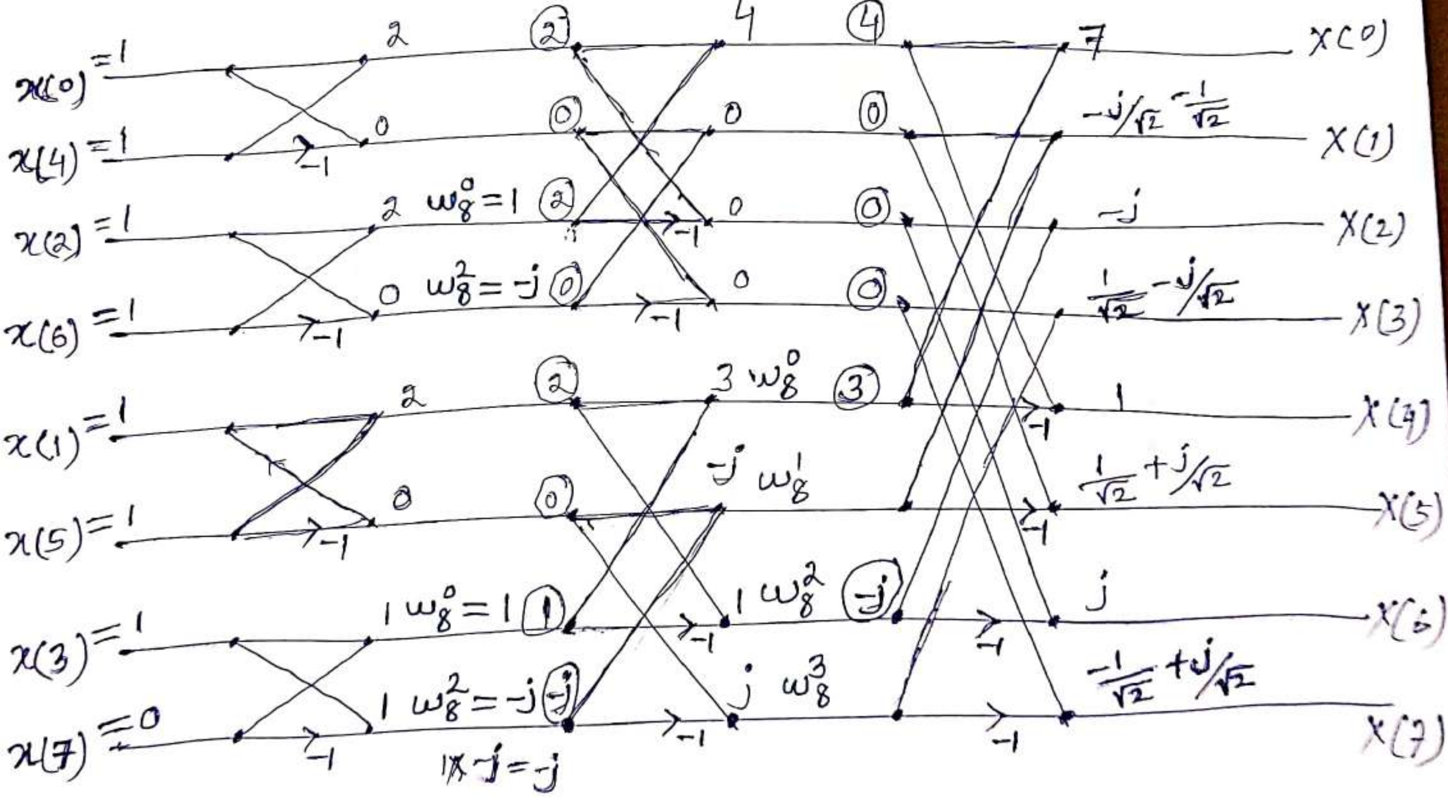
③ $W_N^2 = e^{-j\frac{2\pi}{N}\cdot(2)}$

$$= e^{\frac{-j2\pi}{(N/2)}} = W_{N/2}$$

DIT-FFT [DECIMATION IN TIME]

Que:- Find the 8 point DFT of $x(n) = \{1,1,1,1,1,1,1,0\}$ using radix-2 DIT-FFT algorithm. (or) compute the DFT for the sequence $\{1,1,1,1,1,1,1,0\}$ using DIT-FFT algorithm.

Soln:-



Here $w_8^0 = 1$, $w_8^1 = \frac{1}{\sqrt{2}} - j/\sqrt{2}$, $w_8^2 = -j$, $w_8^3 = \frac{-1}{\sqrt{2}} - j/\sqrt{2}$

$3 \times w_8^0 = 3 \times 1 = 3$, $-j \times w_8^1 = -j\left(\frac{1}{\sqrt{2}} - j/\sqrt{2}\right) = \boxed{\frac{-j}{\sqrt{2}} - \frac{1}{\sqrt{2}}}$,

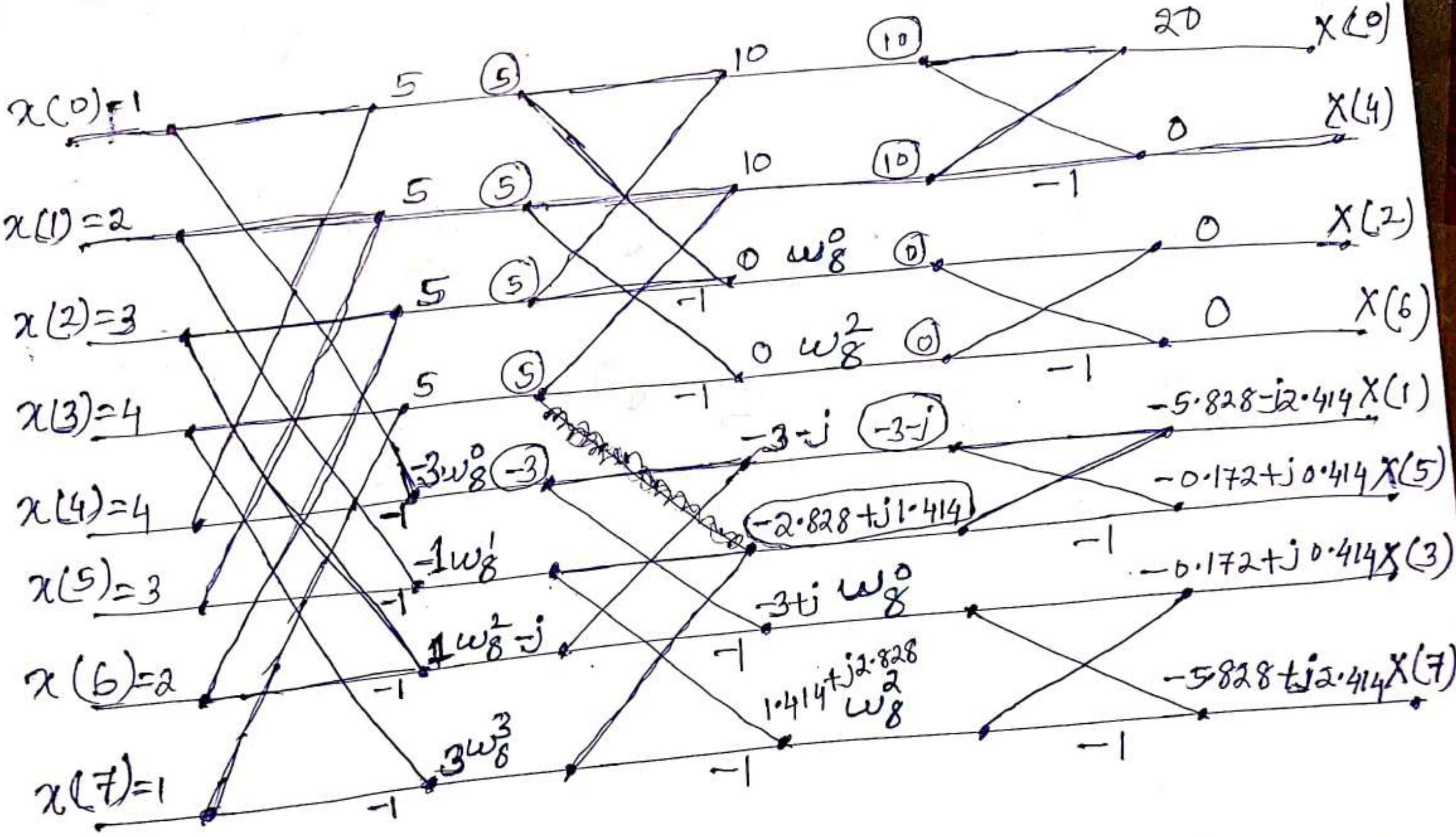
$1 \times w_8^2 = 1 \times -j = -j$, $j \times w_8^3 = j\left(\frac{-1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right) = \boxed{\frac{-j}{\sqrt{2}} + \frac{1}{\sqrt{2}}}$

So, Final DFT is $X(K) = \{7, -0.707 - j0.707, -j, 0.707 - j0.707, 1, 0.707 + j0.707, j, -0.707 + j0.707\}$

— x — x —

Que:- compute the DFT for the sequence {1, 2, 3, 4, 4, 3, 2, 1} using radix-2 DIF-FFT algorithm.

Sol:- Here $x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$



Here $w_8^0 = 1$, $w_8^1 = \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$, $w_8^2 = -j$, $w_8^3 = \frac{-1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$

$-3 \times w_8^0 = -3 \times 1 = -3$, $-1 \times w_8^1 = -1\left(\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right) = \boxed{-\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}}$

$1 \times w_8^2 = 1 \times (-j) = -j$, $3 \times w_8^3 = 3 \times \left(\frac{-1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right)$

$= \boxed{\frac{-3}{\sqrt{2}} - j\frac{3}{\sqrt{2}}}$

$(-3+j) \times w_8^0 = (-3+j) \times 1 = \boxed{-3+j}$

$(1.414 + j2.828) \times w_8^2 = (1.414 + j2.828) \times (-j)$

$= \boxed{2.828 - j1.414}$

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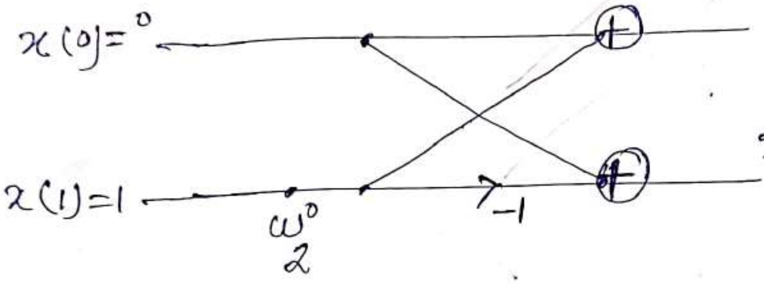
$\therefore X(K) = \{20, -5.828 - j2.414, 0, -0.172 + j0.414,$
$0, -0.172 + j0.414, 0, -5.828 + j2.414\}$

—— x ——

Que:- Find 2-point DIT-FFT Algorithm Butterfly Diagram. If $x(n) = \{0, 1\}$, Find $X(K)$

Sol^n:-



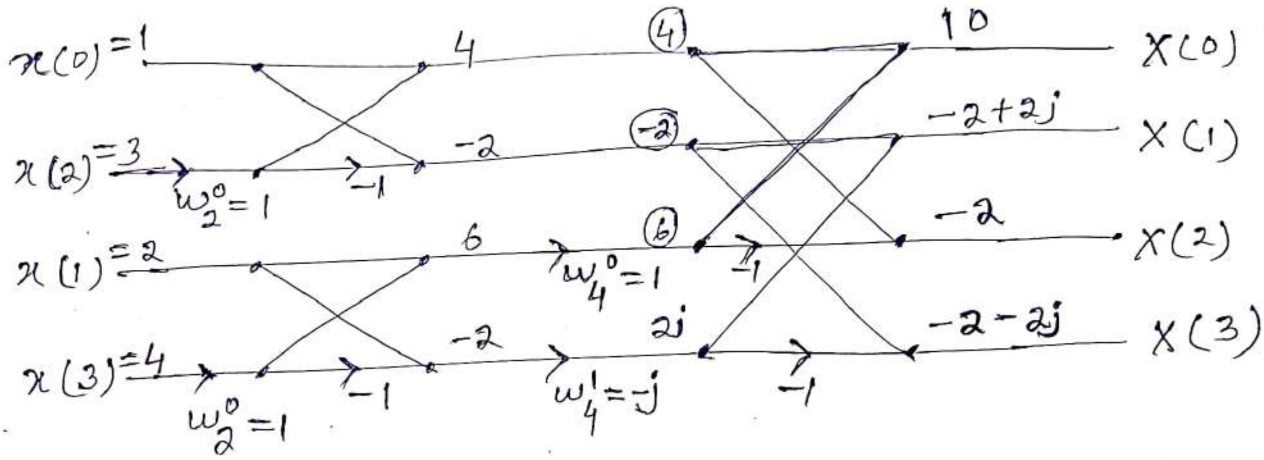
$x(0) + x(1) \cdot W_2^0$
$= 1$

$x(0) + x(1) \times W_2^0 \times (-1)$
$= x(0) - x(1) \cdot W_2^0$
$= -1$

$W_2^0 = 1$

Que:- Find 4-point DIT FFT Butterfly Diagram where $x(n) = \{1, 2, 3, 4\}$

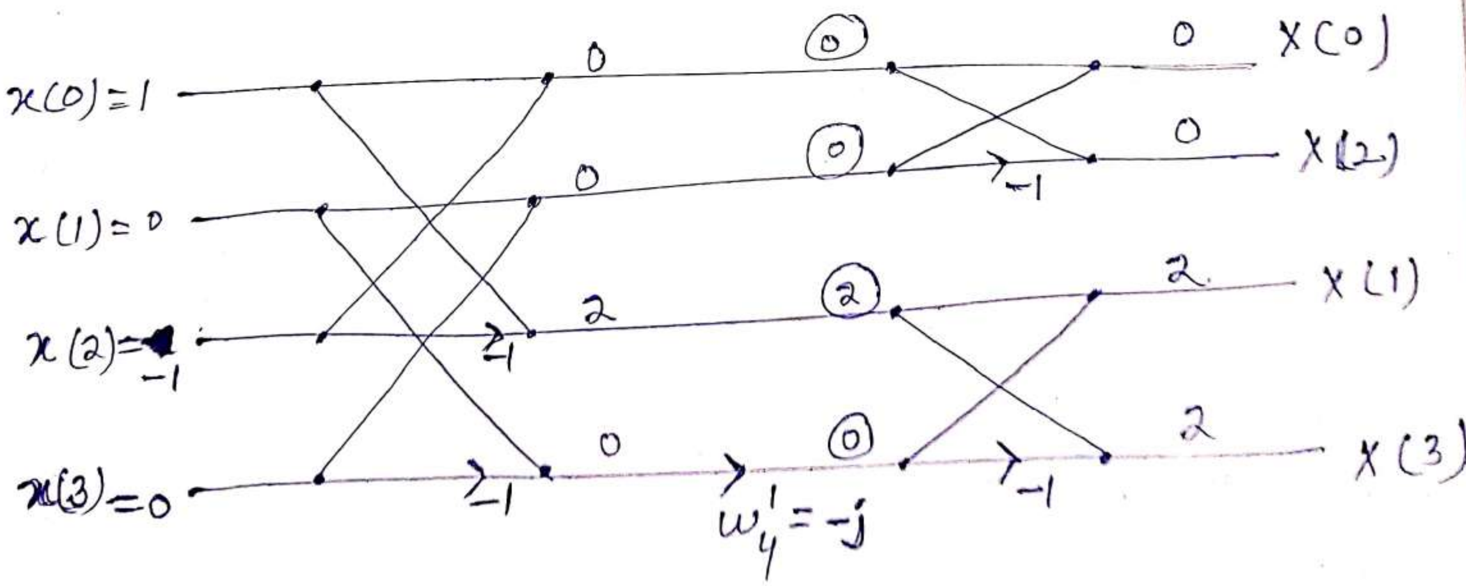
Soln:-



— x — x —

Que:- Find 4-point DIF-FFT Butterfly Diagram where $x(n) = \{1, 0, -1, 0\}$

Soln:-



$\therefore X(K) = \{\underset{\uparrow}{0}, 2, 0, 2\}$

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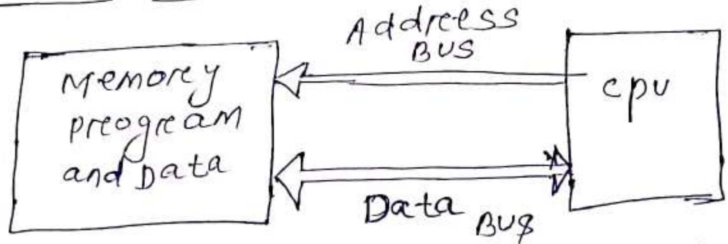
DIGITAL SIGNAL PROCESSING ARCHITECTURES :-

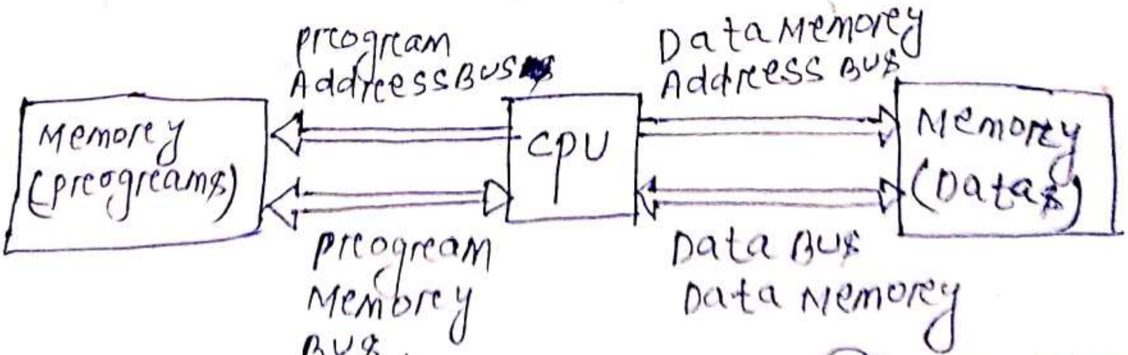
(1) Von-Neumann Architecture :-



↳ There is a common memory to store programs as well as data.
↳ The central processing unit can read an instruction or read/write data from/to the memory.
↳ Both can not occur at the same time as the instruction and data use the same bus system. It has Data bus (Bidirectional), program/Address bus (unidirectional), control bus.

↳ The main characteristics of Von-Neuman Architecture is that it only posses one bus system. The same bus carries all information exchanged between cpu and peripherals including instruction codes as well as data processed cpu.

(2) Harvard Architecture :-



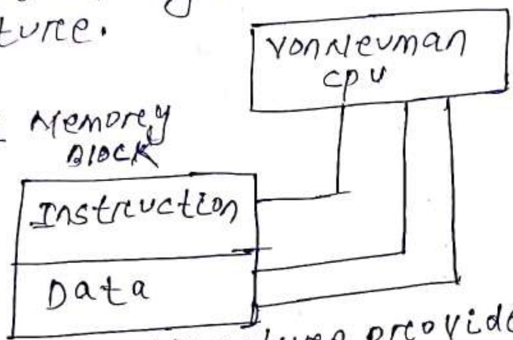
(i) Improve speed of processing. (ii) simultaneously operations can be performed.

(iii) Single memory pathway is there. (iv) physically separate pathway is also there.

—— x ——

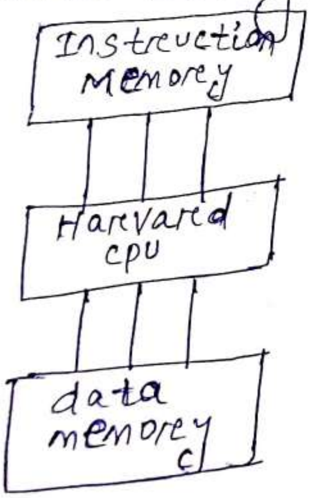
↳ In Harvard architecture, Data and code lie in Different memory Block. In Von Neuman Architecture, Data and code lie in same memory block.

↳ 1 data bus used for both instruction and data. CPU can perform only one operation at a time in Von Neuman Architecture.



All the buses i.e. Address bus, Data bus, control bus are accessing one memory block in Von Neuman Architecture.

↳ Harvard Architecture provides separate buses for both instruction and data. This architecture has data storage entirely contained within the cpu.



↳ In Von-Neuman Architecture, 2 set of clock cycles required. 1 cycle for Data fetch, and 1 cycle for instruction fetch. whereas in Harvard Architecture single set of clock cycle is sufficient.

↳ In Von-Neuman Architecture, pipelining is not possible. In Harvard Architecture pipelining is possible.

↳ Von-Neuman Architecture is simple in design. whereas Harvard Architecture is complex in design.

—— x ——