III-SEM./MECH/AUTO/AERO/DIP IN MECH /MECH(PROD/MECH(MAINT) /MECH(IND INTG) / MECH(SWITCH)/ 2021(W)

TH-II Strength of Materials

Time- 3 Hrs

Full Marks: 80

Answer any **FIVE** Questions including **Q No.1&2**
Figures in the right-hand margin indicates marks

2 x 10

1. Answer **ALL** questions

a. Define Stress.

b. State the Hook's Law.

c. Write down the expression for Strain Energy.

d. Define Hoop stress.

e. What do you understand by Principal Stresses?

f. Write the significance of Mohr's Circle.

g. State different types of beams.

h. What is pure bending?

i. Explain Crippling Load.

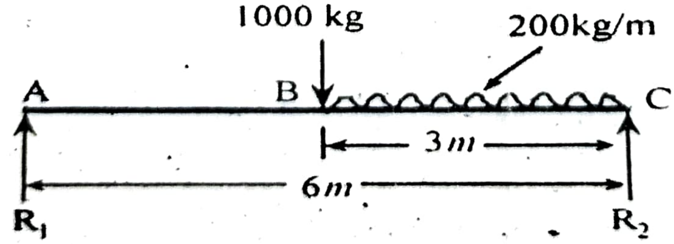
j. Define shaft.

5X6

2. Answer **Any SIX**Questions

a. State the assumptions made in theory of bending.

b. Find the reactions of simply supported beam when a point load of 1000 kg and a uniform distributed load of 200 kg/m is acting on it as shown in figure below:



c. Explain Temperature stress and derive its expression.

d. Derive the torsion equation for a solid circular shaft.

e. A circular bar is subject an axial pull of 120 kN. If the maximum intensity of shear stress on any oblique plane is not to exceed 55 MN/m^2, find the diameter of the bar.

f. Find the generalized equation for Shear Force & Bending Moment of a simply supported beam with Uniformly Distributed Load.

g A steel rod 22mm in diameter and 1.5meters long is subjected to an axial pull of 35 kN. Find i) The intensity of stress, ii) The strain & Elongation. Take $E=2x10^5$ N/mm^2

3 Derive the expression for Hoop Stress & Hoop Strain for thin spherical shells. 10

4 Find out the expression for Section Modulus for a i) Rectangular Section, ii) Hollow Rectangular Section, iii) Circular Section & iv) Hollow Circular Section. 10

5 Derive the relationship between the three modulus (Young's, Bulk & Shear). 10

6 The principal stresses at a point in a bar are 150 N/mm^2 (tensile) and 80 N/mm^2 (compressive). Determine the resultant stress in magnitude and direction on a plane inclined at 60° to the axis of the major principal stress. Also, find the maximum intensity of shear stress in the material at that point. 10

7 Derive the formulae for Crippling Load under various end conditions. 10

S.O.M. ≈ Strength of Materials

1)

a) Define Stress?

A) Stress is defined as the internal resistance or force exerted by a body to oppose the applied external force or load.
Mathematically, Stress = $\frac{\text{Force}}{\text{Area}}$ $\boxed{\sigma = \frac{F}{A}}$

b) State the Hook's law?

A) It states that when a material is loaded within its elastic limit, the stress is proportional to the strain. It may be noted that Hook's law equally holds good for tension as well as compression.

c) write down the expression for strain energy?

$\frac{P}{A}$ A) The expression for strain energy is given as :-

$U = F\delta/2$ Ext. Work = Avg. Load × displacement $W = \frac{P}{2} \times \delta l$

$- = \epsilon$ where, for a perfectly elastic body. Ext work done = Int. work done.

δ = compression, Internal work done is the strain energy stored.

F = Force applied $U = \frac{P}{2} \times \delta l = \frac{\sigma \cdot A}{2} \times \epsilon l$ $[\because V = A \times L]$

$\Rightarrow U = \frac{1}{2} \times \sigma \times \epsilon \times V = \frac{\sigma^2}{2E} \times V$ $\left[\because \epsilon = \frac{\sigma}{E}\right]$ $\Rightarrow \boxed{U = \frac{\sigma^2}{2E} \times V = \frac{P^2 L}{2AE}}$

d) Define Hoop Stress?

A) Hoop or circumferential stress is the tensile stress induced in the material of hollow cylinder or hollow sphere containing same fluid under pressure, the direction of this stress being tangential to the perimeter of the cylinder or sphere.

e) what do you understand by principal stresses?

A) The magnitude of direct or normal stresses (tensile or compressive) as the case may be acting normal to the principal planes are known as principal stresses.

f) write the significance of Mohr's circle?

A) The benefit that mohr's circle provides is that it creates a visual representation of what is happening, and the relative positioning of the stress states on an element relative to a set of coordinate axes

g) State different types of beams?

A) The different types of beams are:-

i) Simply supported beam
ii) cantilever beam
iii) overhanging beam
iv) continuous beam
v) Fixed beam

h) what is pure bending?

A) when a beam is subjected to only bending in such a manner that the other actions are absent, then that type of bending of the beam is called pure bending or simple bending.

i) Explain crippling load?

A) It is the minimum amount of load at which a column or strut will start buckling i.e., it will develop an elastic instability. This load is known as critical or crippling load.

J) Define Shaft?

A) A Shaft is a rotating machine element, usually circular in cross section, which is used to transmit power from one part to another, or from a machine which produces power to a machine which absorbs power.

2

a) State the assumptions made in theory of bending?

A) The assumptions made in theory of bending are:-

i) The beam is subjected to pure or simple bending only and no shear force acts on it

ii) The beam material is stressed within elastic limit and hence obey's hooke's law.

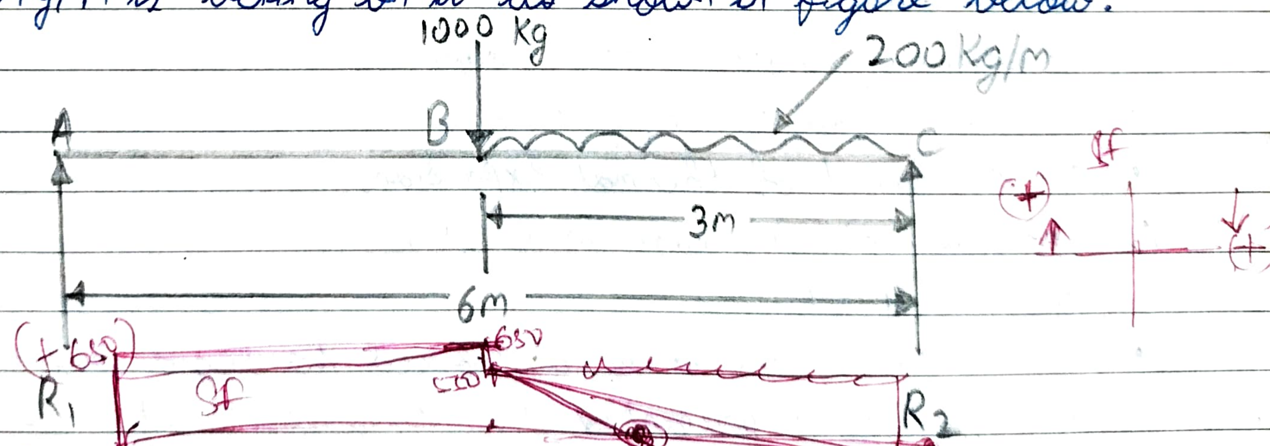
iii) The value of Young's Modulus (E) of the beam material is same for tension as well as ~~compression~~. compression.

iv) The resultant pull or push on a transverse section of the beam is zero.

v) The material of the beam is homogeneous and isotropic.

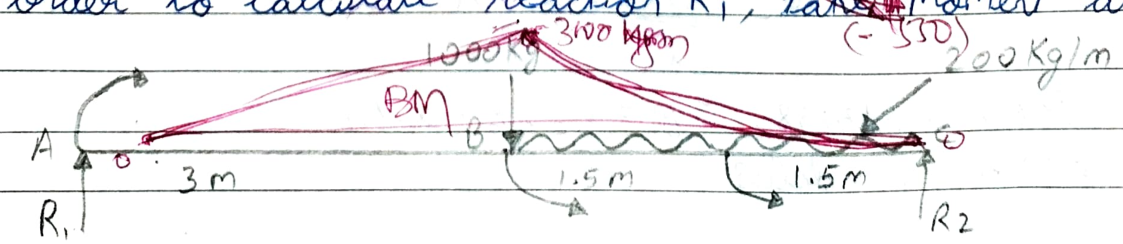
vi) The transverse section of the beam which is plane before bending remains plane after bending.

b) Find the reactions of a simply supported beam when a point load of 1000 Kg and a uniform distributed load of 200 Kg/M is acting on it as shown in figure below:



A) Solution

In order to calculate reaction R_1, take moment at point C



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$\sum M_c = 0$

clockwise moments = Anti clockwise moments

$R_1 \times 6 = 1000 \times 3 + (200 \times 3)\, 3/2$ ~~= 3600~~

$6R_1 = 3000 + 900 = 3900$

$R_1 = \frac{3900}{6} = 650$ Kg ✓

for calculating R_2 i.e. reaction at point c,

$\sum F_v = 0$

$R_1 + R_2 = \cancel{650}\ 1000 + 200 \times 3 = \cancel{1250}\ 1600$

~~$1300 + R_2 = 1400$~~ ~~$1250 - 650 = 600$ kg~~ ~~$= R_2$~~

$R\,2 =$ ~~$1400 - 1300$~~ ~~550 kg~~

~~$R_2 = 100$ kg~~ $R_2 = 1600 - 650 = 950$ kg

$R_B \times 6 = (3 \times 1000) + (200 \times 3) \times 4.5$

$= 3000 + 2700 = 5700$

$R_B = \frac{5700}{6} = 950$ kg

$R_A + R_B = 1000 + (200 \times 3)$

$\Rightarrow R_A = 1600 - 950 =$

$R_A = 650$ kg

c) Explain temperature stress and derive its expression?

A) Temperature/ thermal stress is mechanical stress created by any change in temperature of a material. These stresses can lead to fracturing or plastic deformation depending on the other variables of heating, which include material types and constraints.

- L = Original length of the body,
- t = Rise or fall of temperature,
- a = coefficient of thermal expansion,

∴ Increase or decrease in length,

$\delta l = l \times a \times t$

- If the ends of the body are fixed to rigid supports, so that its expansion is prevented, then compressive strain induced in the body,

$$\varepsilon_c = \frac{\delta l}{l} = \frac{l.\alpha.t}{l} = \alpha.t$$

∴ Thermal stress, $\sigma_{th} = \varepsilon_c \cdot E = \alpha.t.E$

E = Young's Modulus

d) Derive the torsion equation for a solid circular shaft?

A) Consider a solid circular shaft having radius R which is exposed to a torque T at one end and the other end is also under the same torque.

Angle in radius = arc / radius

Arc Ab = Rθ/L Y

$\gamma = R\theta/L$

where,

A and B : these are considered as the two fixed points present in the circular shaft

γ : the angle subtended by AB

$G = \frac{\tau}{\gamma}$ (modulus of rigidity)

where,

τ = shear stress

γ = shear strain

$\frac{\tau}{G} = \gamma$

$\therefore R/L = \tau/G$

consider a small strip of the radius with thickness dr that is subjected to shear stress

$\tau' * 2\pi r\, dr$

where,

r = radius of the small strip

dr = the thickness of the strip

$2\pi\tau' r^2 dr$ (torque at the center of the shaft)

$$T = \int_0^R 2\pi \tau' r^2\, dr$$

$$T = \int_0^R 2\pi G\theta/L\, r^3 dr$$ (Substituting for τ')

$T = (2\pi G\theta/L)\int_0^R r^3 v dr = G\theta/L[(\pi d^4)/32]$

(after integrating and substituting for R)

$(G\theta/L)J$ (Substituting for the polar moment of inertia)

$\therefore \frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$

e) A circular bar is subject an axial pull of 120 KN. If the maximum intensity of shear stress on any oblique plane is not to exceed 55 MN/m², find the diameter of the bar?

A) Given data

$P = 120 KN = \cancel{12\times10^3}\ 120\times10^3 N$

$\tau_{max} = 55 MN/m^2$

$\delta = ?$

we know that, $\sigma = \frac{F}{A}$

Area of circle $= \frac{\pi}{4} \times d^2$

$$\sigma = \frac{120\times10^3}{\frac{\pi}{4}\times d^2}$$

$$\sigma = \frac{120\times10^3\times4}{\pi\times d^2}$$

we also know that,

Maximum shear stress $= \frac{\sigma}{2}$

$$55 = \frac{480000/\pi\times d^2}{2}$$

$$d^2 = \frac{480000/\pi}{2\times55}$$

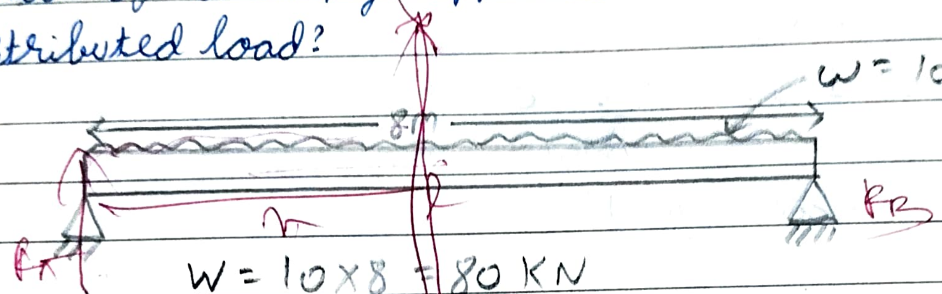
$d^2 = 1388.98$

$d = \sqrt{1388.98}$

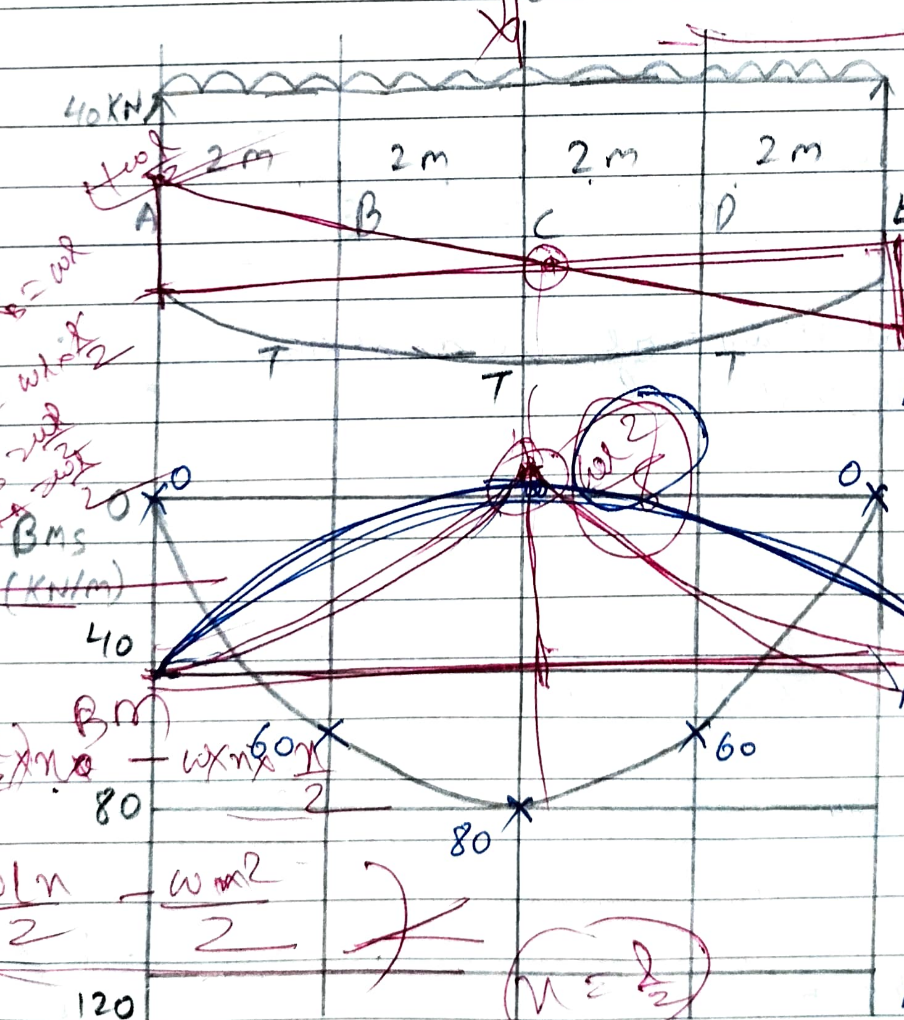
$d = 37.26$ mm

f) Find the generalized equation for shear force & bending moment of a simply supported beam with uniformly distributed load?

A)



$W = 10 \times 8 = 80$ KN



$M_A = M_B = 0$

A — 2m — B, 40, ←1m→

$10 \times 2 = 20$ KN

$M_B = +(40\times2) - (20\times1)$

$= 80 - 20 = 60$ KN

A — 4m — C, 40, ←2m→

$10 \times 4 = 40$ KN

$M_C = +(40\times4) - (40\times2)$

$= 160 - 80 = 80$ KN

A — 6m — D, 40, ←3m→

$10 \times 6 = 60$ KN

$M_D = +(40\times6) - (60\times3)$

$= 240 - 180 = 60$

$R_A + R_B = wl$

$R_A = \frac{wl}{2}$

$M_x = \left(\frac{wl}{2}\right)x - \frac{wx^2}{2}$

$\Rightarrow \left(\frac{wlx}{2} - \frac{wx^2}{2}\right)$

$\frac{dm}{dx} \to 0$

$\frac{wl}{2} - \frac{w}{2} \times 2x = 0$

$\frac{w}{2} \cdot 2x = \frac{wl}{2}$

$x = \frac{l}{2}$

$M_m = \frac{wl}{2} \times \frac{l}{2} - \frac{w}{2} \cdot \frac{l^2}{4} = \frac{wl^2}{4} - \frac{wl^2}{8} = \frac{wl^2}{8}$

g) A Steel rod 22 mm in diameter and 1.5 meters long is subjected

to an axial pull of 35 KN. Find i) the intensity of stress,
ii) The strain & Elongation.

Take $E = 2 \times 10^5 \, N/mm^2$

A) Given data:-

$D = 22 \, mm$

$L = 1.5 \, m = 1.5 \times 10^3 \, mm$

$P = 35 \, KN = 35 \times 10^3 \, N$

i) intensity of stress $= \frac{Force}{Area} = \frac{F/P}{A}$

Area of circle $= \frac{\pi}{4} d^2$

$= \frac{\pi}{4} \times (22)^2 = 380.13 \, mm$

$= \frac{35 \times 10^3}{380.13} = 92.07$

$\sigma = 92.07 \, MPa$

ii) we know that strain $= \frac{\delta l}{l}$, we don't know $\delta l = ?$

we also know that,

$E = \frac{\sigma}{\varepsilon}$

$2 \times 10^5 = \frac{92.07}{\varepsilon}$

$\varepsilon = \frac{92.07}{2 \times 10^5} = 4.6035 \times 10^{-4}$

$\varepsilon = 4.6035 \times 10^{-4}$

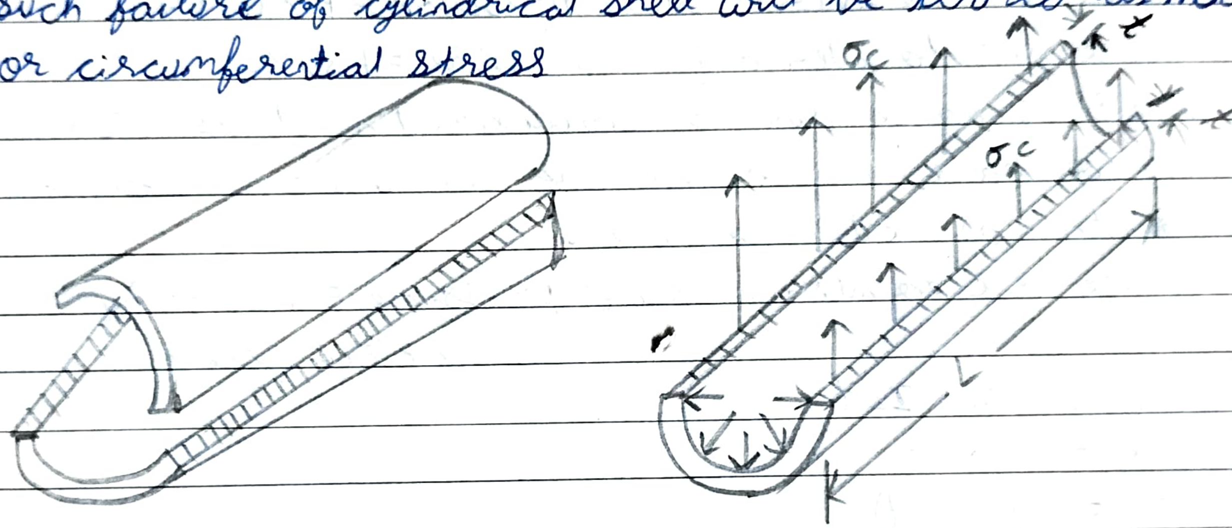
Elongation $(\delta l) = \frac{PL}{AE}$

$\delta l = \frac{35 \times 10^3 \times 1.5 \times 10^3}{380.13 \times 2 \times 10^5}$

$\delta l = 0.69 \, mm$

3) Derive the expression for hoop stress & hoop strain for thin spherical shells?

A) If fluid is stored under pressure inside the cylindrical shell, pressure will be acting vertically upward and downward over the cylindrical wall. pressure vessel will tend to burst as displayed here in following figure and stresses developed in such failure of cylindrical shell will be termed as hoop stress or circumferential stress



Let us consider here following terms to derive the expression for circumferential stress or hoop stress developed in the wall of cylindrical shell

P = Internal fluid pressure

d = Internal diameter of thin cylindrical shell

t = Thickness of the wall of the cylinder

L = length of the cylindrical shell

σ_c = circumferential stress or hoop stress developed in the wall of the cylindrical shell.

cylindrical shell bursting will take place if force due to internal fluid pressure will be more than the resisting force due to circumferential stress or hoop stress developed in the wall of the cylindrical shell.

In order to secure the expression for circumferential stress or hoop stress developed in the wall of the cylindrical

shell, we will have to consider the limiting case i.e. force due to internal fluid pressure should be equal to the resisting force due to circumferential stress or hoop stress.
Force due to internal fluid pressure =
Internal fluid pressure X Area on which fluid pressure will be acting
Force due to internal fluid pressure = $P \times (d \times L)$
Force due to internal fluid pressure = $P \times d \times L$
Resisting force due to circumferential stress = $\sigma_c \times 2Lt$
As we have seen above, we can write following equation as mentioned here.
Force due to internal fluid pressure = Resisting force due to circumferential stress

$$P \times d \times L = \sigma_c \times 2Lt$$

$$P \times d = \sigma_c \times 2t$$

$$\sigma_c = P \times d/(2t)$$

$$\sigma_c = \frac{Pd}{2t}$$

σ_c, σ_a and P stresses are ~~perd~~ perpendicular to each other
Note that P acts as a compressive radial stress on inner surface of cylinder. Stresses σ_c and σ_a are much larger than P; therefore in calculation of strains, the effect of internal pressure P is neglected. Say E is young's modulus and ν is poisson's ratio of the material.
circumferential strain

$$\varepsilon_c = \frac{\sigma_c}{E} - \frac{\nu\sigma_a}{E}$$

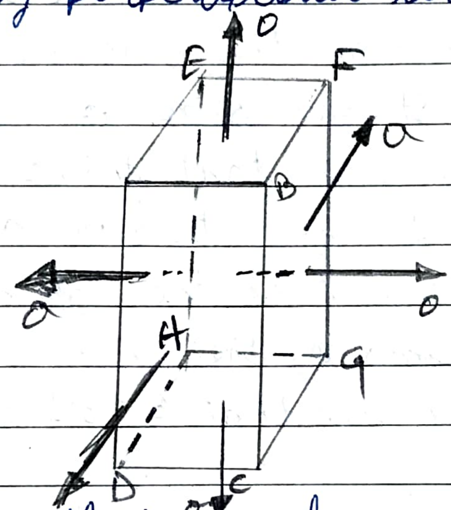
$$= \frac{PD}{2tE} - \frac{\nu PD}{4tE}$$

$$= \frac{PD}{4tE}(1-2\nu)$$

Note that ε_c = circumferential strain

5) Derive the relationship between the three modulus (Young's, Bulk & shear)

A) Let us consider a cube ABCDEFGH as displayed in the following figure, let us assume that cube is subjected with three mutually perpendicular tensile stress σ of similar intensity.



Let us assume that we have following details as mentioned above/here

Length of the cube $= L$

change in length of the cube $= dL$

Young's modulus of elasticity $= E$

Bulk modulus of elasticity $= K$

Tensile stress acting over cube face $= \sigma$

Poisson ratio $= \nu$

longitudinal strain per unit stress $= \alpha$

lateral strain per unit stress $= \beta$

As we have already disscussed the poisson ratio as the lateral strain to longitudinal strain and therefore we can say that poisson ratio, $(\nu) = \beta/\alpha$

Let us recall the young's modulus of elasticity, E = longitudinal stress/longitudinal strain

~~E = 1/[Longitudinal stress/longitudinal~~

E = 1/[Longitudinal strain/ Longitudinal stress]

$\varepsilon = 1/a$

Initial volume of the cube

V = Length X width x height $= L^3$

Now we will secure here the final dimensions of the cube in order to secure the final volume of the cube and finally we will determine the bulk modulus of elasticity. Let us consider first one side of cube i.e. AB. As we have already discussed that three mutually perpendicular tensile stresses of similar intensity are acting over the cube. Let us determine here the effect of tensile stress over dimensions of the cube

As we have already seen that, $\varepsilon = dL/L$

Strain $= dL/L$

$dL = L \times$ Stress $\times a = L \times \sigma \times a$

$dL = L.\sigma.a$

Now we will have to think slightly here to discuss the effect on length of the cube under three mutually perpendicular tensile stresses of similar intensity. When direct tensile stress will be subjected over the face AEHD and BFGC, there will be increase in length due to longitudinal strain developed due to direct tensile stress acting over the face AEHD and BFGC. Simultaneously, we must have to note it here that tensile stress acting over the face AEFB and DHGC will develop the lateral strain in side AB. Similarly, tensile stress acting over the face ABCD and EFGH will also develop the strain in side AB

Final length of the cube, $= L + L.\sigma.a - L.\sigma.\beta - L.\sigma.\beta$

Final side length of the cube, $= L\,[1 + \sigma.(a - 2\beta)$

Final volume of the cube

$V_f = L^3 \times [1+\sigma.(a-2\beta)]^3$

Now we will ignore the product of small quantities in order to easy understanding.

$V_f = L^3 \times [1+\sigma.(a-2\beta)]^3$

$V_f = L^3 + 3\sigma.L^3 (a-2\beta)$

Change in volume of the cube, ~~when~~ when three mutually perpendicular tensile stresses of similar intensity are acting over the cube

$\Delta V = L^3 + 3\sigma.L^3(a-2\beta) - L^3$

$\Delta V = 3\sigma.L^3(a-2\beta)$

Let us see here volumetric strain

Volumetric strain in the specified cube here will be determined as displayed here

Volumetric strain $= \Delta V/V$

$\varepsilon_V = 3\sigma(a-2\beta)$

Now, we will find here Bulk modulus of elasticity (K)

Bulk modulus of elasticity will be defined as the ratio of volumetric stress or hydro static stress to volumetric strain and therefore we will write here as mentioned here

$K = \sigma/[3\sigma(a-2\beta)]$

$K = 1/[3(a-2\beta)]$

$3K(a-2\beta) = 1$

$3K(1-2\beta/a) = 1/a$

As we have already seen above that

Young's modulus of elasticity, $E = 1/a$

Poisson ratio $V = (\beta/a)$

After replacing the value of $1/a$ and (β/a) in above concluded equation, we will have the desired result which will show the relationship between young's modulus of elasticity (E) and bulk modulus of elasticity (K)

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$E = 2G(1+\mu) \Rightarrow G = \frac{mE}{2(m+1)}$

$E = 3K(1-2\mu) \Rightarrow K = \frac{mE}{3(m-2)}$

$3K(1-2v) = E$

$E = 3K(1-2v)$

$E = \frac{9KG}{G+3K}$

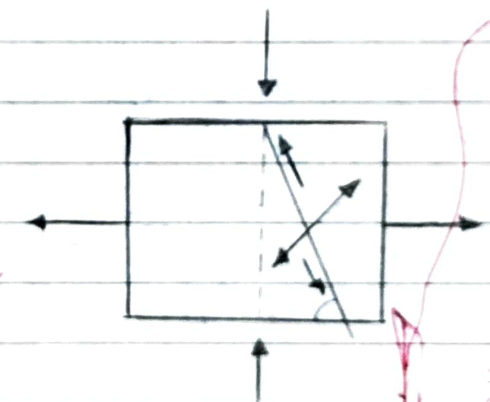
6) The principal stresses at a point ~~bar~~ in a bar are 150 N/mm² (tensile) and 80 N/mm² (compressive). Determine the resultant stress in magnitude and direction on a plane inclined at 60° to the axis of the major principal stress. Also, find the maximum intensity of shear stress in the material at that point

A) Given data

Tensile stress, $\sigma_x = 150\,N/mm^2$

Compressive stress, $\sigma_y = -80\,N/mm^2$

Angle made by the plane in the direction of major principal (tensile) stress, $\theta = 60°$



$\frac{230}{2}$

$115 \times \frac{8}{2}$

$\frac{150-80}{2} + \frac{150+80}{2}\cos 120°$

$\left(-\frac{1}{2}\right)$

$35 - \frac{115}{2}$

$\Rightarrow \frac{70-115}{2} = -\frac{45}{2}$

$\Rightarrow -22.5\ N/mm^2$ (compressive)

Normal stress,

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta$$

$$= \frac{150+80}{2} + \frac{150-80}{2}\cos(2\times 60°)$$

$$= 115 + 35\cos 120° = 97.5\ N/mm^2$$

$= 97.5\ N/mm^2$ (ten)

$\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta$

$\Rightarrow \frac{150+80}{2}\sin 120°$

Shear stress, $\tau = \frac{\sigma_x - \sigma_y}{2}\sin(2\times 60°)$

$$= \frac{150-80}{2}\sin 120° = 30.3\ N/mm^2$$

$\Rightarrow 115 \sin 120°$

$\Rightarrow 99.6$

$\Rightarrow 30.31\ N/mm^2$

$2\theta = \tan^{-1}(0.263)$ ~~$\tan 2\theta = \left(\frac{2\times 30.3}{150+80}\right)$~~

i) Resultant stress. $= 14.761^\circ$

$\sigma_R = \sqrt{\sigma_n^2 + \tau^2}$

$= \sqrt{(97.5)^2 + (30.3)^2}$

$= 102\ N/mm^2$

$\tan 2\theta = \cancel{\frac{2\tau_{xy}}{\sigma_x - \sigma_y}} = \frac{60.6}{230}$ 0.263

$\sqrt{(22.5)^2 + (99.6)^2}$

Direction of the resultant stress,

$\tan\theta = \cancel{\frac{\sigma_n}{\tau}}\ \frac{\tau}{\sigma_n} = \frac{99.59}{22.5} = 4.426$

$= \sqrt{506.25 + \cancel{9918.7}}$

$\theta = \tan^{-1}(4.426)$ 77.26°

$= \frac{97.5}{30.3} \Rightarrow 3.217 = \cancel{72.8}$ ~~37.74~~

$\Rightarrow \sqrt{\cancel{1042}4.94}$

Resultant

$\sigma_R = $ ~~37.74~~ 102.10

~~$\theta = 73^\circ$~~

ii) Maximum shear stress

$\tau_{max} = \frac{\sigma x - \sigma y}{2}$

$= \frac{150+80}{2} = 115$

~~$= 35\ N/mm^2$~~ 115

$\tau_{max} = \sqrt{\left(\frac{\sigma_n - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$\Rightarrow \left(\frac{\sigma_n - \sigma_y}{2}\right) = \frac{150+80}{2}$

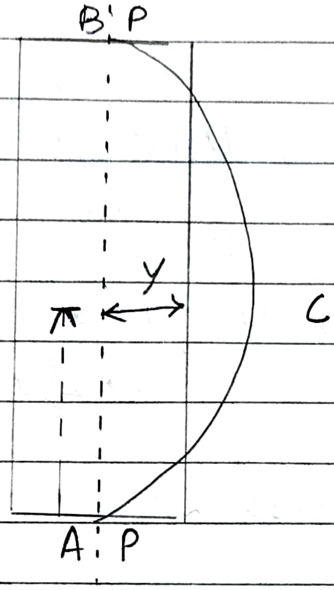
$\Rightarrow \frac{230}{2} = 115$

7) Derive the formula for crippling load under various end conditions?

A) when a column will be subjected to an axial compressive loads, there will be developed bending moment and hence bending stress in the column. column will be bent due to this bending stress developed in the column

Load at which column just bends or buckles will be termed as buckling or crippling load. Let us consider a column AB of length L as displayed in the following figure. Let us consider that both the ends of the column are hinged i.e. end A and end B are hinged.

Let us think that P is the load at which column just bends or buckles or we can also say that crippling load is P and we have displayed in following figure curve ACB indicates the condition of column after application of crippling load or when column buckles



Now, we will consider one Section at a distance x from end A and let us consider that y is the lateral deflection of the column as displayed in above figure.
Now we will determine the bending movement developed across the section and we can write it as mentioned here.
Bending moment, $M = -P \times y$
we have taken negative sign here for bending movement developed due to crippling load across the section
~~and we can~~
As we know the expression for bending moment from deflection equation and we can write as mentioned here
Bending moment, $M = E.I\,[d^2y/dx^2]$
we can also write here the equation after equating both expressions for bending moment mentioned above and we will have following equation

$$E.I \frac{d^2y}{dx^2} = -P \times y$$

$$E.I \frac{d^2y}{dx^2} + P \times y = 0$$

After solving the differential equation, we will following equation

$$Y = C_1 \cos\left[X \cdot \sqrt{\frac{P}{EI}}\right] + C_2 \sin\left[x \cdot \sqrt{\frac{P}{EI}}\right]$$

Above equation will also be termed as lateral deflection equation for column. C_1 and C_2 are the constant of integration, now next step is to determine the value of constant of integration i.e. C_1 and C_2
End conditions for long columns and we will secure the value of constant of integration i.e. C_1 and C_2 by using the respective end conditions.
As we know that for long column with both the ends hinged, we will have following end conditions as mentioned here
At $x = 0$, deflection $y = 0$
At $x = L$, deflection $y = 0$
Let us use the first end condition i.e. at $x = 0$, deflection $y = 0$ in above lateral deflection equation for column, we will have ~~constant~~ value of constant of integration i.e. C_1
At $x = 0$, deflection $y = 0$ and hence after using these value in lateral ~~defle~~ deflection equation for column, we will have constant of integration i.e. $C_1 = 0$
Similarly, we will use second end condition i.e. at $x = L$, deflection $y = 0$ and constant of integration i.e. $C_1 = 0$ in above lateral deflection equation for column and we will have value of constant of integration i.e. C_2.
At $x = L$, deflection $y = 0$ and we have already determined constant of integration i.e. $C_1 = 0$. Therefore constant of integration i.e. C_2 will be determined as displayed here in following figure

$$Y = C_1 \cos\left[x \cdot \sqrt{\frac{P}{EI}}\right] + C_2 \sin\left[x \cdot \sqrt{\frac{P}{EI}}\right]$$

$$0 = 0 + C_2 \sin\left[L\sqrt{\frac{P}{EI}}\right]$$

$$C_2 \sin\left[L\sqrt{\frac{P}{EI}}\right] = 0$$

we can conclude here
Either, $C_2 = 0$
or,
$\sin\left[L\sqrt{\frac{P}{EI}}\right] = 0$

① Both end hinged $= L = L_e$
② Both end fixed $= \frac{L}{2} = L_e$
③ one end fixed end of the free $= 2L$
④ one end fixed & other hinged $= \frac{L}{\sqrt{2}}$

Let us assume that C_2 is zero, we have already concluded that $C_1 = 0$ and in this situation lateral deflection of column i.e. y will also be zero.
we can also say from here that column will not be bend after application of crippling load P and as we know that this statement will never be true and hence our assumption of taking $C_2 = 0$ is wrong.
So only one condition is left as displayed here

$$\sin\left[L\sqrt{\frac{P}{EI}}\right] = 0$$

$$\sin\left[L\sqrt{\frac{P}{EI}}\right] = \sin 0 \text{ or } \sin\pi \text{ or}$$

Let us consider the leat pratical value and we will have $L\sqrt{\frac{P}{EI}} = \pi$ $\Rightarrow \pi^2 = L \times \frac{P}{EI}$ $\Rightarrow P = \frac{\pi^2 EI}{L_e}$

From here we will have expression for crippling load when both the ends of the column are hinged and we have displayed it in following figure

$$P = \frac{\pi^2 EI}{(L_e^2)}$$

effective length.

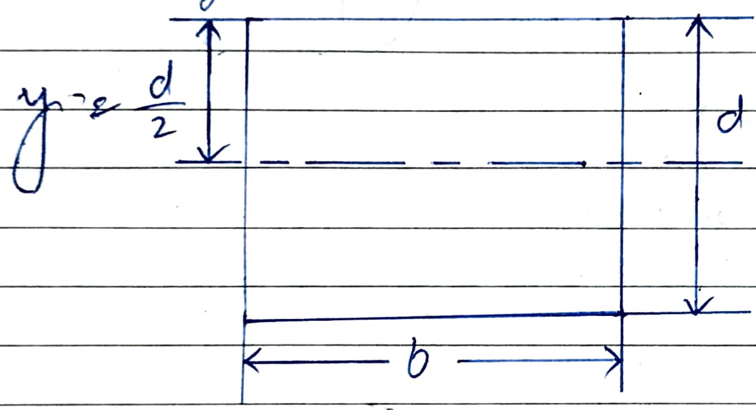
4) Find out the expression for section modulus for a
i) rectangular section ii) hollow rectangular section
iii) circular section & iv) hollow circular section

A) The modulus of section may be defined as the ratio of moment of inertia to the distance to the extreme fibre. It is denoted by Z.

$Z = I/y$; For rectangular section, $I = bd^3/12$ & $y = d/2$

$$I = \frac{bd^2}{6}$$

i) For rectangular section



$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$Z = \left(\frac{I}{Y}\right) \qquad \frac{M}{\sigma} = \frac{I}{Y}, \quad Z = \frac{I}{Y}$$

$$= \left[\frac{\frac{1}{12} bd^3}{\frac{d}{2}}\right]$$

$$= \cancel{\frac{2}{6} bd^2} = \frac{D}{6} bd^2 = \frac{bd^2}{6}$$

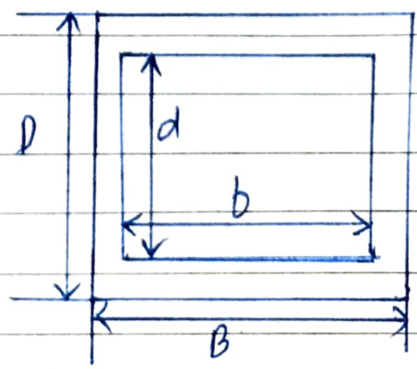
I = moment of inertia
M = Moment of Resistance
σ = bending stress
R = Radius of curvature
y = distance of a point from neutral axis
E = modulus of elasticity

ii) For hollow rectangular section

$$Z = \frac{I}{Y}$$

$$I = \frac{1}{12}(BD^3 - bd^3)$$

$$Y = D/2$$

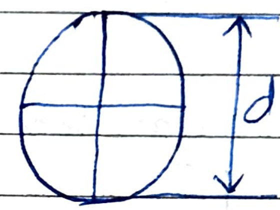


$$Z = \frac{\frac{1}{12}(BD^3 - bd^3)}{\frac{D}{2}}$$

$$= \frac{1}{6}\left(\frac{BD^3 - bd^3}{D}\right)$$

iii) circular section

$$Z = \frac{I}{y}$$



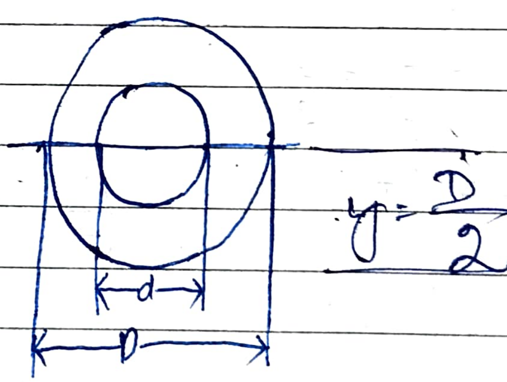
$$I = \frac{\pi d^4}{64} \qquad y = \frac{d}{2}$$

$$Z = \frac{\pi d^4}{64} \times \frac{2}{d} = \frac{\pi d^3}{32}$$

iv) Hollow circular Section

$$Z = \frac{I}{y}$$

$$I = \frac{\pi}{64}(D^4 - d^4)$$



$$y = \frac{D}{2}$$

$$Z = \frac{\pi}{64}(D^4 - d^4) \times \frac{2}{D}$$

$$= \left[\frac{\pi}{32}\,\frac{D^4 - d^4}{D}\right]$$