DISCRETE TIME FOURIER TRANSFORM (DTFT)

↳ If $x(n)$ is Discreete Time signal Then DTFT is given by
$$X(w) = X(e^{jw}) = \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-jwn}$$

↳ In Time Domain The signal is Discreete. But in frequency Domain the signal is continuous and periodic over the range 2π.

↳ In DTFT The Time Domain signal is Discreete and Non periodic and the frequency Domain signal is continuous and periodic.

↳ similarly from $X(w)$ we can obtain Time domain signal a $x(n)$ as:
$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(w) \cdot e^{jwn} \cdot dw$$

↳ DTFT obeys periodicity property, such as
$$X(w + 2\pi K) = X(w)$$
where K is integer.

Que:- If $x(n) = a^n \cdot u(n)$, Then find DTFT of given Signal.

Soln:- $x(n) = a^n$; for $n \geq 0$ [Because $u(n) = 1$; for $n \geq 0$]
$= 0$; $n < 0$

$$X(w) = \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-jwn} = \sum_{n=0}^{\infty} a^n \cdot e^{-jwn}$$

$$= \sum_{n=0}^{\infty} \underbrace{(a \cdot e^{-jw})}_{x}{}^n = 1 + a \cdot e^{-jw} + (a \cdot e^{-jw})^2 + (a \cdot e^{-jw})^3 + \cdots$$

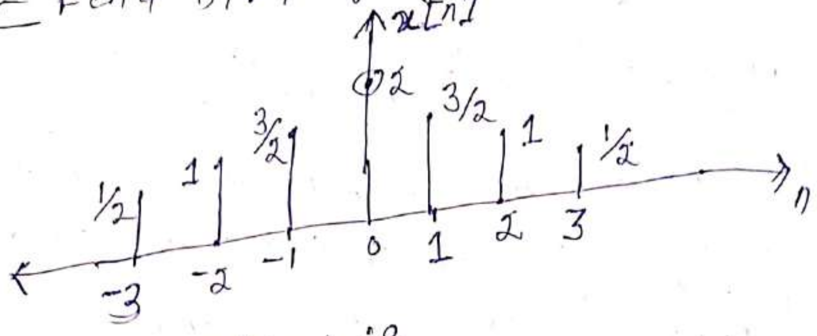
$$= \boxed{\frac{1}{1 - a \cdot e^{-jw}}}$$

$\left[\text{since } \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x} \right]$

similarly $(-a)^n u(n) \xleftrightarrow{DTFT} \dfrac{1}{1+a\cdot e^{-j\omega}}$

—x—

Que:- Find DTFT of given signal $x[n]$



Sol^n:- Given signal is

$$x[n] = \left\{ \underset{\substack{\uparrow \\ n=-3}}{\frac{1}{2}}, \underset{\substack{\uparrow \\ n=-2}}{1}, \underset{\substack{\uparrow \\ n=-1}}{\frac{3}{2}}, \underset{\substack{\uparrow \\ n=0}}{2}, \underset{\substack{\uparrow \\ n=1}}{\frac{3}{2}}, \underset{\substack{\uparrow \\ n=2}}{1}, \underset{\substack{\uparrow \\ n=3}}{\frac{1}{2}} \right\}$$

$$X(\omega) = \sum_{n=-3}^{3} x[n]\cdot e^{-j\omega n}$$

$$= x(-3)\cdot e^{-j\omega(-3)} + x(-2)\cdot e^{-j\omega(-2)} + x(-1)\cdot e^{-j\omega(-1)}$$
$$+ x(0)\cdot e^{-j\omega(0)} + x(1)\cdot e^{-j\omega(1)} + x(2)\cdot e^{-j\omega(2)}$$
$$+ x(3)\cdot e^{-j\omega(3)}$$

$$= \frac{1}{2}\cdot e^{j\omega 3} + 1\cdot e^{j\omega 2} + \frac{3}{2}\cdot e^{j\omega} + 2\cdot e^{0} + \frac{3}{2}\cdot e^{-j\omega}$$
$$+ 1\cdot e^{-j2\omega} + \frac{1}{2}\cdot e^{-j\omega 3}$$

$$= \frac{1}{2}\left[e^{-j\omega 3} + e^{j\omega 3}\right] + 1\cdot\left[e^{j\omega 2} + e^{-j2\omega}\right]$$
$$+ \frac{3}{2}\left[e^{-j\omega} + e^{j\omega}\right] + 2$$

$$= \frac{1}{2}\times 2\left[\frac{e^{j3\omega} + e^{-j3\omega}}{2}\right] + 2\left[\frac{e^{j2\omega} + e^{-j2\omega}}{2}\right]$$
$$+ \frac{3}{2}\cdot 2\left[\frac{e^{j\omega} + e^{-j\omega}}{2}\right] + 2$$

$$= \cos 3\omega + 2\cdot\cos 2\omega + 3\cdot\cos\omega + 2$$

Here signal is Even symmetry.

$\left[\cos\theta = e^{j\theta} + e^{-j\theta}\right]$

Que:- The Signal $x[n] = \left(\frac{1}{2}\right)^n \cdot u(n)$, $y(n) = x^2(n)$
Find DTFT for the signal $y(n)$ i.e. $Y(e^{jw})$ and $Y(e^{j\cdot 0})$

Sol^n:- Given that $x(n) = \left(\frac{1}{2}\right)^n \cdot u(n)$,

$$y(n) = x^2(n) = \left[\left(\frac{1}{2}\right)^n \cdot u(n)\right]^2 = \left(\frac{1}{2}\right)^{2\cdot n} \cdot u(n)$$

$$= \left(\frac{1}{4}\right)^n \cdot u(n)$$

Now DTFT for the signal $y(n)$ is

$$Y(w) = Y(e^{jw}) = \sum_{n=-\infty}^{\infty} y[n] \cdot e^{-jwn}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n \cdot u(n) \cdot e^{-jwn} = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n \cdot 1 \cdot e^{-jwn}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4} \cdot e^{-jw}\right)^n$$

$$= \left(\frac{1}{4} \cdot e^{-jw}\right)^0 + \left(\frac{1}{4} \cdot e^{-jw}\right)^1 + \left(\frac{1}{4} \cdot e^{-jw}\right)^2 + \left(\frac{1}{4} \cdot e^{-jw}\right)^3 + \ldots$$

$$= \frac{1}{1 - \frac{1}{4} e^{-jw}}$$

$\left[\because \sum_{n=0}^{\infty} a^n = a^0 + a^1 + a^2 + \ldots = \frac{1}{1-a}\right.$ Take $a = \frac{1}{4} \cdot e^{-jw}$

$$Y(0) = Y(e^{j\cdot 0}) = \frac{1}{1 - \frac{1}{4} \cdot e^{-j\times 0}} = \frac{1}{1 - \frac{1}{4} \times 1} \quad [e^{-0} = 1]$$

$$= \frac{1}{\frac{4-1}{4}} = \frac{4}{3}$$

$Y(0)$ = Spectrum at origin

$$Y(w) = \sum_{n=-\infty}^{\infty} y[n] \cdot e^{-jwn} \Rightarrow Y(0) = \sum_{n=-\infty}^{\infty} y[n] \cdot e^{-j\cdot 0 \cdot n}$$

$$\Rightarrow Y(0) = \sum_{n=-\infty}^{\infty} y[n]$$

$\Rightarrow$ Frequency Domain signal at origin = sum of Time Domain signals = Area under Time Domain signal.

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DFT (Discrete Fourier Transform) :-

↳ Here Time Domain signal is periodic, Discrete and Frequency Domain signal is also Discrete and periodic.