1 Review of basic concepts

Force

→ vector quantity ($\vec{F}$).

→ $\vec{AB}$ shows direction of force from A to B.

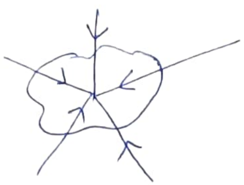
→ unit : Newton

1 kgf = 10 N

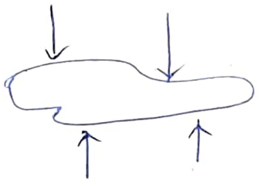
1 N = 1 kg m/s²

→ force system
~~composite set of forces~~

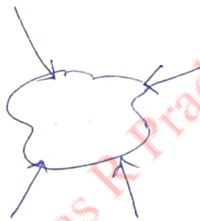
A no. of forces acting over a body forms a force system.



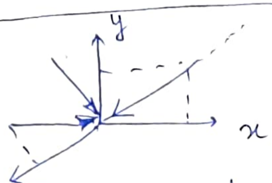
concurrent coplanar



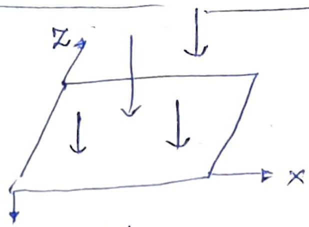
parallel, coplanar



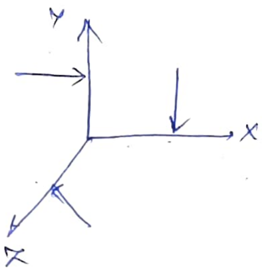
general, coplanar



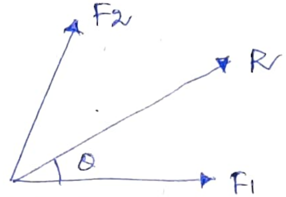
concurrent space



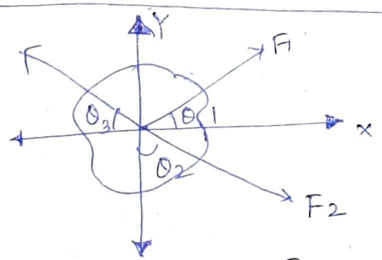
parallel, space



general, ~~cop~~ space



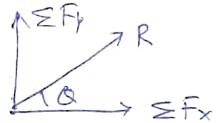
$$\boxed{R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}}$$



Suppose

ΣF_x, ΣF_y can be found out.

$R^2 = (\Sigma F_x)^2 + (\Sigma F_y)^2$

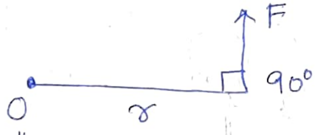


$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$

Moment

→ vector quantity.

→ Moment = force × distance⊥

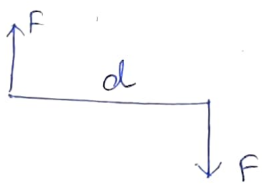


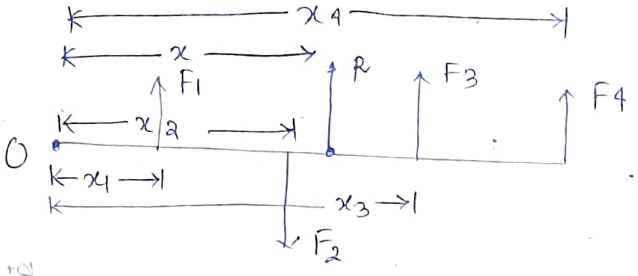
O = moment centre

So moment at O, $\boxed{M_O = F \times r}$

→ unit : SI = KNm

couple
↓
Moment due to two equal forces (F).





$R = \Sigma F = F_1 + F_3 + F_4 - F_2$

~~Moment at O~~ General Concept

$\therefore \boxed{Rx = F_1x_1 + F_2x_2 + F_3x_3 + F_4x_4}$

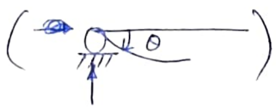
With signs

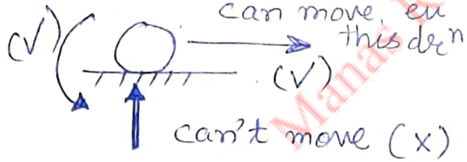
$F_1x_1 + Rx + F_3x_3 + F_4x_4 - F_2x_2 = 0$

$\Rightarrow \cancel{F_2x_2}\ \boxed{Rx = F_2x_2 - F_1x_1 - F_3x_3 - F_4x_4}$

Support conditions

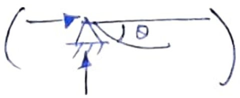
Roller Support :-

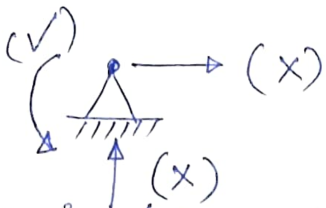




→ 1 restrains (01 reaction)
→ ~~2~~ ~~DL~~ DOF (02)
(degree of freedom)

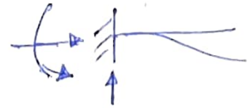
Hinged / pinned support

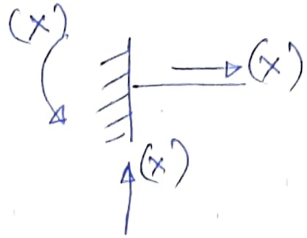




→ 02 restrains (02 reaction)
→ 01 DOF

Fixed Support

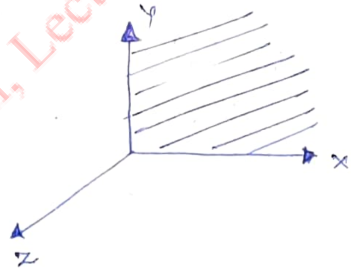




→ 03 ~~S~~ restrains (03 reactions)
→ 00 DOFs.

Equilibrium

→ It means state of rest / motion without acceleration.



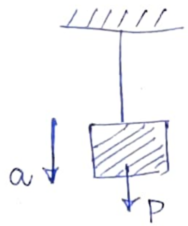
XY plane

$\Sigma F_x = 0$
$\Sigma F_y = 0$
$\Sigma M_z = 0$

} Similarly follows for other planes.

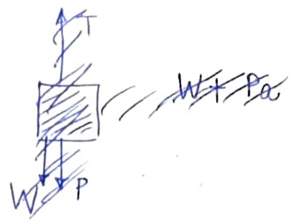
Free Body Diagram (FBD)

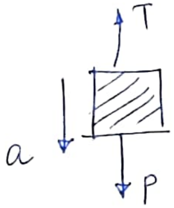
When an isolated body is drawn with all the active & reactive forces acting on it, such diagram is known as FBD.



$P - T = ma$

$\Rightarrow \boxed{a = \frac{P-T}{m}}$



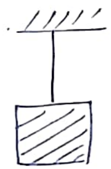


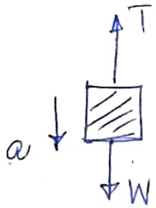
(FBD)

$P - T = ma$

$\Rightarrow \boxed{a = \frac{P-T}{m}}$

Ex: 2





(FBD)

→ force eqm:

$W = T$

$\Rightarrow \boxed{mg = T}$

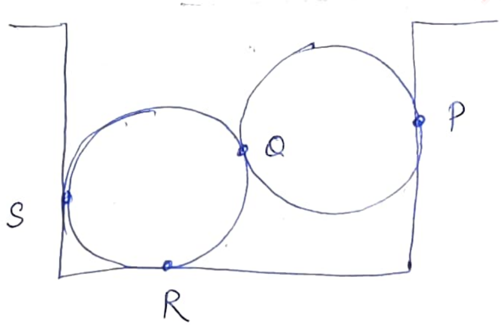
→ If mass goes downward with 'a'

$W - T = ma$

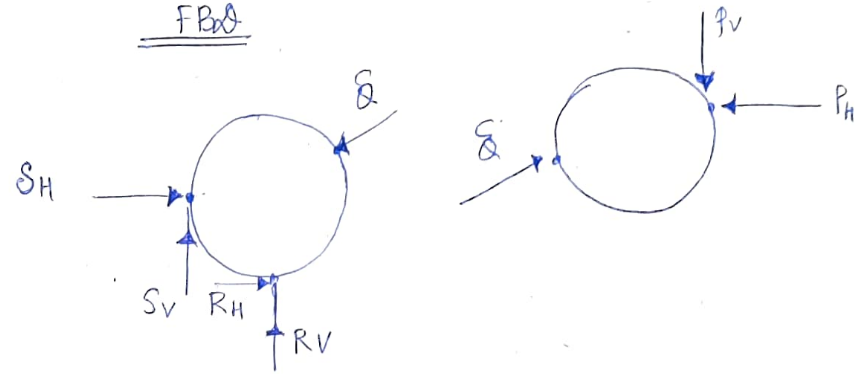
$\Rightarrow mg - T = ma$

$\Rightarrow \boxed{T = m(g-a)}$

Ex: 3



FBD



* Imp: We can assume the force in any dirn. After calculation if F = '+' that means our assumption is right & if F = '−' then the dirn of force will change.

CG & MI

Centre of gravity (CG)

→ It is a point in a body where the whole mass of the body can be assumed to be concentrated.

→ 3D

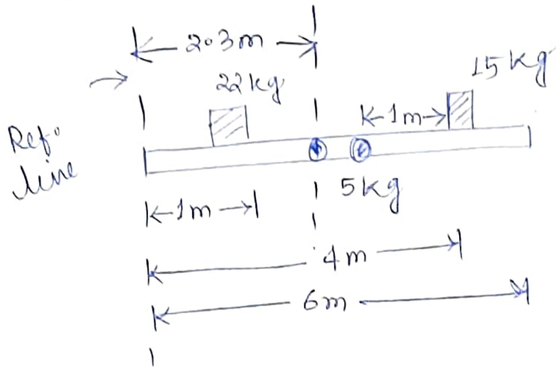
Centroid

It is a point in the area where the whole area can be assumed to be concentrated.

→ 2D

(lamina)

Ex: how to calculate CG.



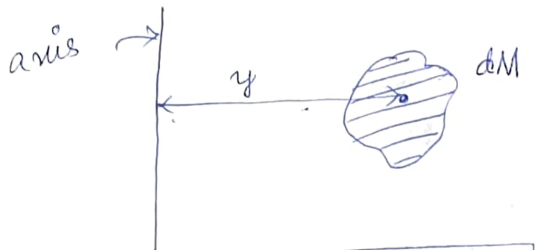
$$\boxed{x = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1+m_2+m_3}}$$

$$= \frac{(22\times1)+(5\times3)+(15\times4)}{22+5+15}$$

$= 2.3m$ (from reference line)

Moment of Inertia (MI)

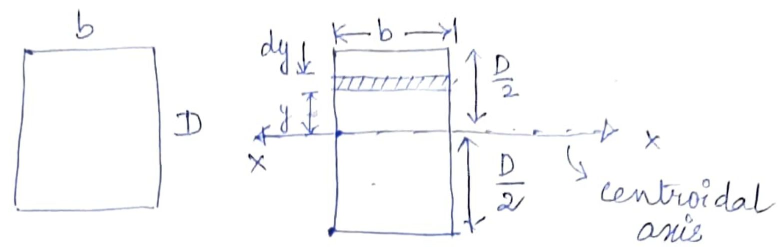
→ It is a rotational analogue of mass, which describes an objects ~~to~~ resistance to translational motion.



$$\boxed{I = \int (dM)y^2}$$

Similarly $\boxed{I = \int (dA)y^2}$

MI of rectangular section



Area of strip (dA) = b dy

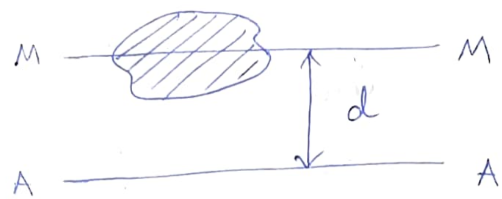
MI about X-X (Ixx) $= \int_{d/2} (dA)y^2$

$= \int_{-d/2} by^2 dy$

$$\boxed{I_{xx} = \frac{bD^3}{12}}$$

Similarly $\boxed{I_{yy} = \frac{Db^3}{12}}$

Parallel Axis Theorem

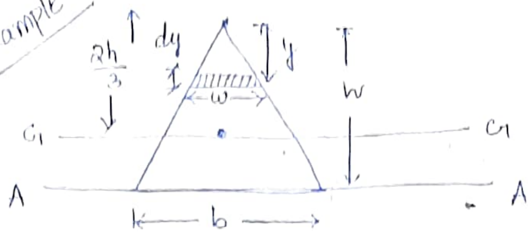


I_{MM} = MI of body through centroid

I_{AA} = MI of body about AA which is at 'd' from body

$$\boxed{I_{AA} = I_{GG} + Ad^2}$$

Example



$$\frac{h}{b} = \frac{y}{w} \Rightarrow w = \frac{by}{h}$$

Area of strip = $\left(\frac{by}{h}\right) \times dy$

$$I_{G_1G_1} = \int_0^h \left(\frac{by}{h}\right) \times \left(\frac{2h}{3} - y\right)^2 dy = \frac{bh^3}{36}$$

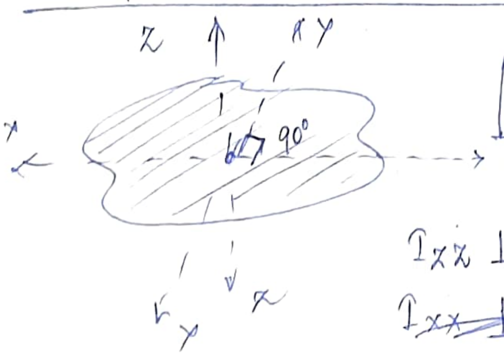
$$\boxed{I_{GG} = \frac{bh^3}{36}}$$

$$\therefore I_{AA} = I_{GG} + A\left(\frac{h}{3}\right)^2 = \frac{bh^3}{36} + \frac{1}{2} \cdot bh \cdot \frac{h^2}{9}$$

$$= \frac{bh^3}{36} + \frac{2bh^3}{36} = \frac{bh^3}{12}$$

$$\boxed{I_{AA} = \frac{bh^3}{12}}$$

Perpendicular Axis Theorem



$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$

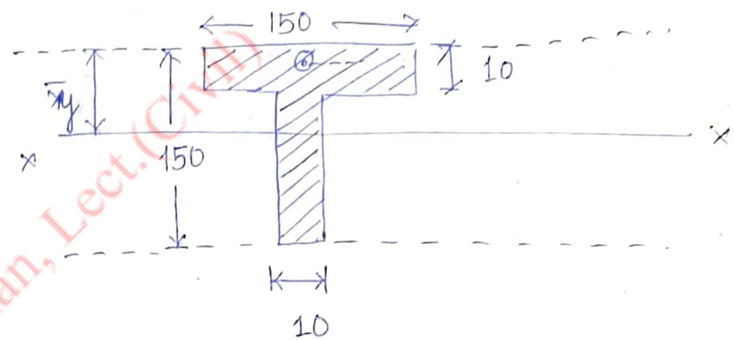
$I_{zz} \perp$ plane of lamina

~~$I_{xx} \perp I$~~ $x \perp y$

It states that the MI of a planar lamina about an axis perpendicular to plane of lamina is equal to sum of ~~MI~~ MIs about two axis at 90° to each other at the same point.

Example

MI of T section



Take top of T-section as reference line.

$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = \frac{(150 \times 10) \times 5 + (140 \times 10 \times 80)}{(150 \times 10) + (140 \times 10)}$$

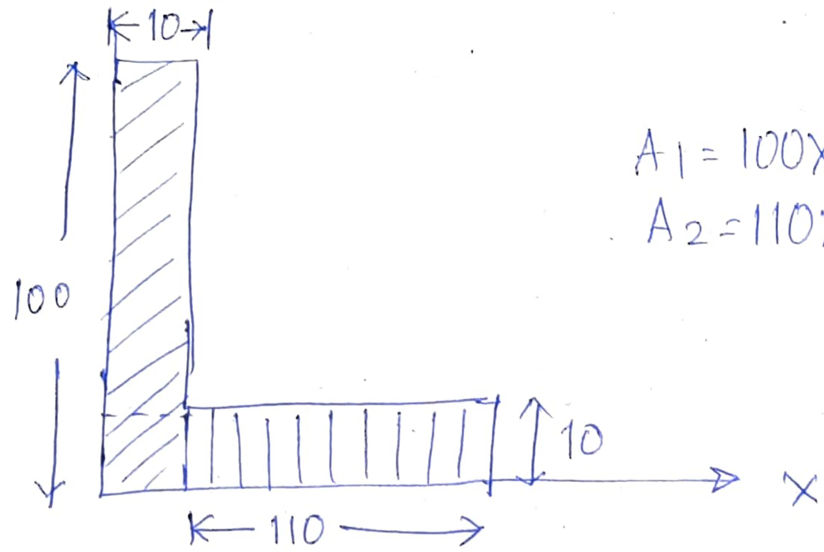
$= 41.2\,mm$

CG of section to N A distance

$$I_{xx} = \frac{150 \times 10^3}{12} + 150 \times 10\,(41.2 - 5)^2 + \frac{10 \times 140^3}{12} + 140 \times 10\,(80 - 41.2)^2 = 6.372 \times 10^6\ mm^4$$

$$I_{yy} = \frac{10 \times 150^3}{12} + \frac{140 \times 10^3}{12} = 2.8242 \times 10^6\ mm^4$$

MI of a 'L' section



$A_1 = 100 \times 10$
$A_2 = 110 \times 10$

As section is unsymmetrical, we have to calculate both $\bar{x}$ & $\bar{y}$.

from Parallel Axis Theorem we know $I_O = I_G + Ay^2$

or $\boxed{(I_O)_x = (I_G)_x + Ay^2}$

$\therefore$ $A_1 = 100 \times 10 = 1000 \text{ mm}^2$
$A_2 = 110 \times 10 = 1100 \text{ mm}^2$

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2} = \frac{1000 \times 5 + 1100 \times 65}{2100}$$

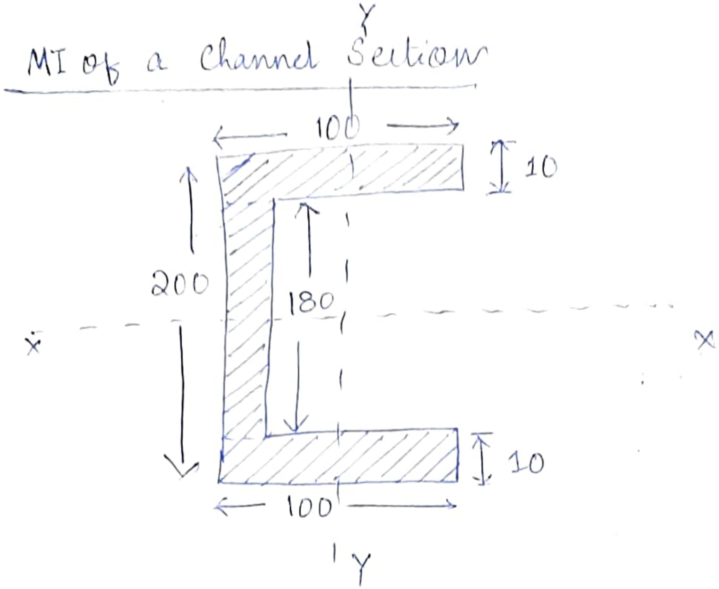
$$= 36.43 \text{ mm}$$

$\bar{y} = 26.43 \text{ mm}$

$$I_{xx} = \frac{10 \times 100^3}{12} + 10 \times 100 (\cancel{100}\ 50 - 26.43)^2 + \frac{110 \times 10^3}{12} + 1100 (26.43 - 5)^2$$

$$I_{yy} = \frac{100 \times 10^3}{12} + 1000 (36.43 - 5)^2 + \frac{10 \times 110^3}{12} + 1100 (65 - 36.43)^2$$

MI of a Channel Section



$A_1 = 100 \times 10 = 1000\,mm^2$

$A_2 = 180 \times 10 = 1800\,mm^2$

$A_3 = 100 \times 10 = 1000\,mm^2$

$$\bar{x} = \frac{(1000 \times 50) + (1800 \times 5) + (1000 \times 50)}{3800}$$

$= 28.68\,mm$ (from extreme left)

$$\bar{y} = \frac{(1000 \times 5) + (1800 \times 100) + (1000 \times 195)}{3800}$$

$= \cancel{18}\ 100\,mm$ (from bottom)

$$I_{xx} = \frac{100 \times 10^3}{12} + 1000\,(\overset{100}{\cancel{28.68}} - 5)^2 + \frac{10 \times 180^3}{12} + 1800\,(90 - \overset{100}{\cancel{28.68}})^2 + \frac{100 \times 10^3}{12} + 1000\,(100 - 5)^2$$

Similarly

$$I_{yy} = \frac{10 \times 100^3}{12} + 1000\,(50 - 28.68)^2 + \frac{180 \times 10^3}{12} + 1800\,(28.68 - 5)^2 + \frac{10 \times 100^3}{12} + 1000\,(50 - 28.68)^2$$

———— x ————

2 Simple & Complex Stress & Strain

2.1 Simple Stresses & Strain

Introduction to stress & strain

Mechanical Properties of Materials

Rigidity

It is the property exhibited by solids to body to resist deformation.

Elasticity

It is the ability of the deformed body to returns to its original shape & size after removal of external force.

Plasticity

It is the ability of the solid material to undergo permanent deformation/ non ~~rem~~ reversible change after ~~application~~ removal of external/applied forces.

Compressibility (β)

It is the measure of relative volume change of fluid/solid as a response to pressure change.

$$\boxed{\beta = \frac{p}{\left(\frac{\Delta V}{V}\right)}}$$

$$\boxed{\text{Bulk Modulus } (K) = \frac{1}{\beta}}$$

Hardness - It is a material's quality to withstand localised deformation/ resistance to identation, deformation or abrasion.

Toughness -

It is the ability of a material to absorb energy and plastic deformation without fracturing.

Stiffness $(F = kx) \Rightarrow \boxed{k = \frac{F}{x}}$

It is defined as the resistance of an elastic body due to unit deflection.

Brittleness (tendency to shatter on receiving shock)

It is defined as the nature of material when subjected to stress, breaks with little elastic deformation and without significant plastic deformation.

ex — glass

Ductility

It is a measure of material's ability to undergo significant plastic deformation before rupture/ breaking.

Malleability

It is defined as the property of metals that defines their ability to be hammered, pressed, rolled into thin sheets without breaking.

Creep

It is defined as the permanent deformation under the influence of ~~con~~ persistent loading.

Fatigue

→ It is a phenomenon when material fails ~~under~~ when subjected to a cyclic load.

→ 03 stages of fatigue

→ initiation
→ propagation
→ final rupture

Tenacity

Minerals toughness/resistance to breaking.

Durability

It is defined as the property to withstand stress (load) over a long period of time.

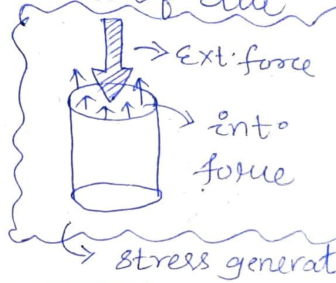
Types of stresses

There are mainly 02 types of stresses -

1. normal stress (tensile & compressive)
2. tangential stress / shear stress

Normal stress (σ) It is a stress acting perpendicular to the cross section of the member.

Normal stress
- → compressive stress
- → tensile stress



tensile stress

P = tensile ~~st~~ force
A = area of c/s

$$\boxed{\sigma_t = \frac{P}{A}}$$

where σ_t = tensile stress

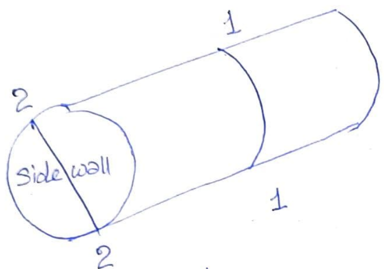
compressive stress

P = compressive force
A = area of c/s

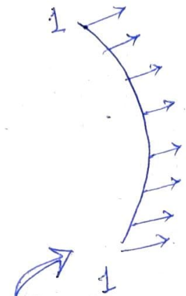
$$\boxed{\sigma_c = P/A}$$

Tangential stress (τ)

If the external forces tend to shear the member along the c/s, internal forces developed act tangentially to c/s.

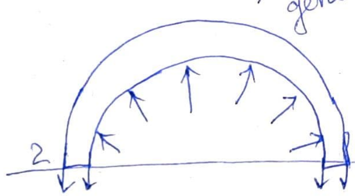


cylinder under internal pressure

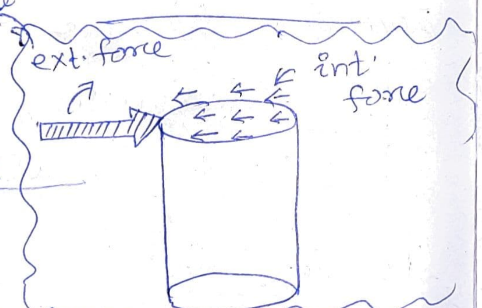


Longitudinal stress

stress generate



hoop stress



$$\boxed{\text{shear stress } (\tau) = \frac{P}{A}}$$

P = shear force
A = area of c/s

unit for stress : $\frac{N}{m^2}$, $\frac{N}{mm^2}$, $\frac{kN}{m^2}$, $\frac{kN}{mm^2}$

$$\boxed{1\ \frac{N}{m^2} = 1 \text{ Pascal}}$$

Types of strain → {Strain is the measure of deformation of body}

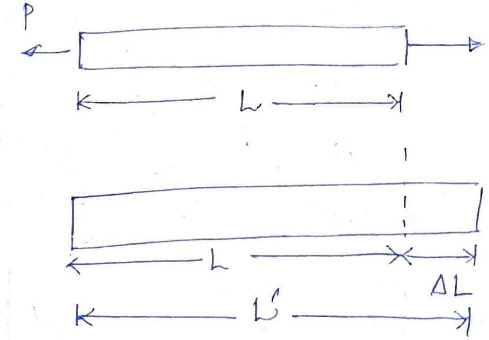
1. Tensile strain
2. compressive strain
3. shear strain

Tensile strain

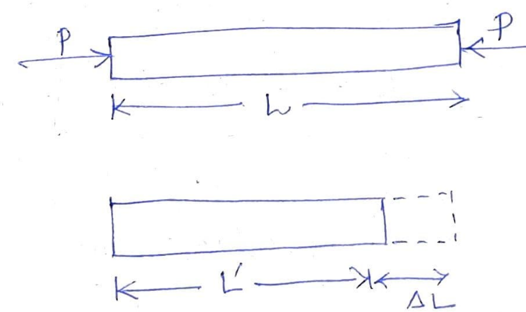
P = tensile force
L = Length of body

$$\boxed{\varepsilon_t = \frac{\Delta L}{L}}$$

$$\boxed{L' > L}$$



compressive strain



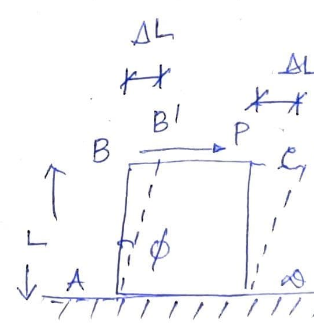
$$\boxed{L' < L}$$

P = compressive

$$\boxed{\varepsilon_c = \frac{\Delta L}{L}}$$

Shear strain (ϕ)

Suppose L = AB
P = shear force on BC
ϕ = shear strain

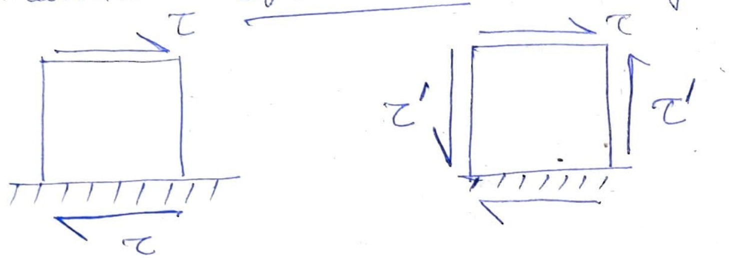


$$\boxed{\phi = \frac{BB'}{AB} = \frac{\Delta L}{L}}$$

Complementary shear stress

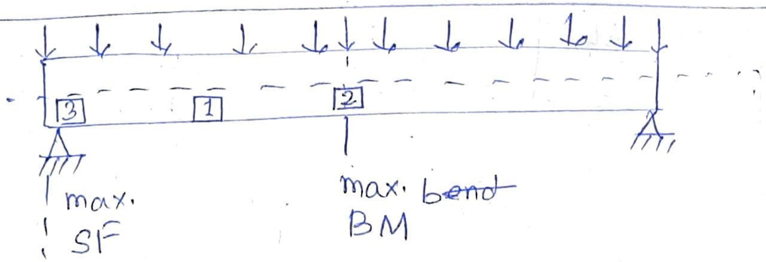
→ When an shear stress acts on upper surface of the cube in a clockwise direction, then the complementary shear stress acts in a plane ⊥ to the plane such as it forms an another ~~cou~~ couple in opposite direction.

→ To maintain equilibrium in body.



τ = Shear stress
τ' = complementary shear stress

Diagonal tensile & compressive stress due to shear

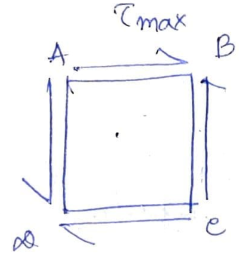


[1] intermediate zone

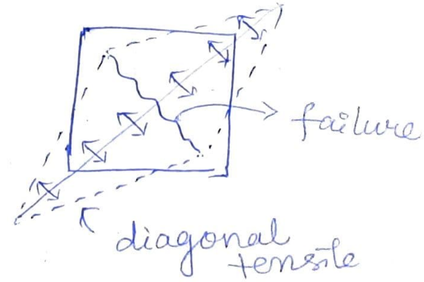
[2] max. BM zone

[3] max SF zone

At the support ([3]) SF is max. So element [3] undergoes max. shear stress & no bending stress as.



element 3



→ due to stress condition the diagonal BD ~~undergo~~ is subjected to tensile stress & AC is subjected to compressive stress.

→ As concrete is weak in tension, diagonal cracks ~~co~~ ~~op~~ occour at 45°.

→ Shear reinf. is provided to counter the crack.

→ Similarly ~~di~~ along AC diagonal compressive stress is generated.

Elongation & Contraction

→ Elongation is a measure of deformation that occurs before a material eventually breaks when subjected to a tensile load.

→ Contraction is the act of decreasing the size of something / shortening it when subjected to a compressive load.

Longitudinal Strain (e).

→ It is defined as the ratio of change in length of material due to applied force to original length.

$$\boxed{\text{longitudinal strain} = \frac{\Delta L}{L} = \frac{\text{change in length}}{\text{original length}}}$$

Lateral strain

→ When the bar is subjected to the axial load, there will be decrease in the dimensions of bar in the ⊥ dirⁿ of ~~axi~~ loading.

So lateral strain is defined as the ratio of decrease in length in ⊥ dirⁿ of loading to that of original length.

$$\boxed{\text{lateral strain} = \frac{\Delta B}{B} \text{ or } \frac{\Delta D}{D}}$$

Poisson's Ratio (μ)

→ It is defined as the ratio of lateral strain to the longitudinal strain.

$$\boxed{\mu = \frac{\Delta D / D}{\Delta L / L}}$$

→ Range : 0 to 0.5

→ concrete : 0.1 – 0.2

steel : 0.27 – 0.3

Rubber : 0.499

~~Corck~~ cork : 0

Volumetric strain

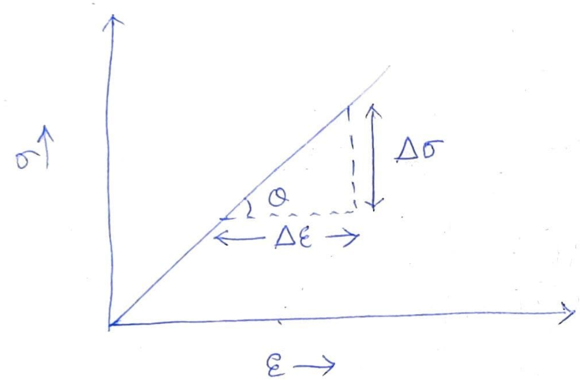
→ It is the ratio of change in volume to the original volume.

→ Bulk modulus (K) = $\frac{\Delta p}{\left(\frac{\Delta V}{V}\right)}$

∴ Bulk Modulus (K) = $\dfrac{\text{change in pressure}}{\text{volumetric strain}}$

Hooke's Law

It says that the stress (σ) is proportional to strain (ε) so long as the material behaves elastically.



$$\boxed{\tan\theta = \frac{\Delta\sigma}{\Delta\varepsilon} = E} = \text{slope}$$

$\Delta\sigma$ = change in stress
$\Delta\varepsilon$ = change in strain
E = Young's Modulus of Elasticity

$$\therefore E = \frac{\sigma}{\varepsilon} = \frac{P/A}{\Delta L/L}$$

$$\Rightarrow E = \frac{\sigma}{\varepsilon} = \frac{P}{A\varepsilon} \Rightarrow \boxed{\varepsilon = \frac{P}{AE}}$$

$$\Rightarrow E = \frac{PL}{A(\Delta L)}$$

$$\Rightarrow \boxed{\Delta L = \frac{PL}{AE}} \quad \text{change in length}$$

Problem 1

The ratio of Youngs Modulus of Elasticity of two materials is 2.35. Find the ratio of the stresses & elongations in two bars of these materials if they are of same length and same area and subjected to the same force P.

Soln $\sigma = P/A$, $\dfrac{E_1}{E_2} = 2.35$

as $A_1 = A_2$

$\sigma_1/\sigma_2 = 1 \Rightarrow \sigma_1 = \sigma_2$

$$\varepsilon_1 = \frac{\sigma_1}{E_1}, \quad \varepsilon_2 = \frac{\sigma_2}{E_2}$$

$$\Rightarrow \frac{\Delta L_1}{L} = \frac{\sigma_1}{E_1}; \quad \frac{\Delta L_2}{L} = \frac{\sigma_2}{E_2}$$

$$\Rightarrow \frac{\Delta L_1}{\Delta L_2} = \frac{E_2}{E_1} = \frac{1}{2.35} = 0.42 \text{ (Ans)}$$

Problem 2

A bar of c/s area 314 mm^2 elongates by 0.8 mm over a length of 600 mm when subjected to a tensile force of 12 kN. Find the E?

Problem 3

A circular bar 20 mm in diameter and 300 mm in length is subjected to a force of 50 kN, find σ, ϵ, ΔL if E = 80 GPa.

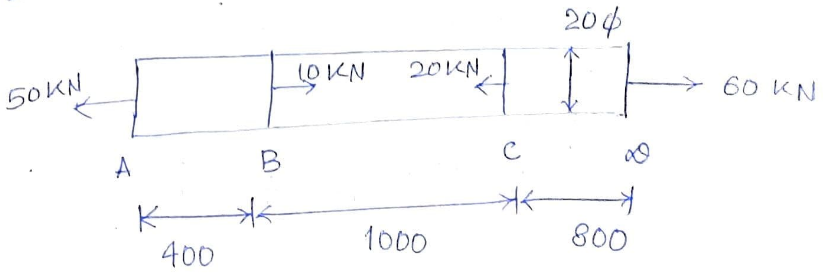
Problem 4

A uniform steel rod 6 mm ϕ and 0.5 m long is subjected to a tensile force of 5 kN. Find the stress in the bar & its elongation. E = 200 GPa.

Problem 5

The length of ~~it~~ an aluminium rod 10 mm ϕ and 400 mm long increases to 400.15 mm when subjected to a tensile force of 5 kN. Find stress in the bar and value of E for aluminium?

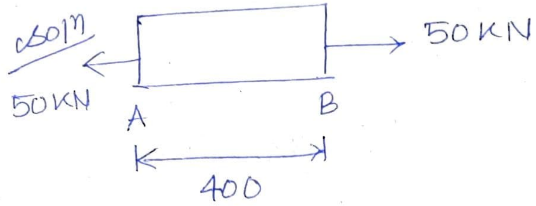
Problem 6



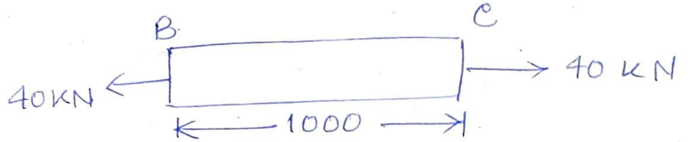
find ΔL?

$E = 200$ GPa
$= 200 \times 10^9 \frac{N}{(10^3)^2 mm^2}$
$= 200000\ N/mm^2$

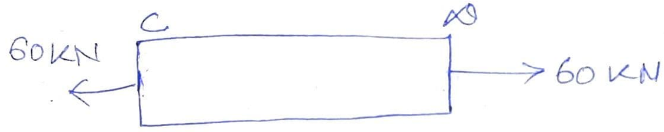
Soln



$$\Delta L_1 = \frac{P_1 L_1}{AE} = \frac{50,000 \times 400}{100\pi \times 200000} = 0.318\ mm$$



$$\Delta L_2 = \frac{P_2 L_2}{AE} = \frac{40000 \times 1000}{100\pi \times 200000} = 0.637\ mm$$



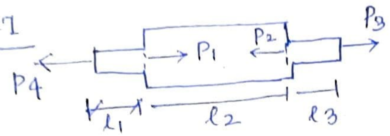
$$\Delta L_3 = \frac{P_3 L_3}{AE} = 0.764\ mm$$

$$\Delta L = 0.318 + 0.637 + 0.764$$
$$= 1.719\ mm$$

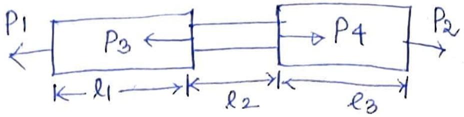
The max. stress will be in section CD.

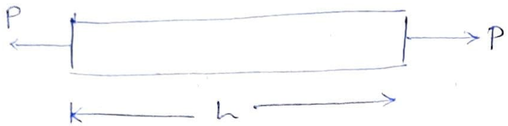
$\sigma = \frac{60,000}{100\,\pi} = 191\ N/mm^2$. (80m)

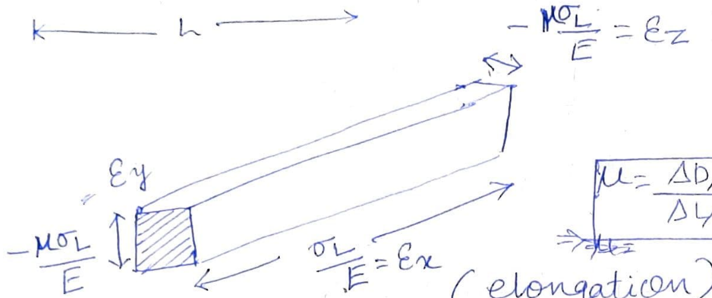
Case-I



Case-II







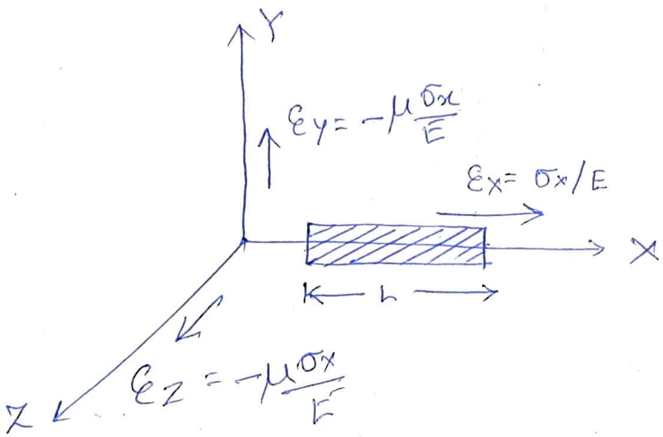
$\boxed{\mu = \frac{\Delta D/D}{\Delta L/L}}$

(elongation)

$\therefore\ \varepsilon_x = \frac{\sigma_L}{E}$

$\varepsilon_y = -\mu\frac{\sigma_L}{E} = -\mu\varepsilon_x$

$\varepsilon_z = -\mu\frac{\sigma_L}{E} = -\mu\varepsilon_x$.

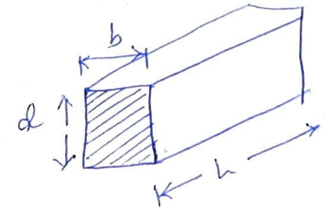


→ On a body stress σ_x acting along x dr^n.

$\varepsilon_x = \frac{\sigma_x}{E}$

So $\varepsilon_y = -\mu\frac{\sigma_x}{E}$, $\varepsilon_z = -\mu\frac{\sigma_x}{E}$

Suppose bar has dimension as follows



Original volume (V) = Lbd

Final volume (V') $= (L + \varepsilon_x L)(d - \varepsilon_y d)(b - \varepsilon_z b)$

$= Lbd\,(1+\varepsilon_x)(1-\varepsilon_y)(1-\varepsilon_z)$

$= Lbd\,[1 + \varepsilon_x(1-2\mu)]$

$\Delta V = V' - V = Lbd\,\varepsilon_x(1-2\mu)$

$\therefore\ \frac{\Delta V}{V} = \varepsilon_x(1-2\mu)$

$\Rightarrow \boxed{\varepsilon_v = \varepsilon_x(1-2\mu)}$

$= \varepsilon_x - 2\varepsilon_x\mu = \varepsilon_x - \varepsilon_x\mu - \varepsilon_x\mu$

$\boxed{\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z}$

For multi axial stresses

$$\varepsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E}$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}$$

poisson's ratio $= \mu$ or ν

$\therefore \varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$

$$\varepsilon_v = (1-2\nu)\left(\frac{\sigma_x+\sigma_y+\sigma_z}{E}\right)$$

→ for hydrostatic case $(\sigma_x = \sigma_y = \sigma_z = \sigma)$

$$\boxed{\varepsilon_v = (1-2\nu)\frac{3\sigma}{E}}$$

$1 - 2\nu \overset{>}{\cancel{\neq}} 0$

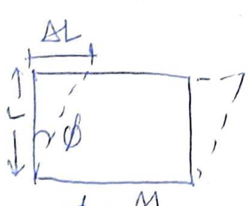
$\Rightarrow \boxed{\nu \ngtr 0.5}$

Again Elastic Constants

↳ Youngs Modulus $(E) = \frac{\sigma}{\varepsilon}$

↳ Bulks Modulus $(K) = \frac{\Delta p}{\left(\frac{\delta V}{V}\right)}$

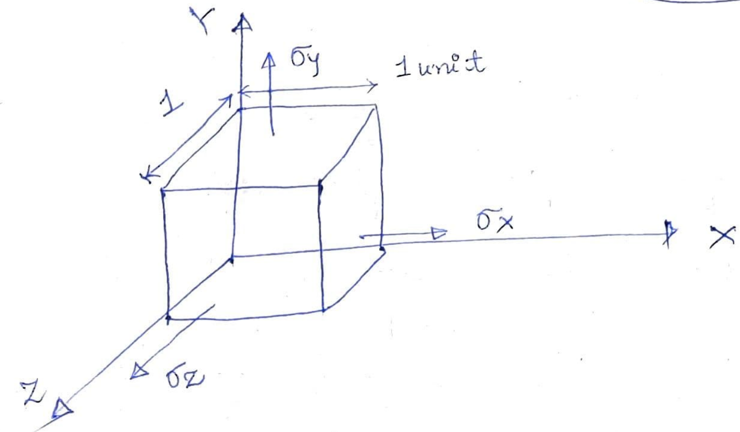
↳ Shear Modulus $(G) = \frac{\tau}{\phi}$ or $\frac{\tau}{\gamma}$



$\phi = \frac{\Delta L}{L}$

Relationship between Elastic Constants (E, G, K)

Part-I



Consider there is a cube of side 1 unit length subjected to tensile stresses as σ_x, σ_y and σ_z.

Original volume

$$V = 1 \times 1 \times 1 = 1$$

New ~~Change in~~ volume when tensile stresses are applied

$$V' = (1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z)$$

$$= 1 + \varepsilon_x + \varepsilon_y + \varepsilon_z + \varepsilon_x\varepsilon_y + \varepsilon_y\varepsilon_z + \varepsilon_z\varepsilon_x + \varepsilon_x\varepsilon_y\varepsilon_z$$

as ~~ε_x, ε_y and~~ $\varepsilon_x\varepsilon_y$, $\varepsilon_y\varepsilon_z$, $\varepsilon_z\varepsilon_x$ & $\varepsilon_x\varepsilon_y\varepsilon_z$ are very small so

$$\boxed{V' = 1 + \varepsilon_x + \varepsilon_y + \varepsilon_z}$$

but we know

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu\sigma_x}{E} - \frac{\nu\sigma_z}{E}$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E}$$

$$\therefore \varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$\boxed{\varepsilon_v = \frac{(\sigma_x + \sigma_y + \sigma_z)}{E}(1 - 2\nu)} = \frac{\Delta V}{V}$$

if $\sigma_x = \sigma_y = \sigma_z = \sigma$

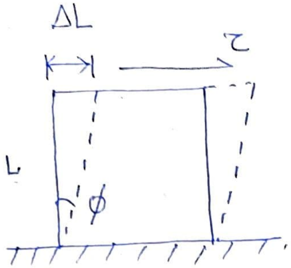
$$\varepsilon_v = \cancel{3E}\ \frac{3\sigma}{E}(1 - 2\nu)$$

$$\Rightarrow E = \frac{3\sigma}{\varepsilon_v}(1 - 2\nu)$$

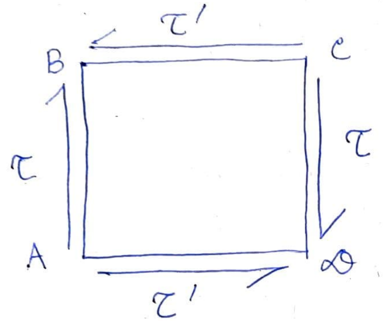
$$\Rightarrow \boxed{E = 3K(1 - 2\nu)} \qquad \boxed{K = \frac{\Delta p}{\left(\frac{\Delta V}{V}\right)}}$$

* Rel^n between E & K *

Extra

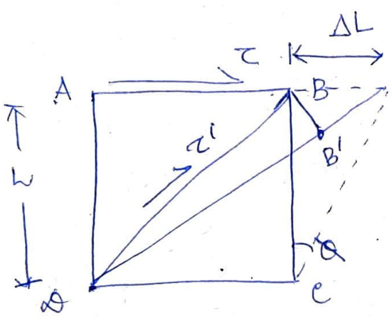


τ = shear stress
L = original length
ΔL = change in length
ϕ = shear strain



> τ = Shear stress
> τ' = complementary shear stress

Suppose 'τ' acting on top of plane in +ve dirⁿ



$$\boxed{\phi = \frac{\Delta L}{L}}$$

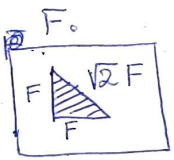
suppose the side of cube $= a$

$\therefore$ force on AB $= \tau \times a \times 1 = F$

force on BD $= \tau' \times \sqrt{2}a \times 1 = F'$

$\therefore$ force on BD in terms of F.

$$\boxed{F' = \sqrt{2} F}$$



$\Rightarrow \tau' \sqrt{2} a \times 1 = \sqrt{2} \times \tau \times a \times 1$

$\Rightarrow \boxed{\tau' = \tau}$

strain along diagonal $= \dfrac{B'B''}{BD}$

$$= \frac{BB'' \cos 45^\circ}{CD \sec 45^\circ}$$

$$= \frac{1}{2}\frac{BB''}{CD} = \frac{1}{2}\frac{BB''}{BC}$$

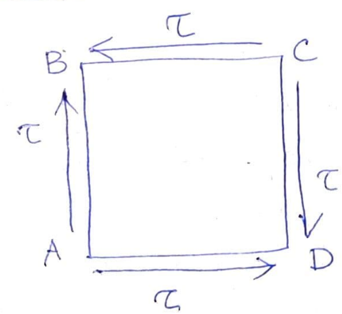
$$= \frac{1}{2}\frac{\Delta L}{L} = \frac{\phi}{2}$$

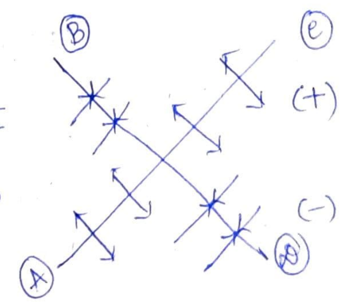
$\therefore$

$$\boxed{\text{strain along diagonals} = \frac{\phi}{2}}$$

part-II

$\varepsilon_{AC} = \dfrac{\phi}{2}$





$$\cancel{\varepsilon_{AC} = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}}$$

$$\cancel{= \frac{\tau}{E} - \frac{\nu(-\tau)}{E}}$$

$$\varepsilon_{AC} = \frac{\tau}{E} - (-)\frac{\tau\nu}{E} = (1+\nu)\frac{\tau}{E}$$

$$\frac{\phi}{2} = \frac{\tau}{E}(1+\nu)$$

$$\Rightarrow \frac{\tau}{2G} = \frac{\tau}{E}(1+\nu)$$

$$\Rightarrow \boxed{E = 2G(1+\nu)}$$

From part-I, part-II

$$\boxed{\begin{aligned} E &= 3K(1-2\nu) \\ E &= 2G(1+\nu) \end{aligned}}$$

$$\Rightarrow \boxed{E = \frac{9KG}{3K+G}}$$

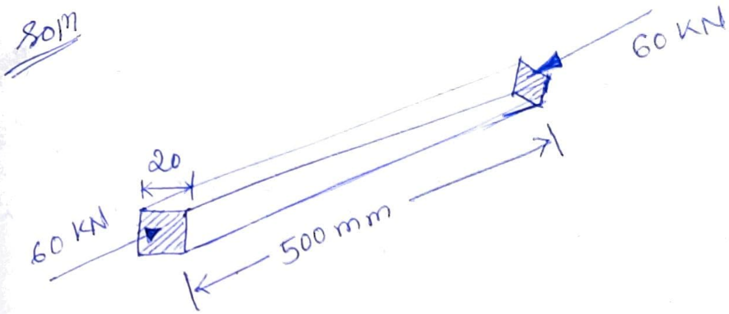
Numericals

1/ ~~St~~ A steel bar 25 mm × 15 mm in c/s is 300mm long and is subjected to a tensile force of 70kN. Find the change in dimension of the bar and change in volume. E = 200 GPa, ν = 0.3.

2/ A steel bar 20mm sq. in section is subjected to an axial compressive load of 60kN. Find the % change in volume if bar is 500mm long.

What are the equal stresses that must be applied to the sides of bar if $\epsilon_v = 0$? ~~E_v~~ E = 200 GPa, ν = 0.3.

Soln



$$\epsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\epsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_z}{E}$$

$$\epsilon_z = \frac{\sigma_z}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E}$$

here $\sigma_x = \frac{P}{A} = \frac{-60{,}000}{20\times20}$ (− = compressive)

$$\therefore \epsilon_x = -\frac{\sigma_x}{E} = \frac{-P}{AE} = -0.00075$$

$$\epsilon_y = -\frac{\nu\sigma_x}{E} = \frac{-\nu(-P)}{AE} = +0.000225 = \epsilon_z$$

$$\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z = -0.3$$

$$\epsilon_v = \frac{\Delta V}{V} \Rightarrow \boxed{\Delta V = -60\ mm^3 \text{ (decrease)}}$$

part-II

volume change

To make the ~~tensile stress 0~~ zero, lest lets apply σ(+) along y & z drⁿ.

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E} \Rightarrow -\frac{\sigma_x}{E} - \frac{2\nu\sigma}{E} = \varepsilon_x$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu\sigma_x}{E} - \frac{\nu\sigma_z}{E} \Rightarrow \frac{\sigma}{E} - \frac{\nu\sigma}{E} + \frac{\nu\sigma_x}{E} = \varepsilon_y = \varepsilon_z$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E}$$

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = 0$$

$$\Rightarrow \sigma_x = 150\ N/mm^2,\ \nu = 0.3$$

$$\therefore \Rightarrow \boxed{\sigma = 75\ N/mm^2}$$

Ans Soln

3) At what depth in sea water will a cube of 1 m side, made of steel, change the volume by 0.05 % ? $E = 200\,GPa$, $\nu = 0.3$

$\gamma = 10.08\ \frac{KN}{m^3}$

Soln

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu\sigma_x}{E} - \frac{\nu\sigma_z}{E}$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_x}{E}$$

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{3\sigma}{E}(1-2\nu)$$

$(\sigma_x = \sigma_y = \sigma_z = \sigma$ hydrostatic case$)$

↳ under water

change in volume (ΔV)

$$= \frac{0.05}{100} \times 1 \times 1 \times 1 = \frac{0.05}{100}$$

$$\frac{\Delta V}{V} = \varepsilon_v$$

$$\Rightarrow \frac{0.05}{100} = \frac{3\sigma}{E}(1-2\nu)$$

$$\Rightarrow \sigma = \frac{0.05}{100} \times \frac{E}{3(1-2\nu)} = 83.33\ N/mm^2$$

$$\therefore \gamma h = 83.33$$

$$\Rightarrow 10.08 \times h = 83.33 \times 10^{-3} \times \frac{1}{(10^{-6})}$$

$$\Rightarrow \boxed{h = 8267\ m}$$

Soln

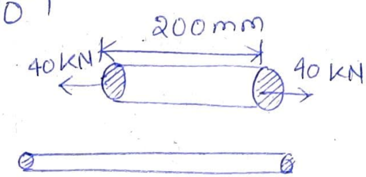
4/ A 25 mm diameter bar when subjected to a force of 40 kN has an extension of 0.08 mm on a gauge length of 200mm. If the diametrical reduction is 0.003mm find the values of E, G, K, ν.

Soln

$A = \frac{\pi}{4}(25)^2 = 491\ mm^2$

$\sigma = \frac{P}{A} = \frac{40,000}{491} = 81.47\ N/mm^2$

$\varepsilon_x = \frac{\Delta L}{L} = \frac{0.08}{200} = 4 \times 10^{-4}$



$E = \frac{\sigma}{\varepsilon} = 203\ GPa$

lateral strain along dia (ε_y)

$= \frac{0.003}{25} =$

$\varepsilon_y = \cancel{\frac{\sigma_y}{E}}^{\,0} - \frac{\nu \sigma_x}{E} - \cancel{\frac{\nu \sigma_z}{E}}^{\,0}$

$\Rightarrow \varepsilon_y = -\nu \varepsilon_x$

$\Rightarrow -\varepsilon_y = -\nu \varepsilon_x$ ($-\varepsilon_y$ = as dia reduced)

$\Rightarrow \nu = \frac{0.003}{25 \times 4 \times 10^{-4}} = 0.3$

$\Rightarrow E = 2G(1+\nu) \Rightarrow G = 78.1\ GPa$

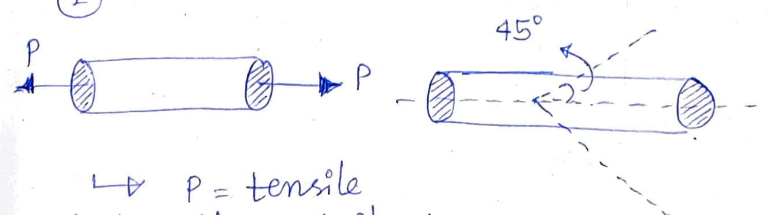
$\Rightarrow E = 3K(1-2\nu) \Rightarrow K = 169.2\ GPa$ (soln)

2.2 Application of simple stress and strain in engineering field

Behaviour of ductile and brittle material under direct load

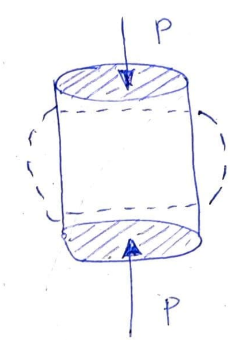
Ductile material under direct load

(I)



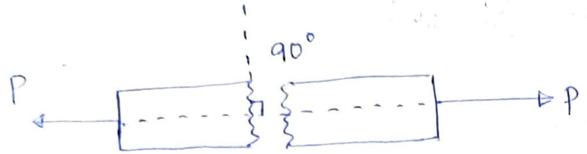
- P = tensile
- shear failure
- failure plane @ 45°
- cup-cone fracture

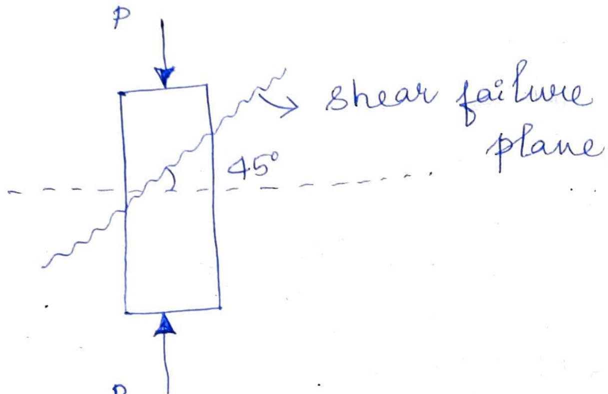
(II)



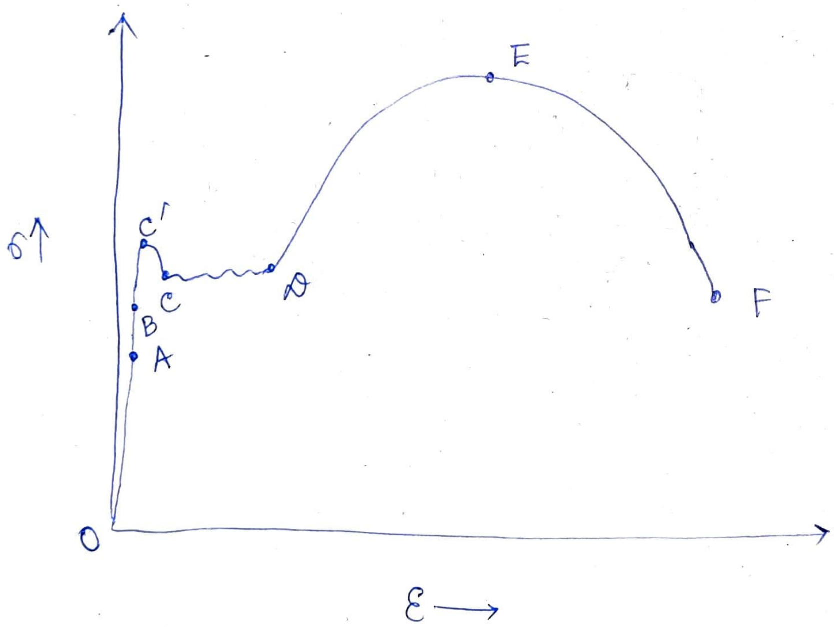
- P = compressive
- failure due to bulging

Brittle material under direct load





Stress- Strain Curve of a ductile Material



OA → Proportional region
→ straight line
→ Hooke's Law is obeyed
($\sigma \propto \varepsilon$)

OB → Elastic region

BC → Elasto plastic region

CD → perfectly plastic region

DE → Strain hardening zone
(material becomes hard when strained)

EF → necking zone

A → limit of proportionality

B → Elastic limit

C → Lower yield point

C′ → Upper "

D → Strain hardening starts

E → Ultimate point

F → Fracture point (metal will break)

Limit of Proportionality (A)

→ It is the point at which stress-strain curve ceases to be straight line.

→ Hooke's Law is valid.

Elastic Limit

It is the the point on the stress-strain
→ curve upto which the materials remain elastic.

→ OB zone

Yield Stress (σ_y)

→ The stress corresponds to the lower yield point on the stress-strain curve of ductile material is called yield stress.

→ σ_y

Yield Point

The point beyond elastic ~~point~~ limit where the specimen undergoes an increase in length without ~~increas~~ further increase in load.

Ultimate Stress

Ultimate stress is the max. stress ordinate in stress-strain diagram.

Breaking Stress

It is the stress at which the specimen ruptures.

Percentage Elongation

$$= \frac{\text{final length} - \text{original length}}{\text{Original length}} \times 100$$

$$= \frac{L_f - L_o}{L_o} \times 100$$

Percentage Reduction in Area

$$= \frac{A_o - A'}{A_o} \times 100$$

A_o = original area

A' = area measured at the neck

Significance of ~~Prete~~ percentage elongation

→ It is a property of the material which provides a value of its ductility.
i.e. the capability of undergoing plastic deformation when kept under load before breaking.

→ The greater the deformation before breaking, the more the material is ductile.

* Significance in reduction in area of c/s *

→ It indicates ductility.

→ It represents a material ability to withstand plastic deformation before experiencing fracture failure.

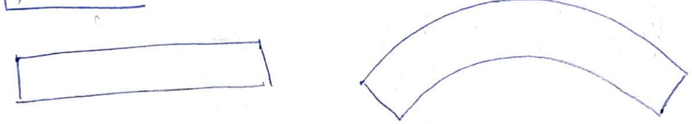
→ A more ductile material will experience a greater reduction in area and less ductile material will experience a less reduction in area.

Deformation of prismatic bar due to uniaxial load :-

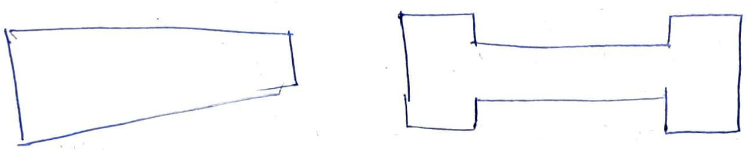
prismatic bar

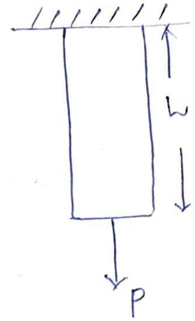
A beam/bar having uniform cross-section at any location across the long axis of the beam.

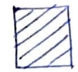
Prismatic



non prismatic







cross section
area = A

→ Let's assume there is a prismatic bar of length L and c/s area A.

→ After application of force 'P', ~~assume~~ the deflection is ΔL.

We know strain $(\varepsilon) = \frac{\Delta L}{L}$

$$\frac{\text{stress}(\sigma)}{\text{strain}(\varepsilon)} = E \text{ (Young's Modulus)}$$

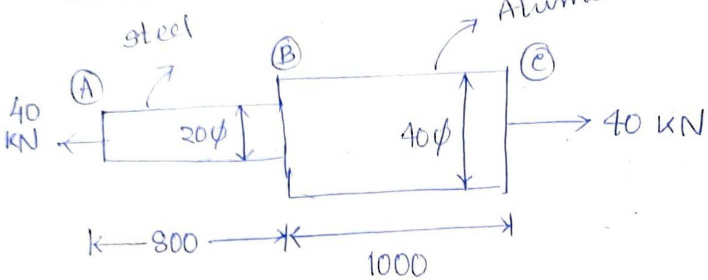
$$\Rightarrow \frac{\sigma}{\varepsilon} = E$$

$$\Rightarrow \frac{P}{A\varepsilon} = E$$

$$\Rightarrow \frac{P}{A\frac{\Delta L}{L}} = E$$

$$\Rightarrow \boxed{\Delta L = \frac{PL}{AE}}$$

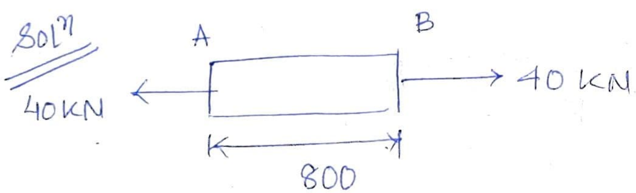
Numericals



A steel rod, 20 mm φ and 800mm long is rigidly attached to an aluminium rod, 40mm φ and 1000mm long. The combination is subjected to a tensile load of 40 KN. Find the stress in the material and total elongation of bar.

$E_{steel} = 200\,GPa$, $E_{Al} = 70\,GPa$.

Soln



$$\Delta_1 = \frac{P_1 L_1}{A_1 E_1} = \frac{40{,}000 \times 800}{\frac{\pi}{4} \times (20)^2 \times 200000} = 0.5093$$

$$\Delta_2 = \frac{P_2 L_2}{A_2 E_2} = \frac{40{,}000 \times 1000}{\frac{\pi}{4} (40)^2 \times 70{,}000} = 0.4547$$

Total deflection $\Delta L = \Delta_1 + \Delta_2$

$= 0.964\,mm$

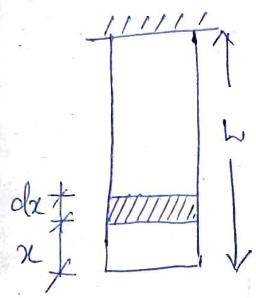
$$\text{Stress in steel} = (\sigma_s) = \frac{40{,}000}{\frac{\pi}{4}(20)^2}$$

$= 127.38\ N/mm^2$

$$\text{Stress in aluminium } (\sigma_{Al}) = \frac{40{,}000}{\frac{\pi}{4}(40)^2}$$

$= 31.8\ N/mm^2$

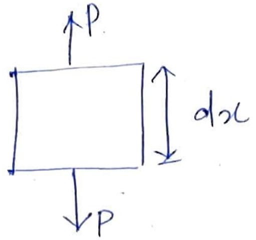
Deformation of prismatic bar due to self weight



L = Length of bar
A = area of c/s
γ = density
E = modulus of elasticity

Take a length 'dx' from distance 'x' from bottom.

$$dx = \frac{PL}{AE}$$



$$= \cancel{\gamma AL}$$

$$= \frac{(\gamma A x)\,dx}{AE}$$

$$= \frac{x\gamma\,dx}{E}$$

for whole bar $\int dx = \int_0^L \frac{x\gamma\,dx}{E}$

$$= \frac{\gamma}{E}\frac{L^2}{2}$$

$$\boxed{\Delta L = \frac{\gamma L^2}{2E}}$$

$$W = \gamma AL \Rightarrow \gamma = \frac{W}{AL}$$

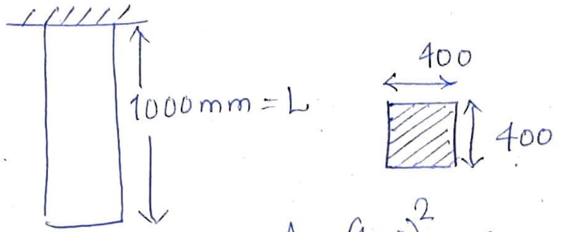
$$\therefore \Delta L = \frac{WL^2}{2ALE} = \frac{WL}{2AE}$$

$$\boxed{\Delta L = \frac{WL}{2AE}}$$

Numerical

A prismatic steel bar of weight ~~5~~ 100kg of size 400×400 hangs from support at top end. The length of bare is 1000mm. find the ~~deflecti~~ deformation, E = 200 GPa.

Soln



$$A = (400)^2 = 160000\ mm^2$$

$$L = 1000mm$$

$W = 100kg = \cancel{100\ N}$ $\left(1\ kg = \cancel{10\ N} = 9.81\ N\right)$

$\cancel{98.1\ N}$ 981 N

$$\boxed{\Delta L = \frac{WL}{2AE}}$$

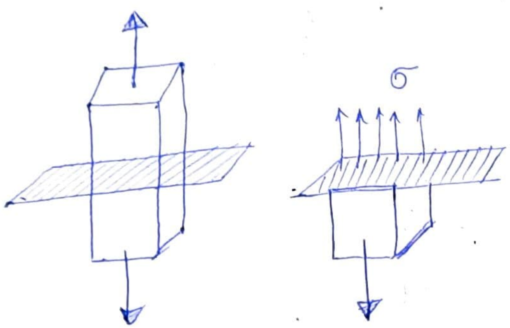
$$= \frac{\cancel{98.1}\ 981 \times 1000}{2 \times 160000 \times 200 \times 10^9 \times \frac{N}{10^6\ mm^2}}$$

$= 0.0000153$ mm $\simeq$ No deflection

2.3 : Complex Stress and Strain

Principal Stresses & Strains concept

- planes that have no shear stress are called as principal planes.
- principal planes carry only normal stresses.



(σ_1 & σ_2)

Principal Stresses :- Principal Stresses are the
→ max. and min. normal stresses on principal plane.

→ ~~Major~~ Maximum ~~princip~~ normal stress = major principal stress (σ_1)

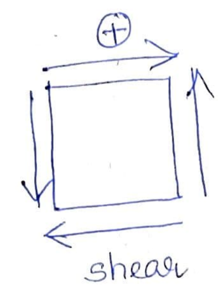
~~Minor~~ Minimum normal stress = minor ~~normal~~ principal stress (σ_2)

Principal Strain

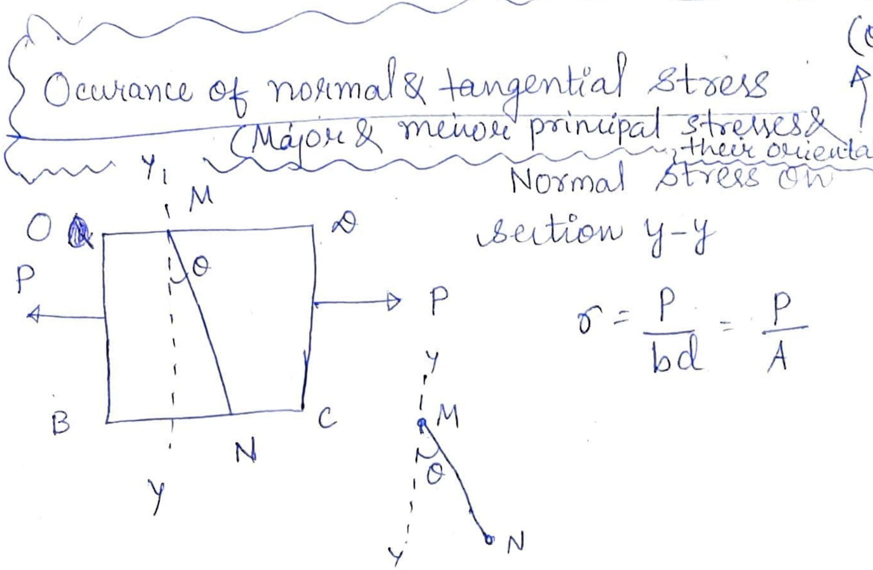
→ Max. and min. normal strain possible for a specific point on a structural element.

→ shear strain is '0' at ~~w~~ the ~~orig~~ orientation where principal strain occurs.

Sign convention

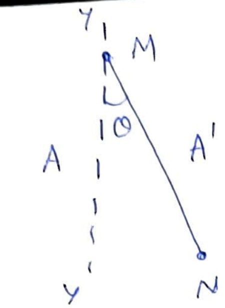


Occurance of normal & tangential stress (Major & minor principal stresses & their orientation)



Normal stress on section y-y

$$\sigma = \frac{P}{bd} = \frac{P}{A}$$



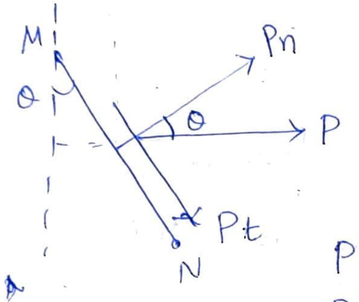
~~Normal~~

Area of section y-y = A

Area of section MN = A'

$\therefore A'\cos\theta = A$

$$\Rightarrow \boxed{A' = \frac{A}{\cos\theta}}$$



$P\cos\theta = P_n$

$P\sin\theta = P_t$

$\therefore \sigma_n$ = normal stress along MN plane

$$= \frac{P_n}{A_{MN}} = \frac{P_n}{A'} = \frac{P\cos\theta}{A/\cos\theta}$$

$$= \frac{P}{A}\cos^2\theta$$

$$\Rightarrow \boxed{\sigma_n = \sigma\cos^2\theta}$$

τ_t = tangential stress along MN

$$= \frac{P_t}{A'} = \frac{P\sin\theta}{A/\cos\theta} = \frac{P}{A}\sin\theta\cos\theta$$

$$\boxed{\tau_n = \frac{\sigma}{2}\sin 2\theta}$$

Case-I : Max. normal stress

$$\boxed{\sigma_n = \sigma \times 1 = \sigma}$$

So $\cos^2\theta = +1$ (-1 is not possible)

$$\Rightarrow \boxed{\theta = 0^\circ}$$

Case-II : ~~Min~~ Max. shear stress

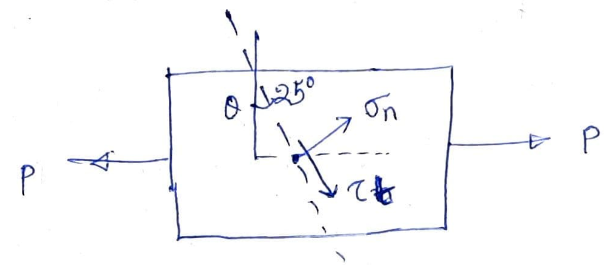
$$\boxed{\tau_n = \frac{\sigma}{2}\times(\pm 1) = \pm\frac{\sigma}{2}}$$

$\sin 2\theta = \pm 1 = \sin 90^\circ$ or $\sin(180^\circ + 90^\circ)$

$$\Rightarrow \boxed{\theta = 45^\circ \text{ or } 135^\circ}$$

Q1 Consider a bar (40mm x 60mm) as shown. Determine the max. value of load 'P' if the stress on plane are limited to a normal stress of $30\frac{N}{mm^2}$ & shear stress of $18\frac{N}{mm^2}$.

Soln



$$\sigma_n = 30\frac{N}{mm^2} = \frac{P}{A}\cos^2\theta = \frac{P}{40\times 60}\cos^2 25^\circ$$

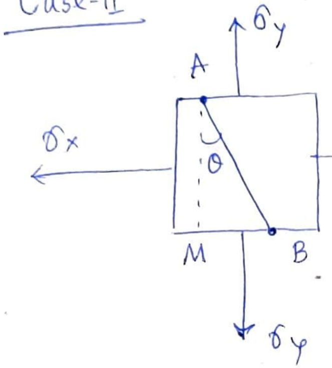
$$\tau = 18\frac{N}{mm^2} = \frac{P}{2(40\times 60)}\times\sin(2\times 25^\circ)$$

$P_1 = 87.6\,kN$
$P_2 = 112.8\,N$
lower is the max. value.

$\boxed{P_1 = 87.6\,kN}$

~~Soln~~ Soln

Case-II



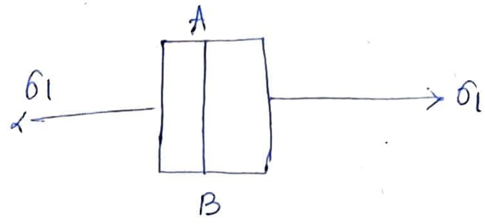
$\boxed{\sigma_x, \sigma_y = \text{normal stresses on } x \,\&\, y \text{ dirn}}$

→ Two normal stresses σ_x & σ_y are acting perpendicular direction on an element.

→ Our aim is to find out the normal stress (σ_n) & shear stress (τ_t) on the plane AB which is oriented at an angle 'θ' with major principal plane. (Simply it can be said inclined θ with Y axis).

→ Major Principal Plane

The plane normal to the major principal stress.

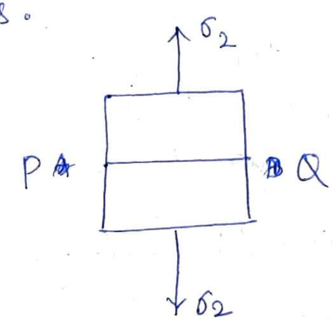


Suppose σ_1 = major principal stress

$\boxed{AB = \text{major principal plane}}$

Minor Principal Plane

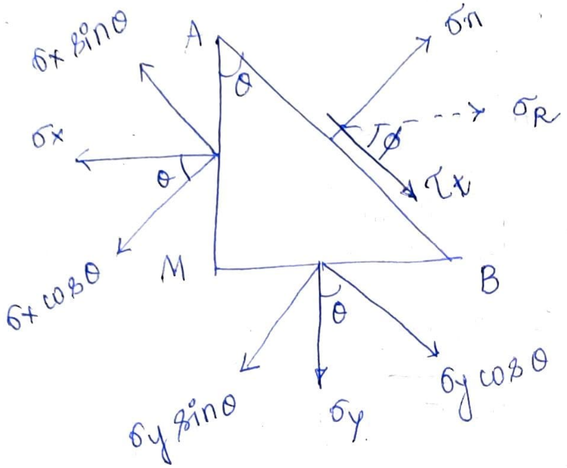
The plane normal to the minor principal stress.



σ_2 = minor principal stress

$\boxed{PQ = \text{minor principal plane}}$

Take Δ AMB for analysis.



c/s of element along ~~AB~~ AM = A_1

c/s of element along MB = A_2

c/s of element along AB = A'

$$\boxed{\begin{aligned} \therefore\ A'\cos\theta &= A_1 \\ A'\sin\theta &= A_2 \end{aligned}}$$

Using condition of equilibrium

$$\sigma_x \cos\theta\,(A'\cos\theta) + (\sigma_y \sin\theta)(A'\sin\theta) = \sigma_n \cdot A'$$

$$\Rightarrow \sigma_x \cos^2\theta + \sigma_y \sin^2\theta = \sigma_n$$

$$\frac{\sigma_x(1+\cos 2\theta)}{2} + \sigma_y\left(\frac{1-\cos 2\theta}{2}\right) = \sigma_n$$

$$\Rightarrow \boxed{\left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta = \sigma_n}$$

Similarly

$$\sigma_x \sin\theta\,(A'\cos\theta) = \overset{\tau_t \times A'}{\cancel{\tau_t}} + \sigma_y \cos\theta\,(A'\sin\theta)$$

$$\Rightarrow \sigma_x \sin\theta\cos\theta - \sigma_y \sin\theta\cos\theta = \tau_t$$

$$\Rightarrow \boxed{\tau_t = \left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta}$$

Max $\sigma_n \Rightarrow \dfrac{d\sigma_n}{d\theta} = 0$

$$\Rightarrow \sin 2\theta = 0 \Rightarrow \boxed{\cancel{\theta = 0 \,\&\, 90^\circ}}$$

$$\Rightarrow \boxed{\theta = 0^\circ \,\&\, 180^\circ}$$

Max $\tau_t \Rightarrow \dfrac{d\tau_t}{d\theta} = 0$

$$\Rightarrow \cos 2\theta = 0$$

$$\Rightarrow \boxed{\theta = 45^\circ \,\&\, 135^\circ}$$

The resultant of stress

$$\sigma_R = \cancel{\sigma_\theta}\sqrt{\sigma_n^2 + \tau_t^2}$$

$$= \sqrt{\sigma_x^2 \cos^2\theta + \sigma_y^2 \sin^2\theta}$$

→ If the resultant stress σ_R makes an angle ϕ with the plane

$$\boxed{\tan\phi = \left(\frac{\sigma_n}{\tau_t}\right)}$$

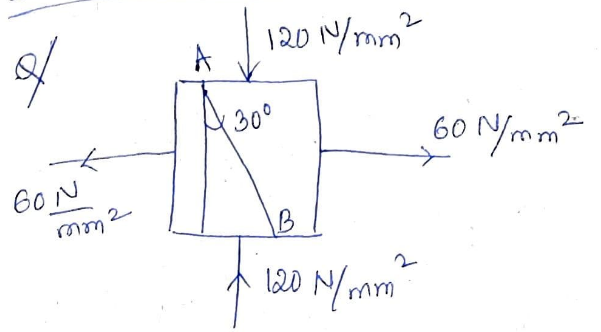
NOTE

In the above calculation both σ_x & σ_y → dirⁿ have been assumed as tensile. So the sign is '+'. But it will change to '–' when ~~the~~ either σ_x or σ_y is compressive.

→ If σ_y is compressive replace σ_y as $(-\sigma_y)$ & vice-versa.

Numerical Problem

Q/



Find out the normal & shear stress along plane AB, resultant stress (σ_R) & ϕ.

solⁿ for σ_x & σ_y both tensile,

$$\boxed{\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta}$$

here $\sigma_x = +60\ N/mm^2$, $\sigma_y = -120\ N/mm^2$

$$\sigma_n = \frac{60-120}{2} + \left(\frac{60-(-120)}{2}\right)\cos 60^\circ$$

$$= -30 + 90 \times \frac{1}{2} = 15\ N/mm^2$$

$$\boxed{\tau_t = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta}$$

$$= \frac{60-(-120)}{2}\sin 2\times 30^\circ$$

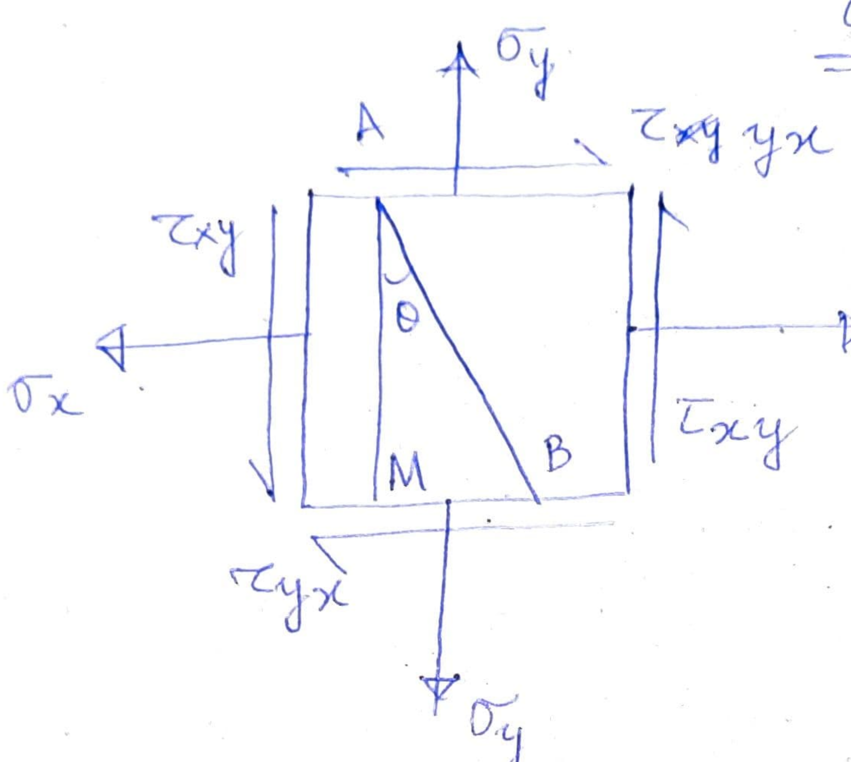
$$= 90 \times \frac{\sqrt{3}}{2} = 45\sqrt{3} = 77.94\ N/mm^2$$

$$\sigma_R = \sqrt{(15)^2 + (77.94)^2} = \boxed{79.37\ N/mm^2}$$

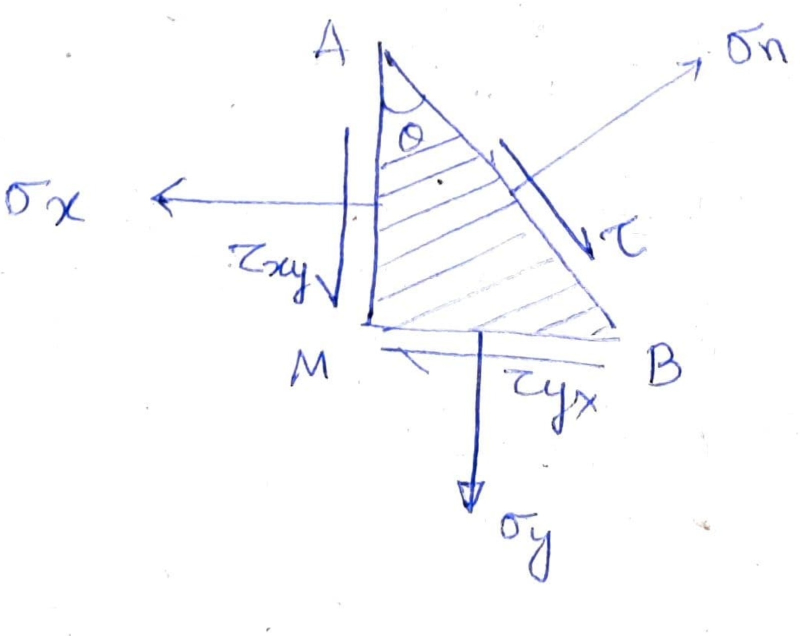
$$\tan\phi = \frac{\tau_t}{\sigma_n} = \frac{77.94}{15} \Rightarrow \boxed{\phi = 79.1^\circ}$$

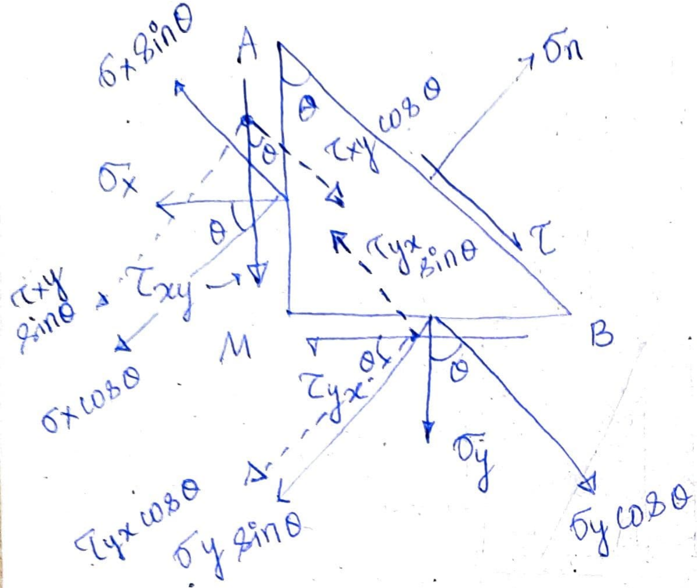
Case-III

(biaxial stress & shear stress)
OR plane stress (No stress in Z-dirⁿ)
↓
only X & Y direction



→ Here aim is to ~~be found~~ find out the normal & shear stress inclined at an angle 'θ' with Y-axis along plane AB.





----▷ shear stress

Area along plane AM $= A_1$

" BM $= A_2$

" AB $= A'$

$$\boxed{A'\cos\theta = A_1 \;,\; A'\sin\theta = A_2}$$

$\therefore \sigma_n \cdot A' = \sigma_x \cos\theta \cdot A_1 + \tau_{xy} \sin\theta \cdot A_1 + \sigma_y \sin\theta \cdot A_2 + \tau_{yx} \cos\theta \cdot A_2$

$\Rightarrow \sigma_n A' = \sigma_x \cos\theta \cdot A' \cos\theta + \tau_{xy} \sin\theta\, A' \cos\theta + \sigma_y \sin\theta\, A' \sin\theta + \tau_{yx} \cos\theta\, A' \sin\theta$

$\Rightarrow \sigma_n = \sigma_x \cos^2\theta + \tau_{xy} \sin\theta\cos\theta + \sigma_y \sin^2\theta + \tau_{yx} \sin\theta\cos\theta$

$\Rightarrow \sigma_n = \dfrac{\sigma_x(1+\cos 2\theta)}{2} + \dfrac{\sigma_y(1-\cos 2\theta)}{2} + 2\tau_{xy} \sin\theta\cos\theta$

$$\Rightarrow \boxed{\sigma_n = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau_{xy} \sin 2\theta}$$

Similarly

$\tau \cdot A' + \tau_{xy} \cos\theta\, A_1 - \tau_{yx} \sin\theta\, A_2 + \sigma_y \cos\theta \cdot A_2 - \sigma_x \sin\theta\, A_1 = 0$

$\Rightarrow \tau + \tau_{xy} \cos^2\theta - \tau_{xy} \sin^2\theta + \sigma_y \cos\theta \sin\theta - \sigma_x \sin\theta\cos\theta = 0$

$\Rightarrow \tau = \tau_{xy} \sin^2\theta - \tau_{xy} \cos^2\theta + (\sigma_x - \sigma_y) \sin\theta\cos\theta$

$$\boxed{\tau = \left(\frac{\sigma_x-\sigma_y}{2}\right) \sin 2\theta - \tau_{xy} \cos 2\theta}$$

$$\sigma_n = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\therefore \boxed{\begin{aligned} \sigma_1 &= \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ \sigma_2 &= \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} \end{aligned}}$$

plane of zero shear stress ($\tau = 0$)

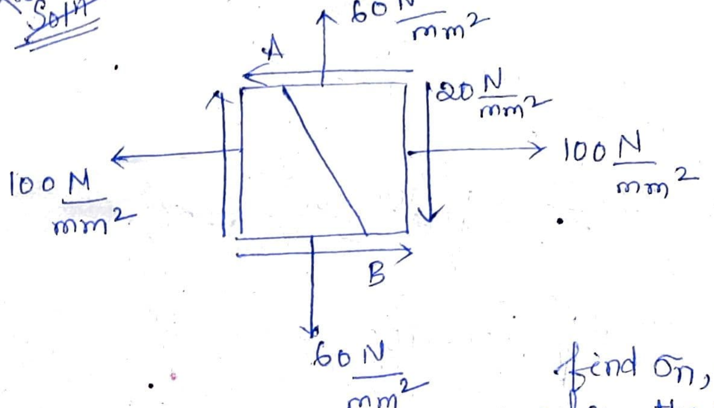
We know

$$\tau = \frac{\sigma_x-\sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$\Rightarrow \tan 2\theta = \frac{2\tau_{xy}}{\sigma_x-\sigma_y}$$

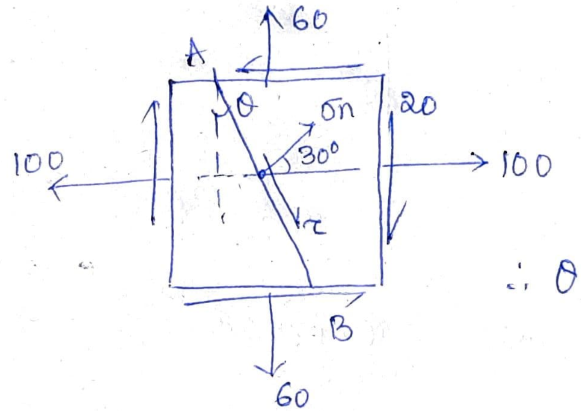
$$\Rightarrow \boxed{\theta = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x-\sigma_y}\right)}$$

Question
Soln



find σ_n, τ when the normal along plane AB makes 30° with X-axis.

Soln



$\therefore \theta = 30^\circ$

We know. $\sigma_n = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$

(when σ_x, σ_y & τ_{xy} as +ve)

$$\therefore \sigma_n = \frac{100+60}{2} + \frac{100-60}{2}\cos 60^\circ + (-)\,20\sin 2\times 30^\circ$$

$$= 80 + 20\times\frac{1}{2} - 20\times\frac{\sqrt{3}}{2}$$

$$= 90 - 10\sqrt{3} = 72.68\ \frac{N}{mm^2}$$

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{100+60}{2} \pm \sqrt{\left(\frac{100-60}{2}\right)^2 + (-20)^2}$$

$$= 80 \pm \sqrt{400+400}$$

$$= 80 \pm 28.28$$

$$\boxed{\sigma_1 = 108.28 \ \frac{N}{mm^2} \qquad \sigma_2 = 51.72 \ \frac{N}{mm^2}}$$

$$\tau = \frac{\sigma_x-\sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$= \frac{100-60}{2}\sin 60^\circ - (-20)\cos 60^\circ$$

$$= 20\times\frac{\sqrt{3}}{2} + 20\times\frac{1}{2}$$

$$\boxed{\tau = 27.32 \ \frac{N}{mm^2}}$$

→ plane of zero shear stress

$$\tan 2\theta = \left(\frac{2\tau_{xy}}{\sigma_x-\sigma_y}\right)$$

$$= \frac{-2\times 20}{100-60}$$

$$\Rightarrow \tan 2\theta = -1 = \tan 135^\circ$$

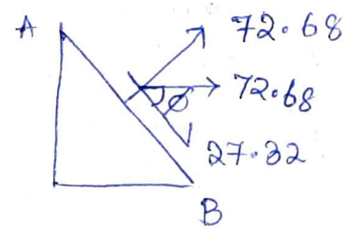
$$\Rightarrow \boxed{\theta = 67.5^\circ}$$

for this orientation, the inclined plane will be zero shear.

Resultant Stress

$$\sigma_R = \sqrt{(72.68)^2 + (27.32)^2}$$

$$= 77.645 \ \frac{N}{mm^2}$$



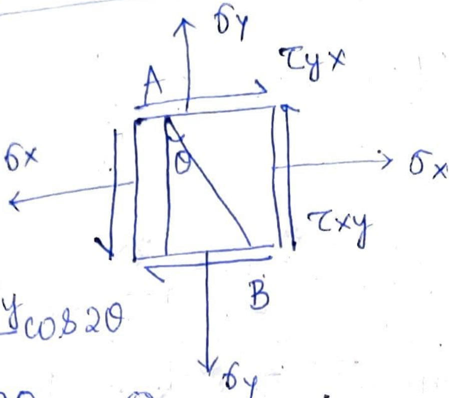
$$\tan\phi = \frac{72.68}{27.32}$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{72.68}{27.32}\right)$$

$$\boxed{\phi = 69.39^\circ}$$

Mohr's Circle & its application to solve problems of complex stresses

We know



$$\sigma_n = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \quad \text{---} (1)$$

$$\tau = \frac{\sigma_x-\sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta \quad \text{---} (ii)$$

from eqⁿ (1)

$$\left(\sigma_n - \frac{\sigma_x+\sigma_y}{2}\right)^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 \cos^2 2\theta + \tau_{xy}^2 \sin^2\theta + 2\cdot\frac{\sigma_x-\sigma_y}{2}\cdot\tau_{xy}\sin 2\theta\cos 2\theta$$

from eqⁿ (ii)

$$\tau^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 \sin^2 2\theta + \tau_{xy}^2\cos^2 2\theta - 2\cdot\frac{\sigma_x-\sigma_y}{2}\tau_{xy}\sin 2\theta\cos 2\theta$$

Adding these two

$$\boxed{\left(\sigma_n - \frac{\sigma_x+\sigma_y}{2}\right)^2 + \tau^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

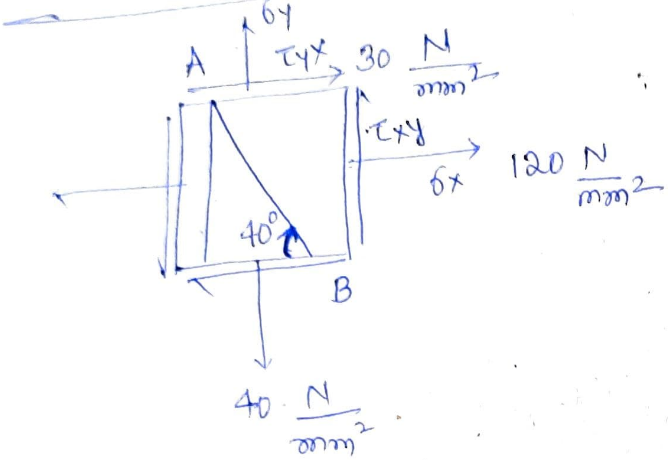
↑ Equation of Mohr's Circle.

$$\boxed{(x-x_0)^2 + (y-y_0)^2 = r^2}$$ eqⁿ of circle

$$r = \text{radius} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

centre of circle $\left(\frac{\sigma_x+\sigma_y}{2}, 0\right)$.

How to draw Mohr's Circle



$\therefore$ Radius of circle $(r) = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{\left(\frac{120-40}{2}\right)^2 + 30^2}$

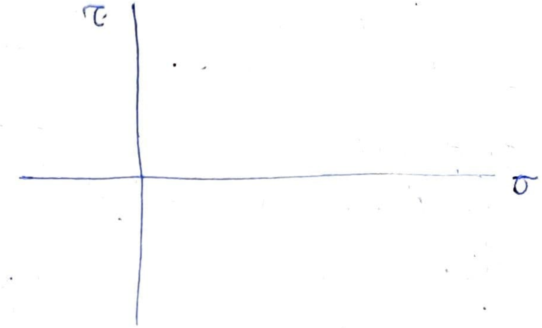
$= 50$ ~~mm~~ unit

centre of circle $\left(\frac{\sigma_x + \sigma_y}{2}, 0\right)$

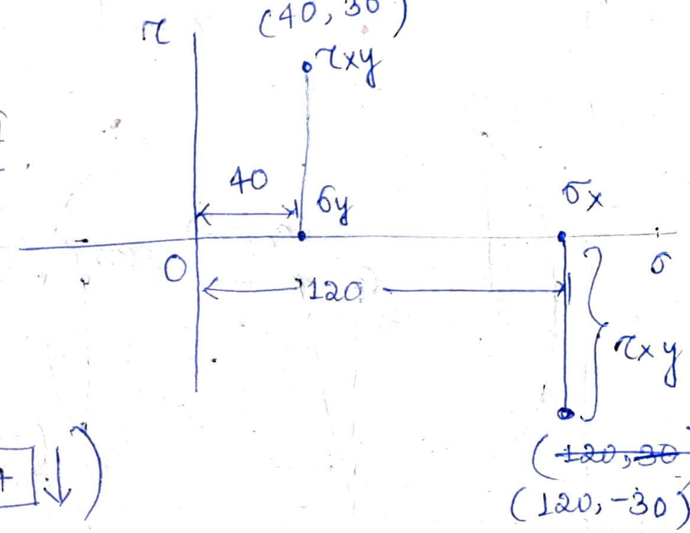
$= \left(\frac{120+40}{2}, 0\right)$

$= (80, 0)$

Step I

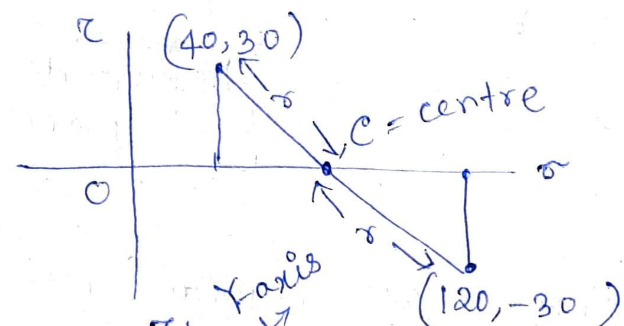


Step-II

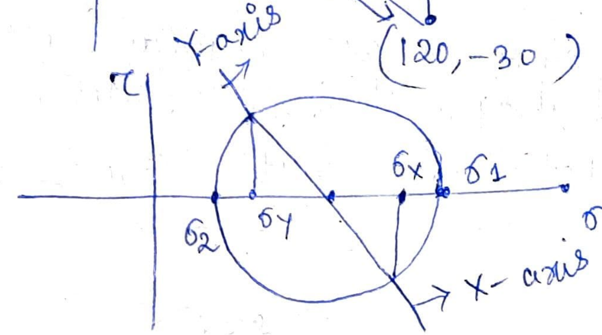


$(\uparrow \boxed{+} \downarrow)$

Step-III

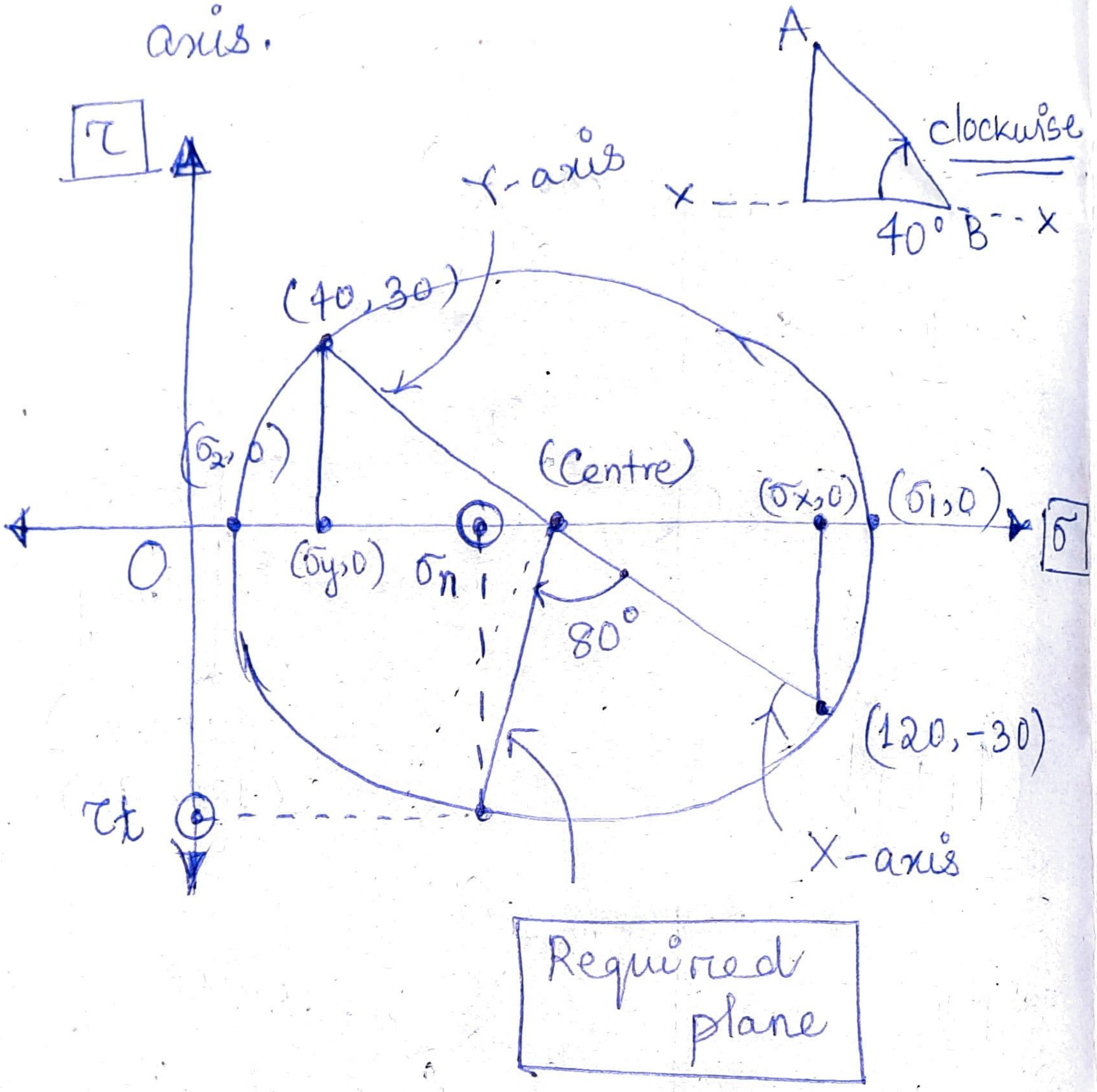


Step-IV



Stress Analysis on Mohr's Circle

Find out stresses along a plane AB which is inclined at 40° wrt X-axis.



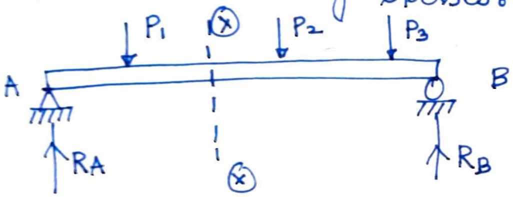
→ σ_n & τ_t are required stresses along a plane AB inclined at ~~40~~ 40° clockwise wrt X-axis.

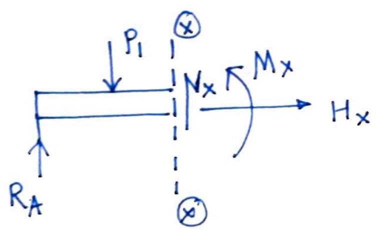
③ STRESSES IN BEAMS & SHAFTS

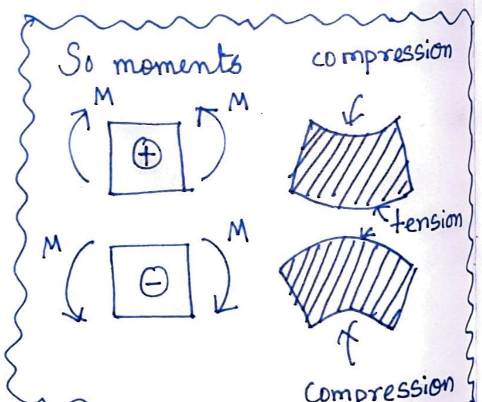
Stresses in beams due to bending

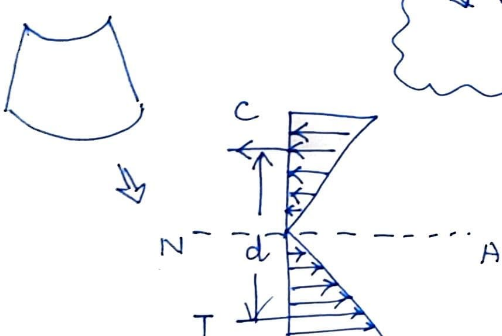
Bending stress in beams

When external loads acts on the beam, the SF & BM are setup at all sections of the beam. Due to SF & BM, the beam undergoes certain deformation. So the beam will offer resistance/stresses against deformation. The stresses introduced by bending moment are known as bending stresses.









C = compressive force
T = tensile force

Moment (M) = C × d
or
T × d

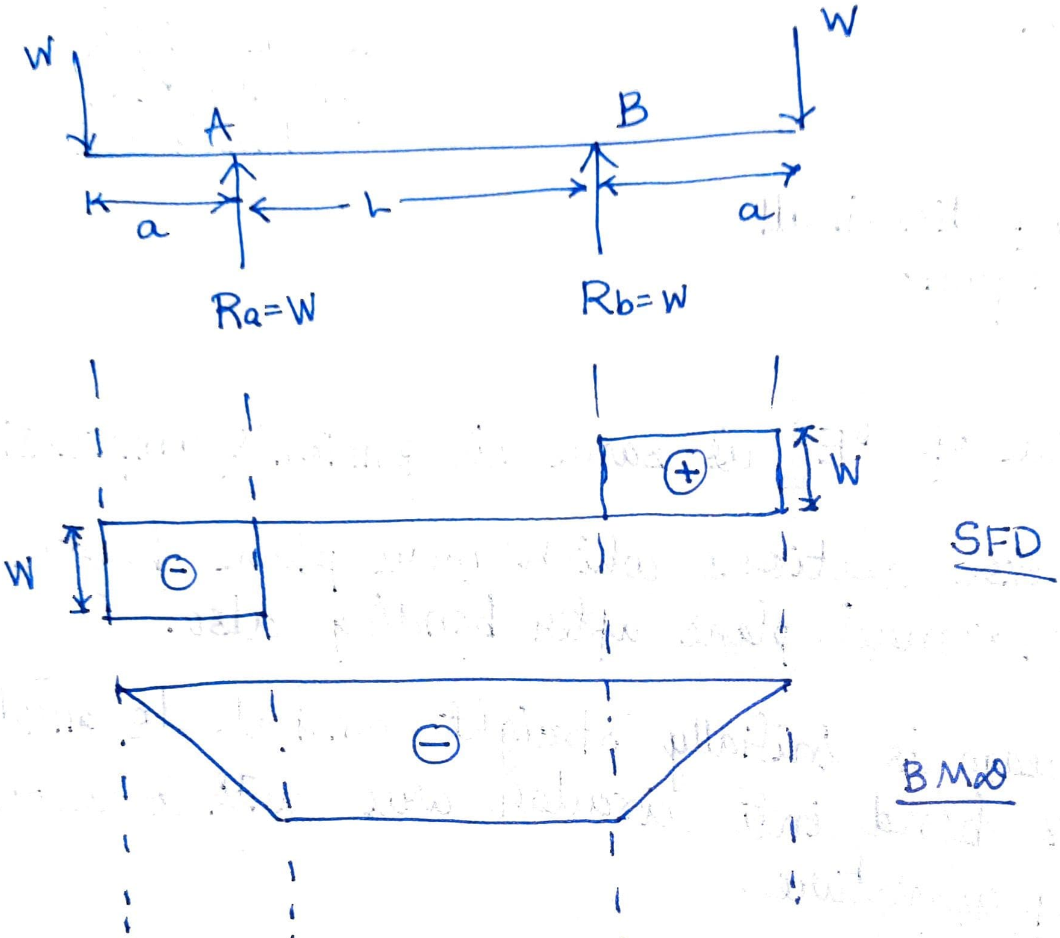
Theory of simple bending (Assumptions)

1. The material of the beam is homogeneous & isotropic.
   - homogeneous → material is same kind throughout
   - isotropic → elastic properties in all d^n are equal.
2. The value of "E" is same in tension & compression.
3. The traverse sections which were plane before bending, remain plane after bending also.
4. The beam is initially straight and all longitudinal filaments bend into circular arc with common centre of curvature.
5. The radius of curvature is large compared with the dimensions of c/s.
6. Each layer of the beam is free to expand/contract.

Moment of Resistance

Neutral Axis: The line of intersection of neutral layer with traverse section is known as neutral axis (NA) of that traverse section.

Pure bending / simple bending

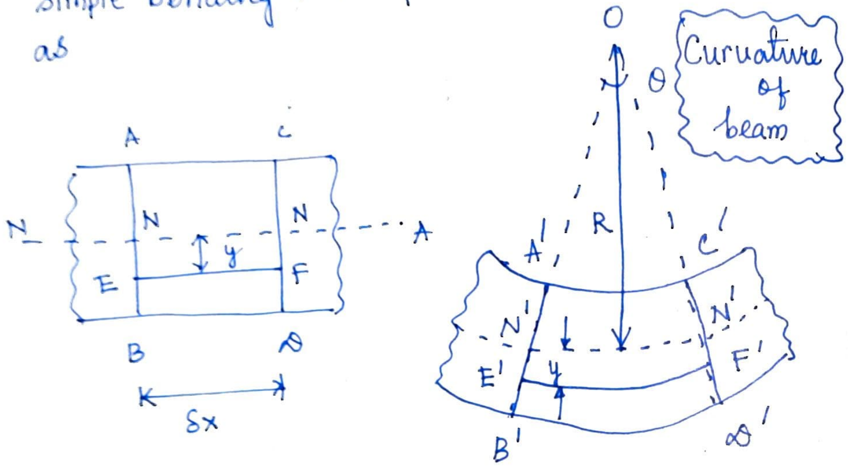


if a length of the beam is subjected to const. bending moment and no shear force, then that portion of the beam is ~~under~~ said to be in "PURE BENDING".

→ ~~Dure~~ Due to pure bending, the layer above NA is under compression & layer below NA is ~~ter~~ in tension stresses. Due to stresses ~~their~~ there will be forces. The total moment of these force about NA for a section is known as MOMENT OF RESISTANCE.

Equation for flexure

A small length 'δx' of beam subjected to a simple bending. The part 'δx' will be deformed as



R = radius of neutral layer N'N'
θ = angle subtended at O by A'B' & C'D'

original length EF = δx
length of neutral layer NN = δx

After bending length of neutral layer = N'N'

$N'N' = NN = \delta x = EF$

$N'N' = R\theta = NN$
$E'F' = (R+y)\theta$

$\therefore \boxed{\delta x = R\theta}$

increase in length of layer EF

$= E'F' - EF = (R+y)\theta - R\theta$
$= y\theta$

strain in layer EF $= \dfrac{\Delta L}{L}$

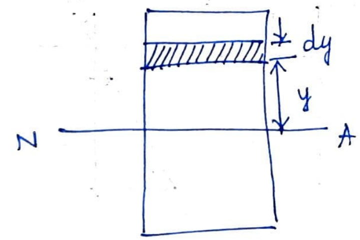
$= \dfrac{y\theta}{R\theta}$

$$\boxed{\varepsilon = \frac{y}{R}}$$

$\Rightarrow \dfrac{\sigma}{E} = \dfrac{y}{R}$

$\Rightarrow \boxed{\dfrac{\sigma}{y} = \dfrac{E}{R}}$

Moment of Resistance



Suppose area of that strip portion = dA

$\dfrac{\sigma}{y} = \dfrac{E}{R} \Rightarrow \sigma = \dfrac{E}{R}y$

$\Rightarrow$ force(F) $= \dfrac{E}{R} \cdot y \cdot dA$

Moment of force about NA (dM) $= \dfrac{E}{R} y \, dA \cdot y$

Total moment (M) $= \displaystyle\int \frac{E}{R} y^2 dA$

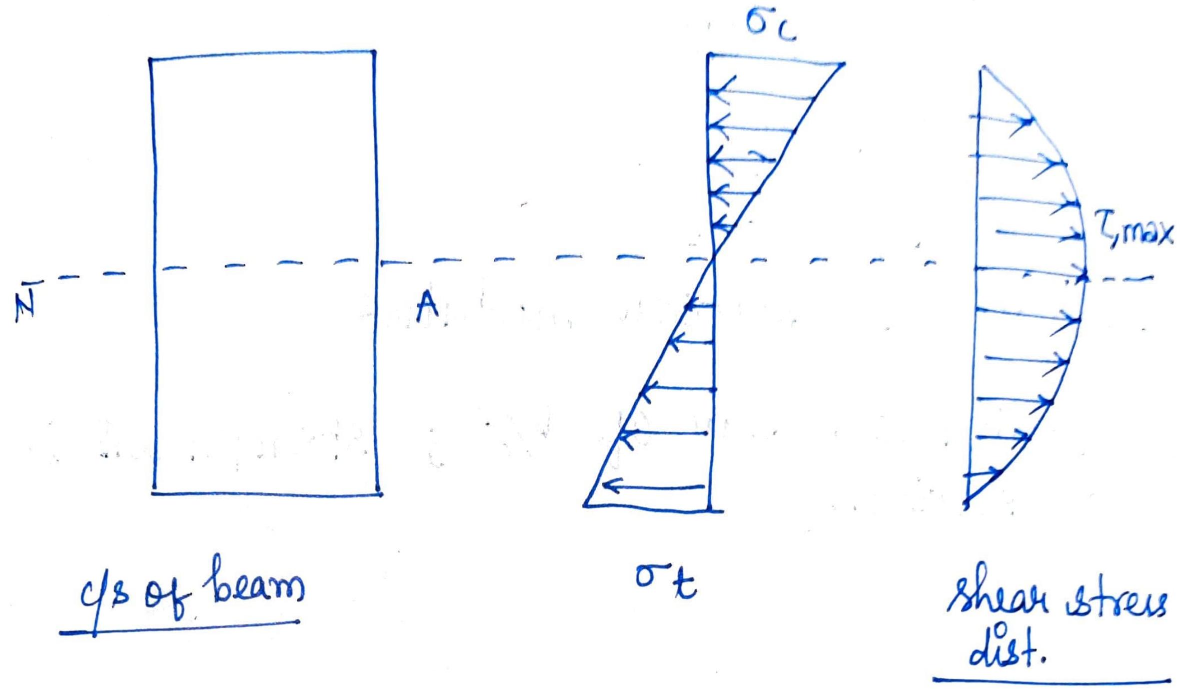
$= \dfrac{E}{R}\displaystyle\int y^2 dA = \dfrac{E}{R} \times I$

$\Rightarrow \dfrac{M}{I} = \dfrac{E}{R}$

but we know $\dfrac{\sigma}{y} = \dfrac{E}{R}$

So $\boxed{\dfrac{\sigma}{y} = \dfrac{M}{I} = \dfrac{E}{R}}$

Flexural Stress Distribution



Flexural Rigidity

→ It is defined as the product of modulus of elasticity (E) & moment of inertia (I).

$$\therefore \boxed{\text{Flexural Rigidity} = EI}$$

→ It is the resistance offered by beam to bending.

Significance of section modulus

we know $\dfrac{\sigma}{y} = \dfrac{M}{I}$

$$\Rightarrow \sigma = \frac{M}{I} \times y$$

$$\Rightarrow \sigma = \frac{M}{(I/y)}$$

$$\Rightarrow \boxed{\sigma = \frac{M}{Z}}$$

→ where Z = section modulus

→ Greater the value of 'Z'; stronger will be the section.

Problem

Q1/ A steel plate of width 120 mm and of thickness 20 mm is bent into a circular arc of radius 10 m. Determine the max. stress induced and the bending moment which will produce the max. stress. Take $E = 2\times10^5\ N/mm^2$.

Soln

$b = 120\ mm,\quad t = 20\ mm$

$$I = \frac{bt^3}{12} = 8\times10^4\ mm^4$$

$R = 10\,m$

$E = 2\times10^5\ N/mm^2$

$$\frac{\sigma}{y} = \frac{E}{R} \Rightarrow \sigma = \frac{E}{R}\times y$$

→ stress will be max. when y will be max.

→ 'y' will be max. at top layer / bottom layer.

$$y_{max} = \frac{t}{2} = \frac{20}{2} = 10\ mm$$

$$\sigma_{max} = \frac{E}{R}\times y_{max}$$

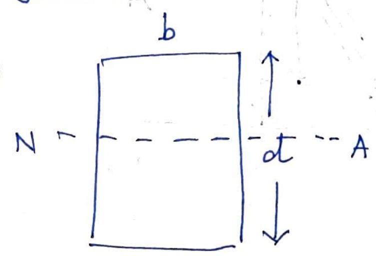
$$= \frac{2\times10^5}{10\times10^3}\times10 = 200\ N/mm^2.$$

$$\frac{M}{I} = \frac{E}{R}$$

$$\Rightarrow M = \frac{E}{R}\times I = \frac{2\times10^5}{10\times10^3}\times 8\times10^4 = 1.6\ kNm$$

Q2/ Section modulus of rectangular, hollow rectangular and hollow circular section.

(i)

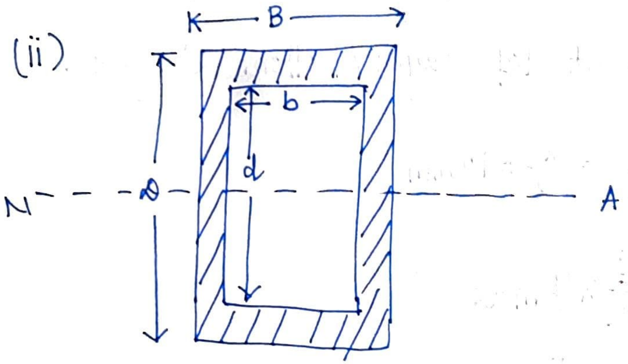


$$I = \frac{bd^3}{12}$$

$$y_{max} = \frac{d}{2}$$

$$Z = \frac{I}{y_{max}} = \frac{bd^3}{12}\times\frac{2}{d} = \frac{bd^2}{6}$$

(ii)

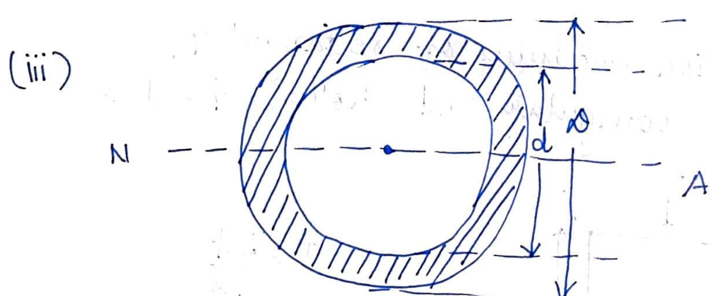


$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$y_{max} = \frac{D}{2}$$

$$Z = \frac{I}{y_{max}} = \frac{\frac{1}{12}(BD^3 - bd^3)}{D/2} = \frac{BD^3 - bd^3}{6D}$$

(iii)



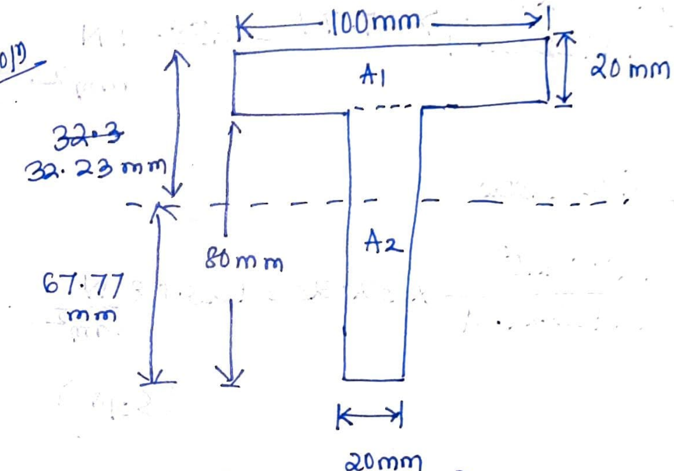
$$I = \frac{\pi}{64}\left[D^4 - d^4\right]$$

$$y_{max} = \frac{D}{2}$$

$$Z = \frac{I}{y_{max}} = \frac{\pi}{32D}\left(D^4 - d^4\right)$$

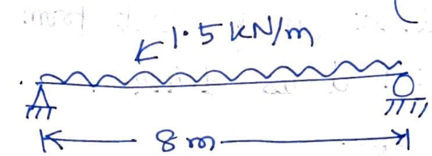
Q3/ A "T" beam is simply supported of span 8m. The beam carries udl of 1.5 kN/m length of entire span. Determine the max. tensile & compressive stresses.

Soln



(c/s of beam)

$\bar{y} = 32.23$ mm $\qquad \left(\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}\right)$



L = 8m

w = 1500 N/m

$\therefore \boxed{y_{max} = 67.77 \text{ mm}}$

$$I = \frac{100 \times 20^3}{12} + 100 \times 20\,(22.23)^2 + \frac{20 \times 80^3}{12} + 20 \times 80\,(27.77)^2 = 3142222.4 \text{ mm}^4$$

$$\text{Max BM} = \frac{wl^2}{8} = \frac{1500 \times 8^2}{8} = 12000 \text{ Nm}$$

using the relation (max. tensile stress)

$$\frac{\sigma}{y} = \frac{M}{I}$$

$$\Rightarrow \sigma = \frac{12000 \times 10^3}{3142222.4} \times 67.77 = 258.81 \ N/mm^2$$

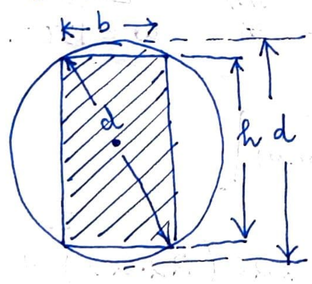
Max. compressive stress

$$\sigma = \frac{12000 \times 10^3}{3142222.4} \times 32.23 = 123.08 \ N/mm^2$$

(soln)

Q4) Prove that ratio of depth to width of the strongest beam that can be cut from a circular log of diameter d is 1.414.

soln



suppose d = dia. of log.
b = width of strongest section
h = depth

Section modulus of rectangular section

$$Z = \frac{I}{y}$$

$$= \frac{(bh^3/12)}{(h/2)} = \frac{bh^2}{6}$$

from Δ law we can get

$$b^2 + h^2 = d^2$$

$$\Rightarrow h^2 = d^2 - b^2$$

$$\therefore \text{So } Z = \frac{bh^2}{6} = \frac{b(d^2-b^2)}{6} = \frac{bd^2 - b^3}{6}$$

here 'd' is constant & 'b' is variable.

For beam to be strongest, Z to be maximum.

$$\frac{dZ}{db} = 0 \Rightarrow \frac{d^2}{6} - \frac{3b^2}{6} = 0$$

$$\Rightarrow \boxed{d^2 = 3b^2}$$

but we know $b^2 + h^2 = d^2$

$$\Rightarrow 2b^2 = h^2$$

$$\Rightarrow \frac{b^2}{h^2} = \frac{1}{2}$$

$$\Rightarrow h^2/b^2 = 2$$

$$\Rightarrow \boxed{\frac{h}{b} = \sqrt{2} = 1.414}$$

soln

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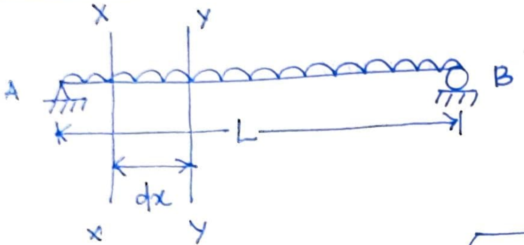
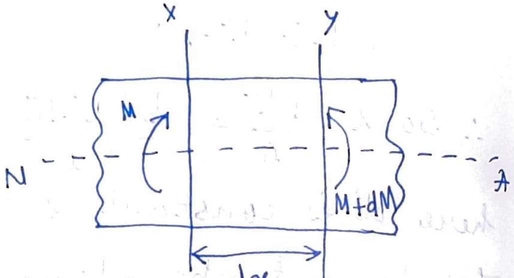
3.2 SHEAR STRESSES IN BEAMS

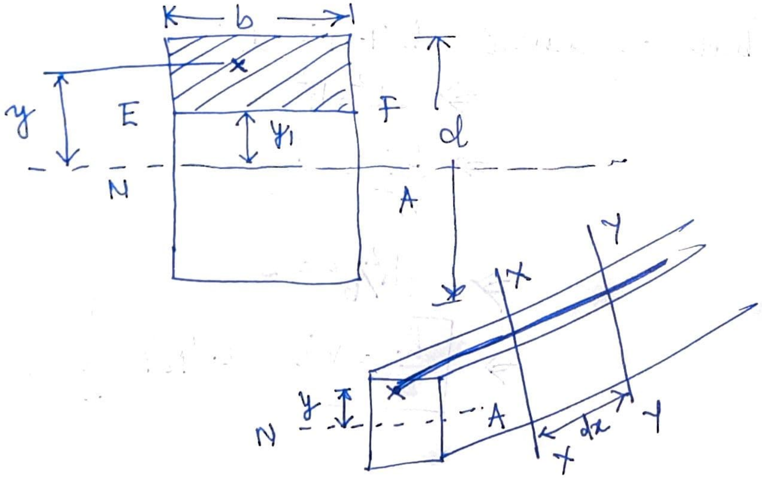
Shear stress distribution in beams of rectangular section





Section XX	Section YY
SF = F	SF = F+dF
BM = M	BM = M+dM
Stress = σ	stress = $\sigma+d\sigma$

So take the Section XX of beam & find out the shear stress on the section AB at a distance y_1 from NA.



we know $\frac{M}{I} = \frac{\sigma}{y}$

$$\Rightarrow \sigma = \frac{M}{I} \cdot y$$

where y = dist. of elemental cylinder from NA

Similarly stress on section Y-Y

$$\sigma + d\sigma = \frac{M+dM}{I} \times y$$

∴ Net unbalanced force on elemental cylinder

$$\frac{M+dM}{I} \times y \times dA - \frac{M}{I} y \, dA$$

$$= \frac{dM}{I} \times y \times dA$$

$$\text{Total unbalanced force} = \int \frac{dM}{I} \times y \times dA$$

$$= \frac{dM}{I} \times A \times \bar{y} \quad \text{——— (1)}$$

A = area of section above EF.

$\bar{y}$ = distance of CG of area A from NA.

τ = Shear stress at level EF.

$$SF = b \times dx \times \tau \quad \text{——— (2)}$$

from (1) & (2)

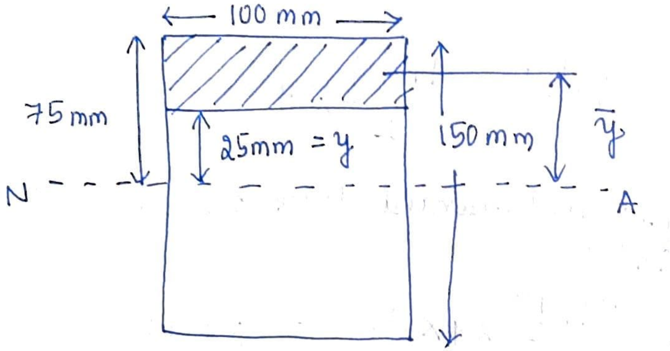
$$\tau \times b \times dx = \frac{dM}{I} \times A \times \bar{y}$$

$$\Rightarrow \boxed{\tau = \frac{FA\bar{y}}{Ib}} \qquad \boxed{F = \frac{dM}{dx}}$$

Q2/

A wooden beam of 100 mm width and 150 mm deep is simply supported over a span of 4 m. If SF at a section of beam is 4500 N, find shear stress at a dist. of 25 mm above NA.

soln



$b = 100$ mm
$d = 150$ mm
$SF = F = 4500$ N
τ = shear stress at a distance of 25 mm above NA.

we know $\tau = \dfrac{FA\bar{y}}{Ib}$

$A = 100 \times 50 = 5000 \text{ mm}^2$

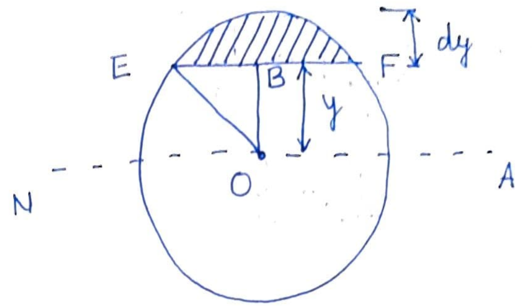
$\bar{y} = 25 + \dfrac{50}{2} = 50$ mm

I = MOI of total section $= \dfrac{100 \times 150^3}{12}$

$= 28125000 \text{ mm}^4$

$\tau = \dfrac{FA\bar{y}}{Ib} = 0.4 \text{ N/mm}^2$ (Ans)

Shear stress distribution in circular section



Shear force at a section EF /

$\tau = \dfrac{FA\bar{y}}{Ib}$

Consider a ~~section~~ Level EF at 'y' from NA.

$dA = EF \times dy = b \times dy = 2 \times EB \times dy$

$= 2\sqrt{R^2 - y^2}\, dy$

Moment of this area ~~at a distance~~ about NA

$= y \times dA$

$= y \times 2\sqrt{R^2-y^2}\, dy = 2y\sqrt{R^2-y^2}\, dy$

Total area moment $= A\bar{y} = \int_{y}^{R} 2y\sqrt{R^2-y^2}\, dy$

$\therefore A\bar{y} = \dfrac{2}{3}\left(R^2 - y^2\right)^{3/2}$

$\therefore \tau = F \times \dfrac{2}{3}\left(R^2-y^2\right)^{3/2} \Big/ I \times 2\sqrt{R^2-y^2}$

$$\boxed{\tau = \frac{F}{3I}\left(R^2 - y^2\right)}$$

at NA, $y=0$

$$\tau_{max} = \frac{F}{3I} \times R^2 = \frac{F}{3 \times \frac{\pi D^4}{64}} \times \left(\frac{D}{2}\right)^2$$

$$= \frac{F}{\pi R^2} \times \frac{4}{3}$$

$$\tau_{avg} = \frac{F}{\pi R^2}$$

$$\therefore \boxed{\tau_{max} = \frac{4}{3}\tau_{avg}}$$

Problem 2

A circular beam of 100mm dia is subjected to a SF of 5kN. Find τ_{avg}, τ_{max} and τ at 40 mm from NA.

Soln

$$\tau_{avg} = \frac{F}{\pi R^2} = \frac{5000}{\pi \times (50)^2} = 0.6366\ \frac{N}{mm^2}$$

$$\tau_{max} = \frac{4}{3}\tau_{avg} = \frac{4}{3} \times 0.6366 = 0.8488\ \frac{N}{mm^2}$$

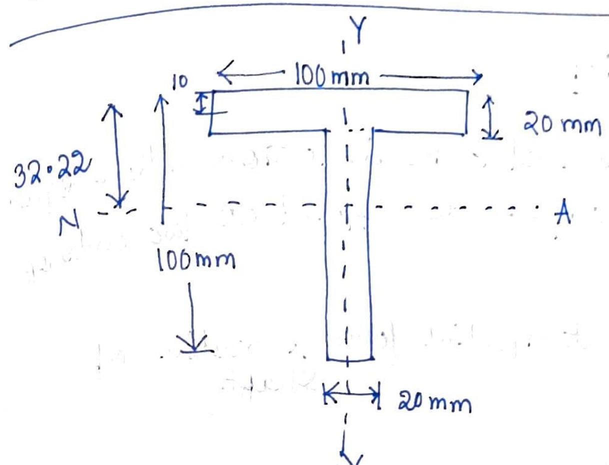
τ at ~~30~~ 40mm from NA $= \frac{F}{3I}(R^2 - y^2)$

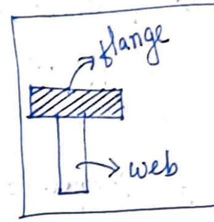
$$= \frac{5000}{3 \times \frac{\pi D^4}{64}}(50^2 - 40^2)$$

$$= 0.3055\ \frac{N}{mm^2}$$

(NA & junction)

Shear stress distribution in beams of standard sections symmetrical about vertical axis





given $F = 50\,kN$, $I = 314.221 \times 10^4\ mm^4$
(if not given find out MOI)

$$y^* = \text{CG from top} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$= \frac{(100 \times 20) \times 10 + 20 \times 80 \times 60}{2000 + 1600}$$

$$= 32.22\ mm \text{ (from top)}$$

τ at ~~flange~~ (junction) $\tau = \frac{FA\bar{y}}{Ib} = \frac{\cancel{2000} \times 5000 \times 2000 \times 22.22}{314.221 \times 10^4 \times 100}$

$$= 7.07\ N/mm^2$$

20
12.22

{τ at ~~web~~ NA} $\tau = \frac{5000 \times \{100\ \cancel{80} \times 20 \times 22.22 + 12.22 \times 20 \times 6.11\}}{314.221 \times 10^4 \times 20}$

(max at junction of flange & web)

(soln)

—x—

3.3 ▶ Stresses in shafts due to torsion

Concept of torsion

→ A shaft is said to be in torsion when equal & opposite torques are applied at two ends of shafts.

→ Torque = tangential force × radius of shaft

Basic assumption of pure ~~to~~ torsion

→ The material is homogeneous and isotropic.

→ Hooke's Law is obeyed by the material.

→ The shaft is circular in section.

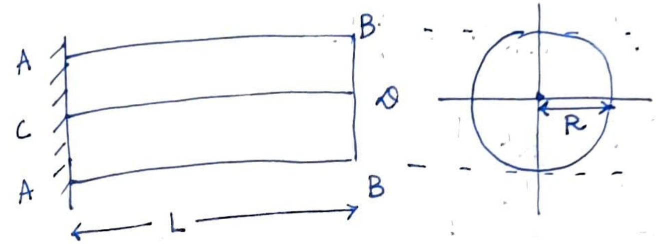
→ The c/s of the shaft remains uniform throughout.

→ The shaft is subjected to pure torsion only.

→ The shaft is not subjected to any initial torque.

→ The transverse section which were plane before appⁿ of torque remain plane after appⁿ of torque.

Torsion of circular sections



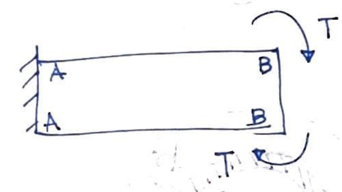
AA = fixed end ~~at~~

BB = free end

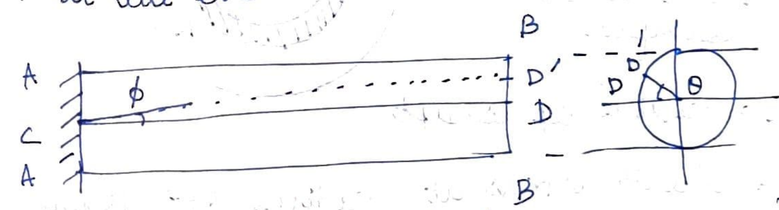
CD = any line ~~of~~ at outer surface of the shaft.

R = radius of shaft

L = length of shaft



Now let the shaft subjected to torque 'T' at the end BB.



(after application of torque)

ϕ = shear strain

θ = angle of twist

shear strain $\phi = \frac{DD'}{L}$

but $DD' = R \times \theta = \phi L$

$$\Rightarrow \boxed{\phi = \frac{R\theta}{L}}$$

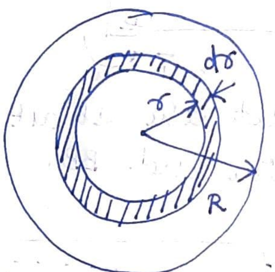
Modulus of Rigidity $(G) = \frac{\tau}{\phi}$

$$\Rightarrow G = \frac{\tau}{\left(\frac{R\theta}{L}\right)}$$

$$\Rightarrow \boxed{\frac{\tau}{R} = \frac{G\theta}{L}} \text{ ——— (1)}$$

Consider a ring of thickness dr at r from centre.

τ = max. shear stress induced at the outer surface



R = radius of shaft

q = shear stress at radius 'r' from the centre.

$dA = 2\pi r \cdot dr$

As we know $\frac{\tau}{R} = \frac{q}{r}$

$\Rightarrow q = \frac{\tau}{R} \cdot r$

Turning force on circular ring

$= q \times dA$

$= \frac{\tau}{R} \cdot r \cdot (2\pi r)\, dr$

$= \frac{\tau}{R} \cdot 2\pi r^2\, dr$

Torsion $dT = \cancel{dF} F \times r$

$= \frac{\tau}{R} \cdot (2\pi r^2)\, dr \times r$

$= \frac{\tau}{R} \times 2\pi r^3\, dr$

$\therefore$ Torsion $T = \int_0^R dT$

$= \int_0^R \frac{\tau}{R}\, 2\pi r^3\, dr$

$= \frac{\tau}{R} \cdot 2\pi \left[\frac{r^4}{4}\right]_0^R$

torsion of solid circular section $= \frac{\tau}{R} \cdot 2\pi \cdot \frac{R^4}{4} = \frac{\tau \pi R^3}{2}$

$= \frac{\tau}{2} \times \pi \left(\frac{D}{2}\right)^3 = \cancel{\frac{\tau \pi D^3}{4}} = \cancel{\frac{\tau \pi D^4}{128 D}}$

$= \cancel{\tau \frac{16}{D} \times \left(\frac{\pi D^4}{64}\right)}$

$$\therefore \boxed{T = \frac{\tau}{2} \pi \frac{D^3}{8} = \frac{\tau \pi D^3}{16}}$$

$$\cancel{\frac{T}{I_p} = \frac{16\tau}{D}}$$

from previous discussion we know

$$dT = \frac{\tau}{R} 2\pi r^3 . dr$$

$$= \frac{\tau}{R} . 2\pi r . dr . r^2$$

$$= \frac{\tau}{R} r^2 dA$$

$\therefore$ Total torque $T = \int_0^R dT = \frac{\tau}{R} \int_0^R r^2 dA$

$$\Rightarrow \boxed{\frac{T}{I_p} = \frac{\tau}{R}} \quad —(2)$$

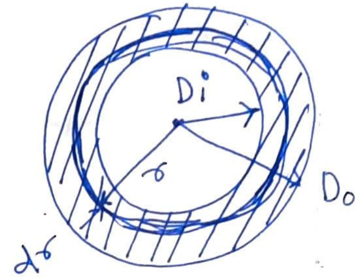
where I_p = polar moment of inertia = $\int_0^R r^2 dA$

from eqⁿ (1) & (2)

$$\boxed{\frac{\tau}{R} = \frac{T}{I_p} = \frac{G\theta}{L}}$$

$\hookrightarrow$ **EQᴺ OF TORSION**

Torsion of hollow circular section



Di = inner diameter
Do = outer diameter

for solid circular section, we know

$$T = \frac{\tau \pi D^3}{16}$$

for hollow section take a circular ring section at 'r' distance.

$$\boxed{\frac{\tau}{R_o} = \frac{q}{r}} \rightarrow \text{shear of small ring}$$

$$\Rightarrow q = \frac{\tau r}{R_o}$$

force on that ring $= q \times dA$

$$= \frac{\tau r}{R_o} \times (2\pi r) \times dr$$

torque $dT = \frac{2\pi\tau}{R_o} r^3 dr$

$$\therefore T = \int_{R_i}^{R_o} dT = \frac{2\pi\tau}{R_o} \left[\frac{r^4}{4} \right]_{R_i}^{R_o}$$

$$\Rightarrow \boxed{T = \frac{\pi}{2} \times \tau \times \left[\frac{R_o^4 - R_i^4}{R_o} \right]}$$

total torque T

$$= \frac{\pi}{2} \times \tau \left[\frac{R_o^4 - R_i^4}{R_o} \right]$$

$$\boxed{T = \frac{\pi}{16} \times \tau \left[\frac{D_o^4 - D_i^4}{D_o} \right]}$$

Polar Modulus (Z_p) : $\boxed{Z_p = \frac{I_p}{R}}$

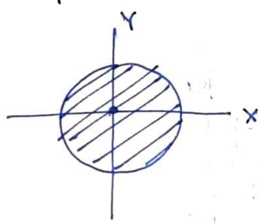
Torsional Rigidity $= \boxed{G \times I_p}$

We know $\frac{\tau}{r} = \frac{T}{I_p} = \frac{G\theta}{L}$

$\therefore \boxed{G I_p = \frac{TL}{\theta}}$

Polar moment of Inertia (I_p)

→ It is the measure of an objects ability to resist torsion as a function of its shape.



$\boxed{I_p = I_{xx} + I_{yy}}$

we know $I_{xx} = \frac{\pi D^4}{64}$

$I_{yy} = \frac{\pi D^4}{64}$

$\therefore I_p = 2 \times \frac{\pi D^4}{64} = \frac{\pi D^4}{32}$

Power transmitted by shaft

$$\boxed{P = \frac{2\pi N T}{60} \text{ watts}}$$

where N = rpm of shaft

T = mean torque transmitted in (N-m)

ω = angular speed of shaft

Numerical Problems

Q1 A solid steel shaft has to transmit 75 kW at 200 rpm. Taking allowable shear stress as 70 N/mm^2. find suitable dia of shaft if max. torque transmitted at each rev. exceeds the mean by 30%.

Soln

$P = 75000$ W

$N = 200$ rpm

$\tau = 70\ N/mm^2$

$T_{max} = T \times 1.3 = 1.3T$

$P = \frac{2\pi N T}{60} \Rightarrow T = 3580980$ Nmm

$T_{max} = 1.3T = 4655274$ Nmm

max torque transmitted

$T_{max} = \frac{\pi}{16} \tau D^3 \Rightarrow \boxed{D = 70 \text{ mm}}$

Q2/

A hollow shaft having inside diameter 60% of its outer diameter is to replace a solid shaft transmitting the same power at same speed. Calculate the % saving in material, if the material to be used is also the same.

Soln Out dia = D_o

inside dia = D_i = 60% D_o

$= 0.6 D_o$

$\therefore P = \frac{2\pi NT}{60}$

$T = \frac{\pi}{16}\tau D^3$ (solid shaft)

$T = \frac{\pi}{16}\tau\left[\frac{D_o^4 - D_i^4}{D_o}\right]$

$= \frac{\pi}{16}\tau \times 0.8704\, D_o^3$

Since torque transmitted is the same

$\frac{\pi}{16}\tau D^3 = \frac{\pi}{16}\tau \times 0.8704\, D_o^3$

$\Rightarrow D = 0.9548\, D_o$

Area of solid shaft $= \frac{\pi}{4}D^2$

$= 0.716\, D_o^2$

Area of hollow shaft

$= \frac{\pi}{4}(D_o^2 - D_i^2)$

$= \frac{\pi}{4}(D_o^2 - (0.6D_o)^2) = 0.502\, D_o^2$

for the shaft of same material, the weight of the shaft is proportional to the areas.

Saving in material = saving in area

$= \frac{\text{Area of solid shaft} - \text{Area of hollow shaft}}{\text{Area of solid shaft}}$

$= \frac{0.716 D_o^2 - 0.502 D_o^2}{0.716 D_o^2}$

$= 0.2988$

$\therefore$ Percentage saving in material

$= 0.2988 \times 100 = 29.88\%$.

(soln)

$\boxed{\text{Note}}$:

→ Numerical problems on ~~tosi~~ torsion eqn, polar modulus and torsional rigidity can be practiced.

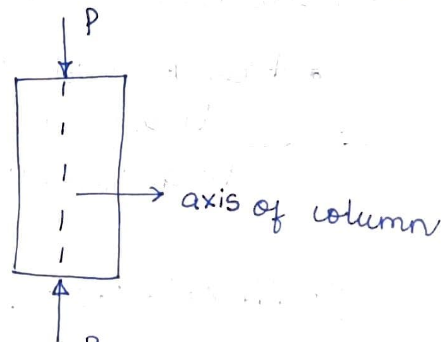
$$\sigma_b = \frac{M}{Z} = \frac{M}{I/(\pm y)}$$

3.4 ▶ Combined bending & direct stresses

Combination of stresses

Combined direct & bending stresses

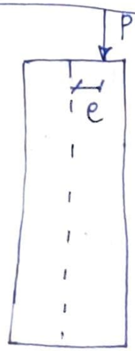
→ ~~It is~~ the combination of stresses include both direct stress and bending stress.



P = axial compressive load on column

A = c/s area of column

$$\boxed{\text{direct stress } (\sigma_0) = \frac{P}{A}}$$

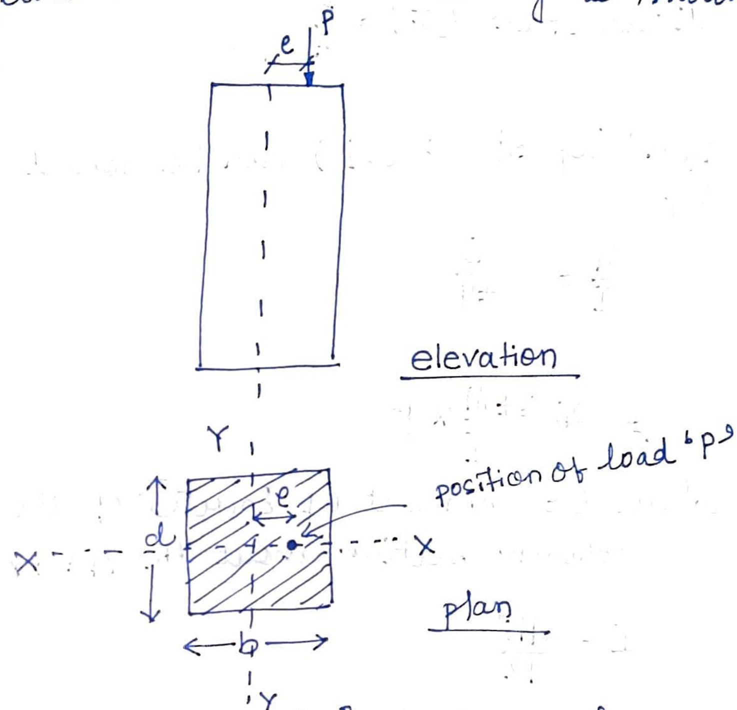


Suppose load P acts at a distance of 'e' from axis of the column.

BM at the axis (M) = Pe

$$\boxed{\text{bending stress } (\sigma_b) = \pm\frac{M}{Z} = \pm\frac{Pe}{I}\times y}$$

Suppose a column of rectangular section subjected to an eccentric loading as shown.



where P = ~~et~~ eccentric load on column

e = eccentricity of the load

σ_0 = direct stress

σ_b = bending stress

b = width of column

d = depth of column

A = area of column

= b × d

Moment due to eccentric load 'P'

$M =$ load $\times$ eccentricity

$= P \times e$

direct stress $(\sigma_0) = \frac{P}{A}$

bending stress (σ_b) can be found out by

$$\frac{M}{I} = \frac{\sigma_b}{\pm y}$$

$$\Rightarrow \sigma_b = \pm \frac{M}{I} \times y$$

where I = moment of inertia of the column section about the NA YY

$$I = \frac{db^3}{12}$$

$$\therefore \sigma_b = \pm \frac{M}{\frac{db^3}{12}} \times y = \pm \frac{12M}{db^3} \times y$$

The bending stress at the extreme is obtained by ~~y = d/2~~ $y = \frac{b}{2}$

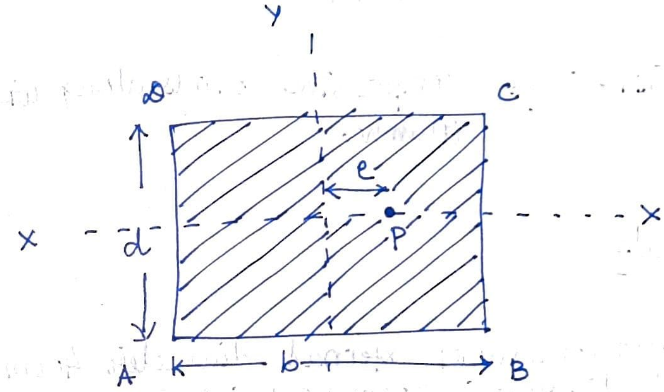
$$\sigma_b = \pm \frac{12M}{db^3} \times \frac{b}{2} = \pm \frac{6M}{db^2}$$

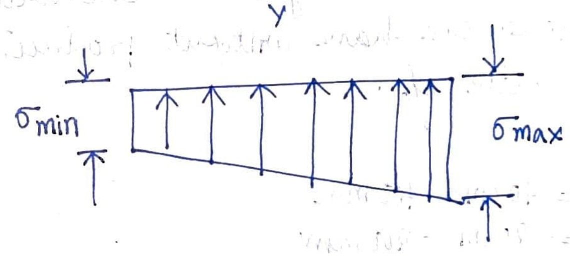
$$\boxed{\sigma_b = \pm \frac{6 \times P \times e}{d \times b \times b} = \pm \frac{6 P \times e}{A \times b}}$$

Max. & min stresses in section

let σ_{max} = max. stress

σ_{min} = min stress





$$\sigma_{max} = \sigma_0 + \sigma_b \quad (\text{along BC})$$

$$= \frac{P}{A} + \frac{6Pe}{Ab}$$

$$= \frac{P}{A}\left(1 + \frac{6e}{b}\right)$$

$$\sigma_{min} = \sigma_0 - \sigma_b \quad (\text{along AD})$$

$$= \frac{P}{A} - \frac{6Pe}{Ab}$$

$$= \frac{P}{A}\left(1 - \frac{6e}{b}\right)$$

NOTE

* $\sigma_{min} < 0 \Rightarrow$ stress along AD tensile.
* $\sigma_{min} = 0 \Rightarrow$ no tensile stress along width of column.
* $\sigma_{min} > 0 \Rightarrow$ compressive stress along width of column.

Numericals

Q/ A short column of external diameter 40 cm & internal diameter 20 cm carries an eccentric load of 80 kN. Find the greatest eccentricity which the load can have without producing tension on the c/s.

Soln

$D = 40\text{ cm} = 400\text{ mm}$

$d = 20\text{ cm} = 200\text{ mm}$

Area of c/s

$$A = \frac{\pi}{4}(D^2 - d^2)$$

$$= \frac{\pi}{4}(400^2 - 200^2)$$

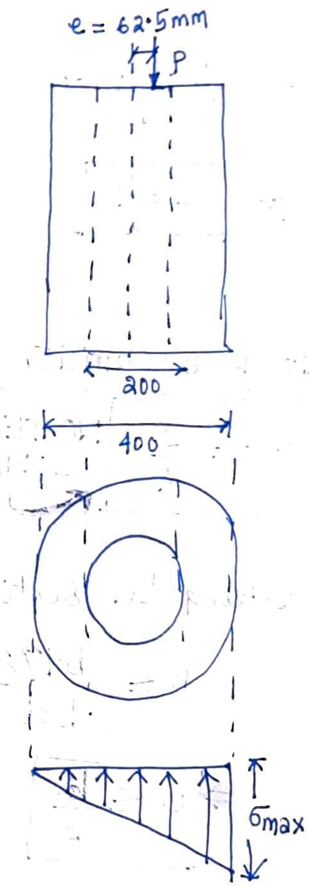
$$= 30000\pi \text{ mm}^2$$

Moment of inertia

$$I_p = \frac{\pi}{64}(D^4 - d^4)$$

$$= 3.75 \times 10^8 \pi \text{ mm}^4$$

Eccentric load $P = 80\text{ kN} = 80000\text{ N}$



$$\sigma_0 = \frac{P}{A} = \frac{80000}{30000\pi}$$

$M = Pe$

$$\Rightarrow \frac{M}{I} = \frac{\sigma_b}{y}$$

$$\Rightarrow \sigma_b = \frac{M}{I}(\pm y)$$

$$= \frac{M}{I}(\pm 200)$$

(where $y = \pm \frac{D}{2}$)

$$\sigma_b = \frac{80000 \times e \times 200}{3.75 \times 10^8 \times \pi}$$

$\sigma_{min} = \sigma_0 - \sigma_b$

for NO TENSION, $\sigma_{min} = 0$

$\Rightarrow e = 62.5\text{ mm}$

(soln)

Condition for no tension

$\sigma_{min} = 0 \Rightarrow \boxed{\sigma_0 - \sigma_b = 0}$

$\Rightarrow \frac{P}{A}\left(1 - \frac{6e}{b}\right) = 0$

$\sigma_0 = \frac{P}{A}$

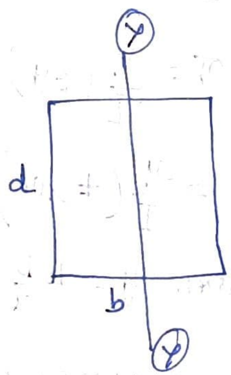
$\sigma_b = \frac{6Pe}{Ab}$

$\Rightarrow 1 - \frac{6e}{b} = 0$

$\Rightarrow \boxed{e = \frac{b}{6}}$

Limit of eccentricity

$\boxed{e \le \frac{Z}{A}}$ NO TENSION CONDITION

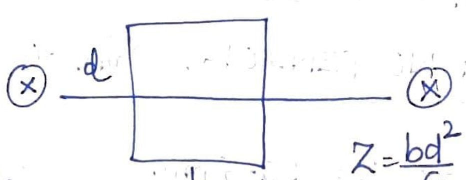


for rectangular section

$e \le \frac{\cancel{bd^2/6}\ db^2/6}{bd}$

$\Rightarrow \boxed{e \le \frac{b}{6}}$

or for x-x

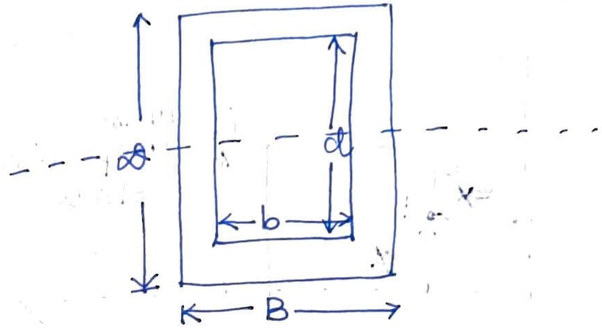


$Z = \frac{bd^2}{6}$

$e \le \frac{bd^2/6}{bd}$

$\rightarrow \boxed{e \le d/6}$

for hollow rectangular section



$Z = \cancel{\frac{BD^2 - bd^2}{6}}$

$Z = \frac{I}{y_{max}}$

$= (BD^3 - bd^3)/12$

$y_{max} = \frac{D}{2}$

$\therefore Z = \frac{(BD^3 - bd^3) \times 2}{12 \times D} = \frac{BD^3 - bd^3}{6D}$

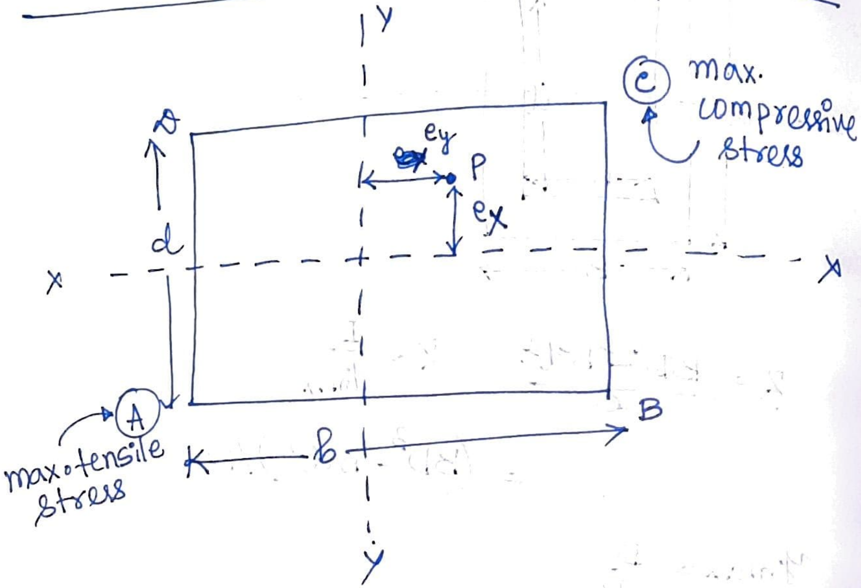
$A = BD - bd$

$\therefore e \le \frac{Z}{A}$

$\Rightarrow \boxed{e \le \frac{(BD^3 - bd^3)}{6D \times (BD - bd)}}$

Note: Similarly limit of 'e' can be found out of circular section.

column is subjected to load which is eccentric to both axes



$$I_{xx} = \frac{bd^3}{12}$$

$$I_{yy} = \frac{db^3}{12}$$

$$\sigma_0 = \frac{P}{A}$$

$$\sigma_{bx} = \frac{M_x y}{I_{xx}} = \frac{P e_x y}{I_{xx}}$$

here 'y' varies from $-\frac{d}{2}$ to $+\frac{d}{2}$.

$$\sigma_{by} = \frac{M_y x}{I_{yy}} = \frac{P e_y x}{I_{yy}}$$

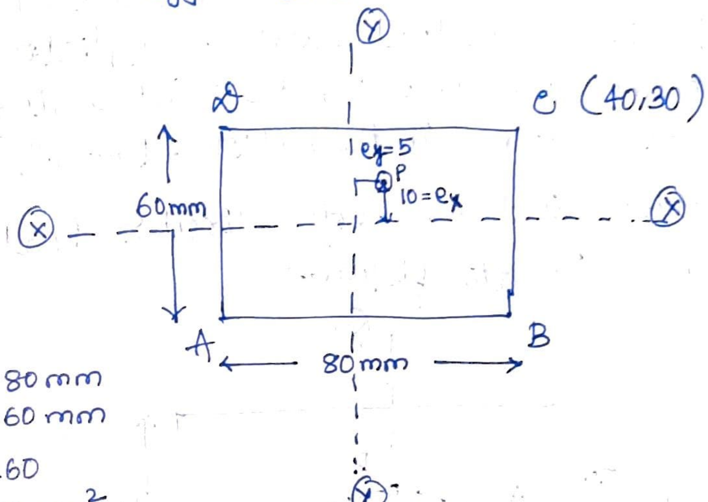
'x' varies from $-\frac{b}{2}$ to $+\frac{b}{2}$.

Resultant stress

$$= \sigma_0 \pm \sigma_{by} \pm \sigma_{bx}$$

$$= \frac{P}{A} \pm \frac{P e_y x}{I_{yy}} \pm \frac{P e_x y}{I_{xx}}$$

Q.



where b = 80 mm
d = 60 mm

$A = 80 \times 60$
$= 4800\ mm^2$

$P = 40\ KN = 40000\ N$

$e_x = 10\ mm,\ e_y = 5\ mm$

$M_x = Pe_x = 400000\ Nmm$

$M_y = Pe_y = 40000 \times 5 = 200000\ Nmm$

$$I_{xx} = \frac{bd^3}{12} = \frac{80 \times 60^3}{12} = 1440000\ mm^4$$

$$I_{yy} = \frac{db^3}{12} = \frac{60 \times 80^3}{12} = 2560000\ mm^4$$

at 'C' (40,30) stress $= \frac{P}{A} + \frac{M_y x}{I_{yy}} + \frac{M_x y}{I_{xx}}$

$= 19.785\ N/mm^2$ (soln)

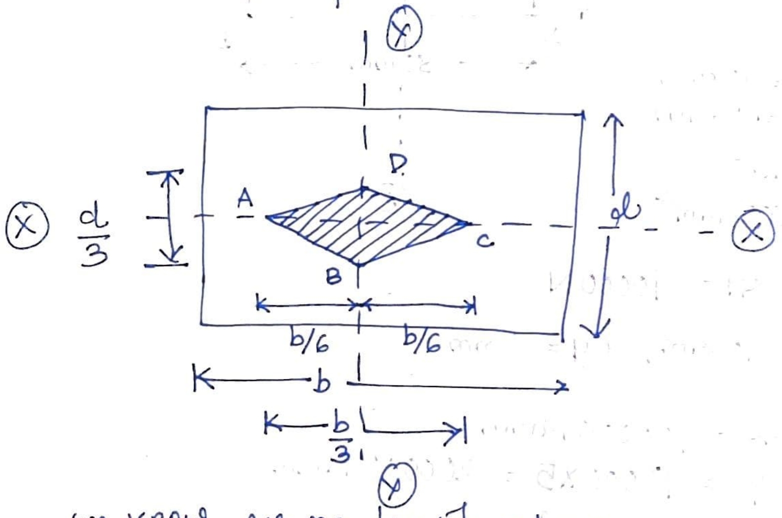
Similarly stress at 'A' can be found out.

'3'

Middle $\frac{1}{4}$ Rule / Middle '4' Rule

(Kernel Section)

→ The cement concrete columns are weak in tension. Hence load must be applied on these column in such a way that there is no tension stress anywhere in the section. But when eccentric loading is there, both direct & bending stress came in the picture.



we know for no tensile stress

$\sigma_{min} \geq 0$

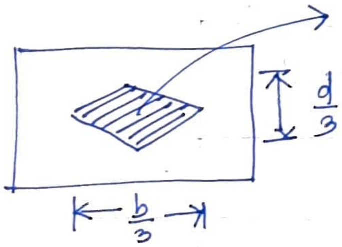
$\Rightarrow \frac{P}{A}\left(1 - \frac{6e}{b}\right) \geq 0$

$\Rightarrow \boxed{e \leq \frac{b}{6}}$

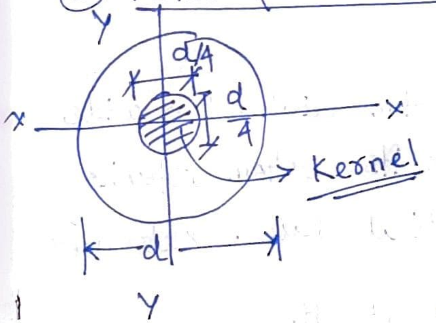
hence for any load is applied at any distance less than '$\frac{b}{6}$'; on either side of Y-Y the stress are wholely compressive. In combined that is $\frac{b}{6} + \frac{b}{6} = \frac{b}{3}$ (Middle third Rule).

→ Similarly from X-X axis that is $\frac{d}{6}$ on either side. In combined the value is '$\frac{d}{3}$'.

Note



(II) Middle Quarter Rule



$\sigma_{min} \geq 0$

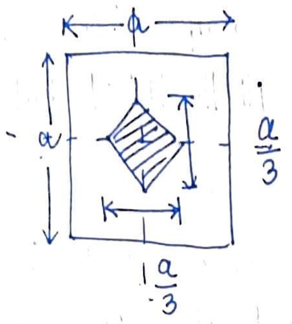
$\Rightarrow \frac{4P}{\pi d^2} - \frac{32 Pe}{\pi d^3} \geq 0$

$\Rightarrow \boxed{e \leq \frac{d}{8}}$ from either side of axis XX & YY.

→ In combined $\frac{d}{8} + \frac{d}{8} = \frac{d}{4}$

Middle '4' Rule

Sq. section



$$e_{min} = \frac{P}{A} - \frac{6Pe}{Ab}$$

$$\therefore e_{min} = \frac{P}{a^2} - \frac{6Pe}{a^3} \geq 0$$

$$\Rightarrow \boxed{e \leq \frac{a}{6}}$$

Numericals of ~~r~~ on Dams &, Retaining Walls, Chimney

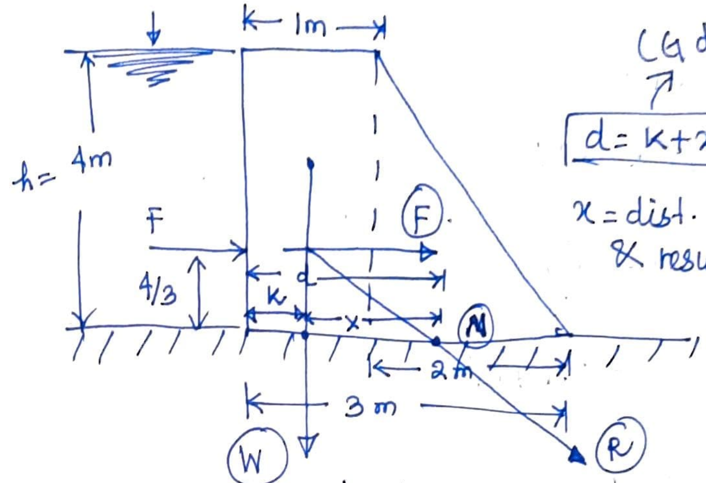
Q1

A masonary trapezoid dam 4 m high, 1 m wide at its top and 3 m width at its bottom retains water on its ~~vertifeal~~ vertical face. Determine the max. & min. stresses at the base.

(i) when reservoir is full.

(ii) when " empty

$\gamma_{masonary} = 19.62\ kN/m^3$



CG dist

$\boxed{d = k + x}$

x = dist. betⁿ W & resultant 'R'

when the reservoir is full

the force exerted by water on vertical face of the dam,

$$F = \gamma_w A\bar{h} = 9810(4 \times 1) \times \frac{4}{2}$$

$$= 78480\ N$$

weight of dam/m length

W = weight density of dam × area of trapezoid × 1

$$= 19.62 \times \frac{1+3}{2} \times 4 \times 1$$

$$= 156960\ N$$

$$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = \frac{(4 \times 1 \times 0.5) + (\frac{1}{2} \times 2 \times 4 \times \frac{5}{3})}{8}$$

$$= \left(2 + \frac{20}{3}\right)/8 = 1.08\ m$$

$\Sigma M_N = 0$

$\Rightarrow F\left(\frac{h}{3}\right) = Wx$

$\Rightarrow x = \frac{Fh}{W3}$

$= 0.67\,m$

$d = K + x$

$= 1.08 + 0.67 = 1.75\,m$

Eccentricity $(e) = d - \frac{b}{2}$

$= 1.75 - \frac{3}{2} = 0.25\,m$

$\sigma_{max} = \frac{W}{b}\left(1 + \frac{6e}{b}\right) = 78480\ N/m^2.$

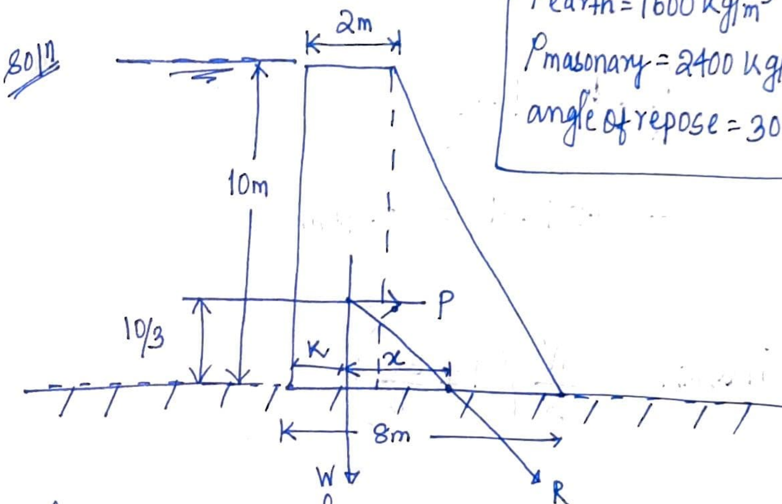
$\sigma_{min} = \frac{W}{b}\left(1 - \frac{6e}{b}\right) = 26163\ N/m^2$

Soln

2/ A masonary retaining wall of trapezoid section of 10 m ~~ht~~ height which retains the earth upto top level. The width at the top is 2m & bottom 8m. Find the max. & min. intensities of normal stress at the base.

$\rho_{earth} = 1600\ kg/m^3$
$\rho_{masonary} = 2400\ kg/m^3$
angle of repose $= 30°$

Soln



height of wall $h = 10m$

$a = 2m$

$b = 8m$

$\phi = 30°$

Thrust of earth on vertical face of wall

$$P = \frac{1}{2}\gamma h^2\left(\frac{1-\sin\phi}{1+\sin\phi}\right) = \frac{80000 \times 9.81}{3}\ N$$

weight of 1m length of trapezoidal wall

W = weight density of wall $\times$ vol. of wall

$= 2400 \times 9.81 \times \frac{1}{2}(2+8) \times 10 \times 1$

$= 120000 \times 9.81\ N$

$CG\ (k) = 2.8m$

$d = k + x \quad \left(x = \frac{Ph}{3W} = 0.74m \right)$

$= 3.54m$

$e = \cancel{\frac{b}{2}} - d \qquad d - \frac{b}{2}$

$= 3.54 - 4 = 0.46m$

$\sigma_{max, min} = \frac{W}{b}\left(1 \pm \frac{6e}{b}\right)$

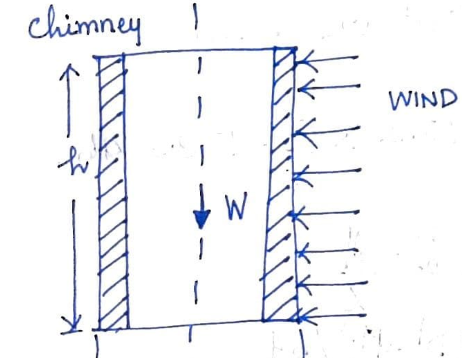
$= 197916.76\ N/m^2$

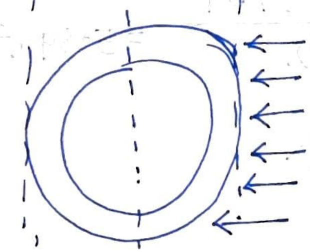
&

$96383.25\ N/m^2$.

3/ Determine the max. & min stresses at the base of hollow circular chimney of height 20m with external diameter of 4m & internal dia of 2m. The chimney is subjected to a horizontal wind pressure intensity of $1\ kN/m^2$. $\gamma_{mat} = 22\ \frac{kN}{m^2}$.

Soln





$h = 20m,\ D = 4m,\ d = 2m,\ p = 1\ \frac{kN}{m^2}$

W = weight of chimney

$= \rho g \times$ Vol. of chimney

$= 22 \left[\frac{\pi}{4}(D^2 - d^2) \right] \times h = 4146.9\ kN$

$\sigma_0 = \frac{W}{A} = 440\ kN/m^2$

wind force $(F_w) = K \times p \times A$

$$= \frac{2}{3} \times 1 \times (D \times h)$$

$$= \frac{2}{3} \times 1 \times 80 = 53.33 \text{ kN}$$

bending moment at the base,

$$M = F \times \frac{h}{2} = 53.33 \times \frac{20}{2} = 533.3 \text{ kNm}$$

bending stress $(\sigma_b) = \frac{M}{Z}$

$$Z = I/y = \frac{\pi (D^4 - d^4)/64}{D/2} = 5.89 m^3$$

$$\sigma_b = 90.54 \text{ kN/m}^2.$$

$$\sigma_{max} = \sigma_0 + \sigma_b = 530.54 \text{ kN/m}^2$$

$$\sigma_{min} = \sigma_0 - \sigma_b = 349.46 \text{ kN/m}^2.$$

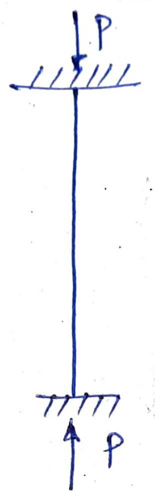
Soln

—— x ——

4. Column & Struts

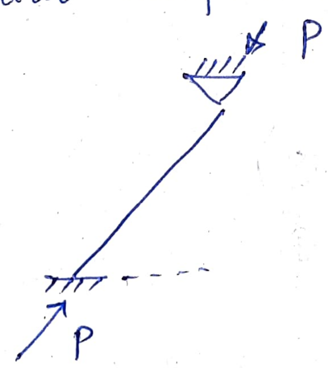
Column

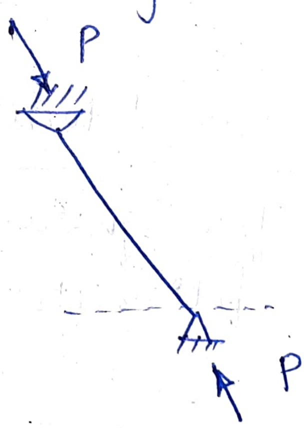
A column is defined as the member of the structure which is vertical & both ~~ends of the~~ of its ends are fixed rigidly when subjected to axial compressive load.



Strut

A strut is defined as the member of the structure which is not vertical and one or both of its ends are hinged when subjected to axial compressive load.

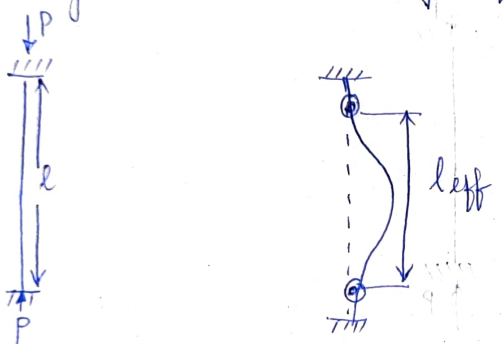


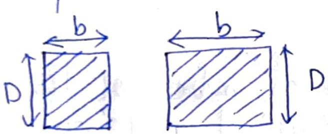


Short column

A short column is the one whose ratio of effective length to its least lateral dimension is less than or equal to 12.

Effective length (l_{eff}) – effective length is nothing but the length between buckling inflection points.





← plan of column

(sq. column) (rectangular column)

∴ least lateral dimension

→ for square column = b (= ∞)
→ for rectangular column = D

$$\boxed{\therefore \text{short column} = \frac{l_{eff}}{\infty} \leq 12}$$

→ short column fails by crushing.

Long column

→ A long column is one whose eff ratio of effective length to its least lateral dimension is greater than 12.

$$\boxed{\text{long column} = \frac{l_{eff}}{\infty} > 12}$$

→ fails by buckling.

End conditions

→ both ends of column are hinged/pinned.

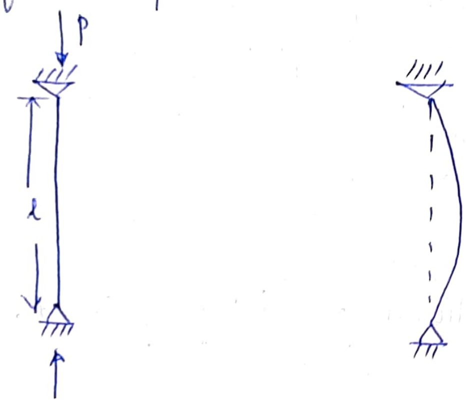
→ One end is fixed & other end is free.

→ both ends are fixed.

→ One end is fixed & other is pinned.

Effective length (l_{eff})

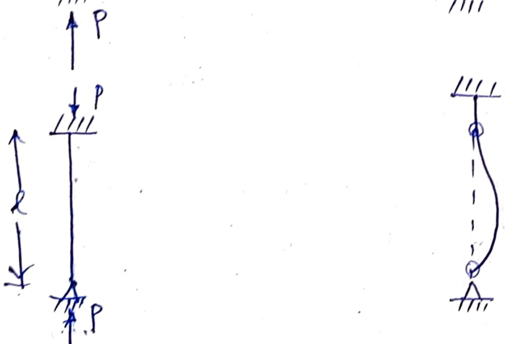
In a long column when axial compressive load acts, it undergoes buckling. The effective length is nothing but the length between buckling inflection points.



$\boxed{l_{eff} = l}$

$\boxed{l_{eff} = 2l}$

$\boxed{l_{eff} = \frac{l}{2}}$



$\boxed{l_{eff} = \frac{l}{\sqrt{2}}}$

Slenderness Ratio (λ)

→ It is defined as the ratio of effective length (l_{eff}) of a column to the least rad radius of gyration (r_{min}).

$$\boxed{\lambda = \frac{l_{eff}}{r_{min}}}$$

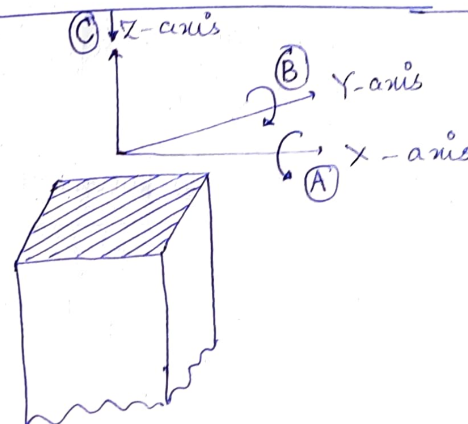
→ λ short column ≤ 50

→ ~~λ~~ $50 \leq \lambda$ intermediate column < 200

→ λ long column > 200

} classification of column based on 'λ'

Axially loaded short column & long column



axial load → C

uniaxial " → either A or B

biaxial " → both A & B

→ A column whose ratio of effective length of least lateral dimension ~~with axial~~ is equal to or less than 12 with axial compressive load, known as axially loaded short column.

→ A column whose ratio of effective length of least lateral dimension is greater than 12 with axial compressive load is known ~~as~~ as axially loaded long column.

Euler's theory of long column

→ The column is initially perfectly straight & load is applied axially.

→ The c/s of column is uniform throughout its length.

→ The column material is perfectly elastic, homogeneous, isotropic and obeys Hooke's Law.

→ The length of the column is very large as compared to its lateral dimension.

→ The direct stress is very small as compared to bending stress.

→ The column will fail by buckling alone.

→ The self-weight of the column is negligible.

Critical load of column with different end condition.

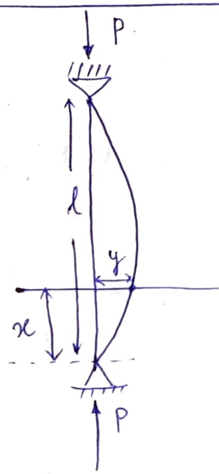
~~Cas~~

Critical load - It is load that will not cause lateral deflection / buckling.

→ A load beyond the critical load causes the column to fail by buckling.

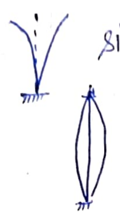
Case-I

both ends of columns are hinged



Moment at a distance 'x' from the hinge support —

$$M = -Pxy$$

[ sign '+' sign '-']

$$\Rightarrow EI\frac{d^2y}{dx^2} = -Py$$

$$\Rightarrow EI\frac{d^2y}{dx^2} + Py = 0$$

$$\Rightarrow \frac{d^2y}{dx^2} + \frac{P}{EI}y = 0$$

$$\Rightarrow \frac{d^2y}{dx^2} + k^2y = 0 \qquad \left[\text{where } k = \sqrt{\frac{P}{EI}}\right]$$

$$\Rightarrow \boxed{y = A\sin kx + B\cos kx}$$

$\therefore x = 0,\ y = 0$

$$\Rightarrow \boxed{B = 0}$$

$$\therefore \boxed{y = A\sin kx}$$

when $x = l,\ y = 0$

$$A\sin kl = 0$$

$$\therefore \Rightarrow \sin kl = \sin\pi$$

$$\Rightarrow kl = \pi$$

$$\Rightarrow \sqrt{\frac{P_{cr}}{EI}} \times l = \pi$$

$$\Rightarrow \frac{P_{cr}}{EI} = \frac{\pi^2}{l^2}$$

$$\Rightarrow \boxed{P_{cr} = \frac{\pi^2 EI}{l^2}}$$

critical load for hinged-hinged column

$$\Rightarrow \boxed{P_{cr} = \frac{\pi^2 EI}{l_{eff}^2}}$$

standard critical load formulation

Case-II : One side is free & other side fixed

$\therefore l_{eff} = 2l$

$$\boxed{P_{cr} = \frac{\pi^2 EI}{(2l)^2} = \frac{\pi^2 EI}{4l^2}}$$

Case-III : both side fixed.

$l_{eff} = l/2$

$$\therefore \boxed{P_{cr} = \frac{\pi^2 EI}{(l/2)^2} = \frac{4\pi^2 EI}{l^2}}$$

Case-IV : One of side fixed & other hinged.

$l_{eff} = l/\sqrt{2}$

$$\therefore P_{cr} = \frac{\pi^2 EI}{(l/\sqrt{2})^2} = \frac{2\pi^2 EI}{l^2}$$

Numerical Problem

Q1/ A solid round bar 3m long and 5cm in diameter is used as a strut with both ends hinged. Determine the crippling load where $E = 2\times10^5\ N/mm^2$.

Sol^n $l = 3m$

both end hinged.

$l_{eff} = l = 3m = 3000\ mm$

$\therefore P_{cr} = \dfrac{\pi^2 EI}{l^2}$

$I = \dfrac{\pi d^4}{64} = \dfrac{\pi \times (50)^4}{64} = 0.306\times10^6\ mm^4$

$\therefore P_{cr} = \dfrac{\pi^2 \times 2\times10^5 \times 0.306\times10^6}{(3000)^2}$

$\boxed{P_{cr} = 67.11\ kN}$

Q2/ Determine the ratio of buckling strength of two circular column one hollow and one solid. Both the columns are made of same material and have same length, c/s area and end conditions. The internal diameter of the hollow column is half of the external dia.

Sol^n d = dia of solid column

D_i = internal dia. of hollow column

D_e = external "

Area of c/s of hollow column

$A_h = \dfrac{\pi}{4}(D_e^2 - D_i^2)$

$= \dfrac{\pi}{4}\left(D_e^2 - \left(\dfrac{D_e}{2}\right)^2\right)$ ~~= 0.866 De~~

$A_s = \dfrac{\pi}{4} d^2$

Since both column are made of same material & hence same length, c/s area and end conditions.

So $A_h = A_s$

$\Rightarrow \dfrac{\pi}{4}\left(D_e^2 - \left(\dfrac{D_e}{2}\right)^2\right) = \dfrac{\pi}{4} d^2$

$\Rightarrow d = 0.866\ D_e$

$\therefore P_s = \dfrac{\pi^2 E I_s}{l_{eff}^2}$, $P_h = \dfrac{\pi^2 E I_h}{l_{eff}^2}$

$\boxed{\dfrac{P_s}{P_h} = \dfrac{I_s}{I_h} = \dfrac{(\pi d^4)/64}{\pi(D_e^4 - D_i^4)/64} = 1.67}$

~~5: Shear Force~~

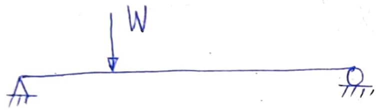
5. Shear Force & Bending Moment

5.1 Types of loads & beams

Type of loads

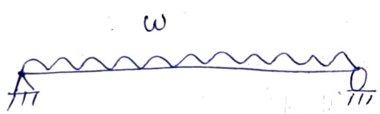
Concentrated / point load

A concentrated load is one which is considered to act a point.

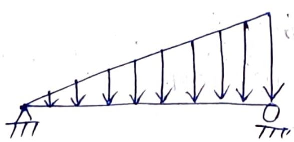


uniformly distributed load

It is the load which is spread over a beam in such a manner that the rate of loading is uniform throughout the ~~lor~~ length.



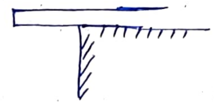
uniform varying load is one which is spread over a beam in such a manner that the rate of loading varies from point to point along the beam.



Types of Support

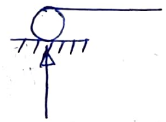
Simple Support : structural member rest on a external supp structure.

(Restrain vertically)



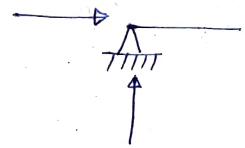
Roller support :

(Restrain vertically)



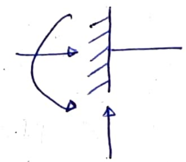
Hinged support :

(Restrain horizontally & vertically)

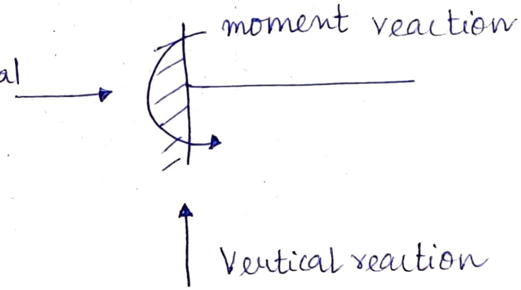


fixed Support :

(Restrain horizontally, vertically & rotation)



Type of Reaction



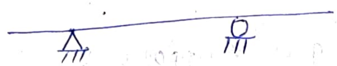
Types of beams based on support conditions

Simply supported beam

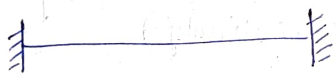
cantilever

continuous beam

overhanging beam

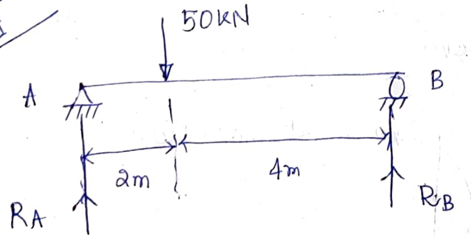


fixed beam



Calculation of support reaction using equation of static equilibrium

Case-I



Find out R_A & R_B ?

We know $\Sigma F_y = 0$

$\Rightarrow R_A + R_B = 50$ —— (i)

As 'A' is hinged & 'B' is roller support.

$\Rightarrow \boxed{\Sigma M_A = 0 \ \& \ \Sigma M_B = 0}$

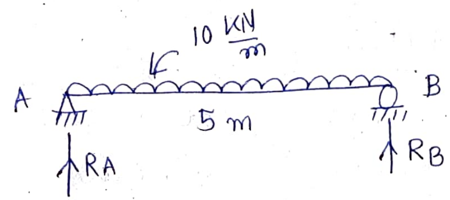
$\therefore$ Take $\Sigma M_A = 0$

$\Rightarrow 50 \times 2 = 6 \times R_B$

$\Rightarrow R_B = \frac{50}{3}$ KN.

$\therefore R_A = 50 - R_B = 50 - \frac{50}{3} = \frac{100}{3}$ KN

Case-II



$\therefore R_A + R_B = 10 \times 5 = 50$ KN

$\Sigma M_A = 0$

$\Rightarrow 5 \times R_B = 50 \times 2.5$

$\Rightarrow R_B = 25$ KN

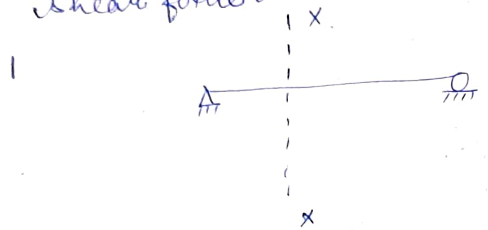
$\therefore R_A = 50 - R_B = 25$ KN.

—x—

5.2: Shear Force & Bending Moments in beams

Shear Force (SF)

The resultant internal force is called shear force.



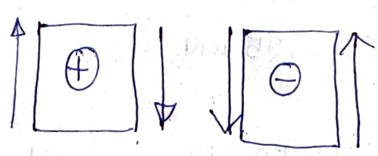
The algebraic sum of all the forces on either side of a section is known as shear force at that section.

Bending Moment (BM)

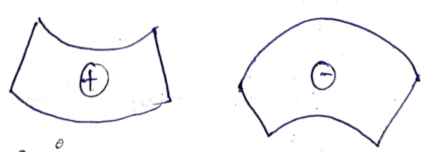
The resultant moment at a section of the beam is known as bending moment at that section.

Sign Convention

Shear force



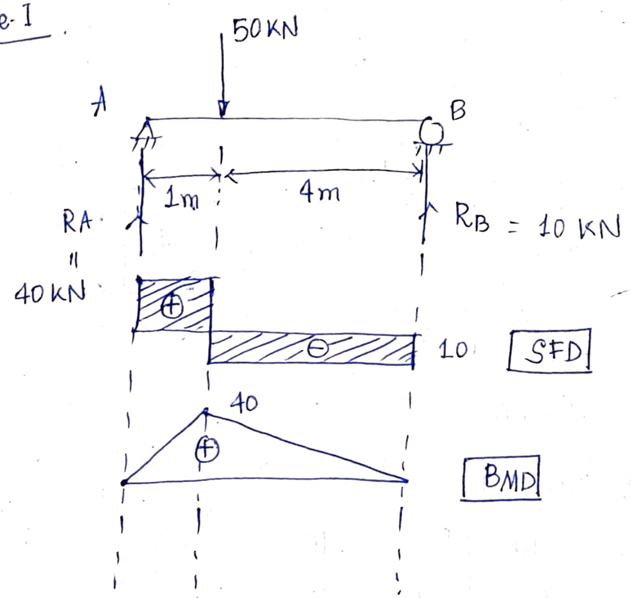
Bending Moment



sagging   hogging

SF & BM of a beam with concentrated load

Case-I



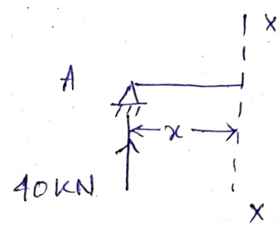
We know $R_A + R_B = 50$

$\Sigma M_A = 0$

$\Rightarrow 5 R_B = 50 \Rightarrow R_B = 10\ KN$

$\therefore R_A = 40\ KN$

Take a section X–X at a distance x from A.



$SF_{xx} = +40\ KN$

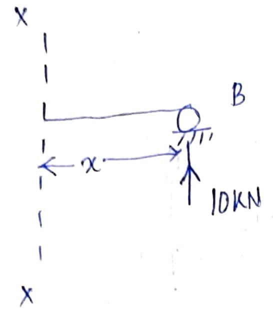
$BM_{xx} = +40 \times x$

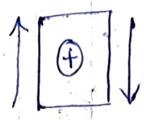
$\therefore$ $0 \leq x \leq 1$ (boundary condition for taken section)

$SF_0 = 40\ KN$, $SF_1 = 40\ KN$

$BM_0 = 0$, $BM_1 = 40\ KNm$

Similarly take a section at a distance x from B.





$SF_{xx} = -10\ kN$

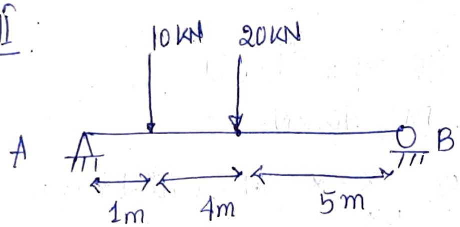
$BM_{xx} = +10x$

for $0 \le x \le 4$

$SF_0 = -10$, $SF_4 = -10$

$BM_0 = 0$, $BM_4 = +10 \times 4 = +40$

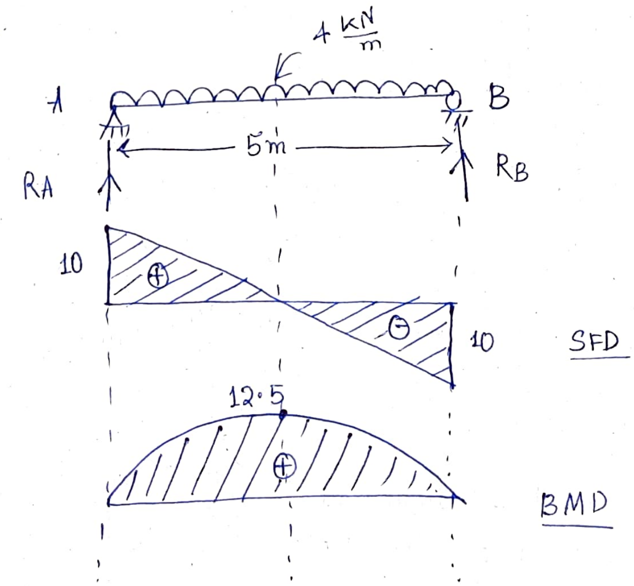
Case-II



Draw the SFD & BMD ?

SFD & BMD of a beam with udl

Case-I



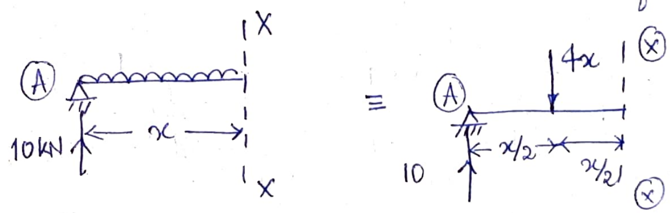
$RA + RB = 5 \times 4 = 20\ kN$

$\Sigma MA = 0$

$\Rightarrow 5 \times RB = 20 \times 2.5$

$\Rightarrow RB = \frac{20 \times 2.5}{5} = 10\ kN$ $\therefore RA = 20 - 10 = 10\ kN$

Take a section at a distance x from A.



$SF_x = +10 - 4x$

$BM_x = +10x - 4x \times \frac{x}{2}$

$= 10x - 2x^2$

SF will be '0' at

$10 - 4x = 0$

$\Rightarrow x = \frac{10}{4} = 2.5m$

BM will be '0' at $10x - 2x^2 = 0$

$\Rightarrow x(10 - 2x) = 0$

$x = 0$ & $x = \frac{10}{2} = 5m.$

$0 \leq x \leq 5$

$SF_0 = 10 - 4 \times 0 = 10$ KN

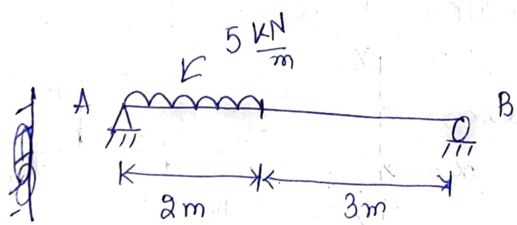
$SF_5 = 10 - 4 \times 5 = -10$ KN.

$BM_0 = 10 \times 0 - 2 \times 0^2 = 0$

$BM_5 = 10 \times 5 - 2 \times 5^2 = 0$

$BM_{2.5} = 10 \times 2.5 - 2 \times 2.5^2 = 25 - 12.5 = 12.5$ KNm

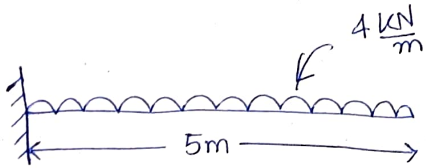
Case-II



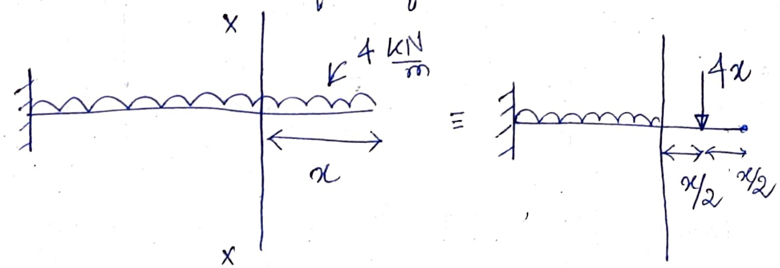
Draw SFD & BMD ?

SFD & BMD for cantilever

Case-I



Take a section X-X from free end.



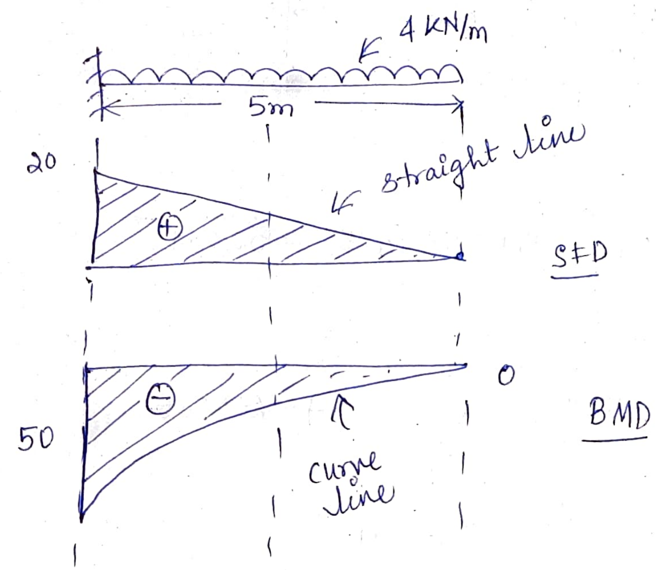
$SF_{xx} = +4x$

$BM_{xx} = -4x \times \frac{x}{2} = -2x^2$

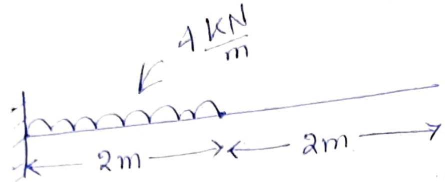
for $0 \leq x \leq 5.$

$SF_0 = +4 \times 0 = 0$, $SF_5 = +4 \times 5 = 20$ KN

$BM_0 = -2 \times 0^2 = 0$, $BM_5 = -2 \times 5^2 = -50$ KNm

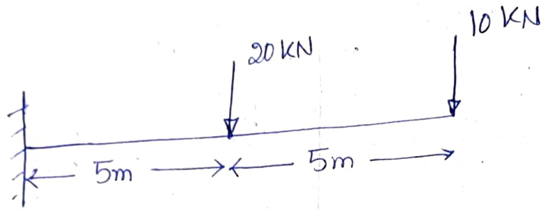


Case II



Draw SFD & BMD ?

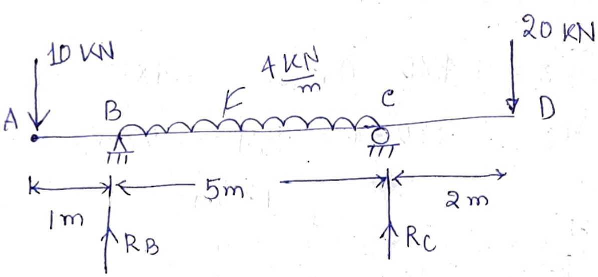
Case III



Draw SFD & BMD ?

SFD & BMD of overhanging beams

Case-I



$\therefore R_B + R_C = 10 + 20 + (4\times5) = 50$ KN.

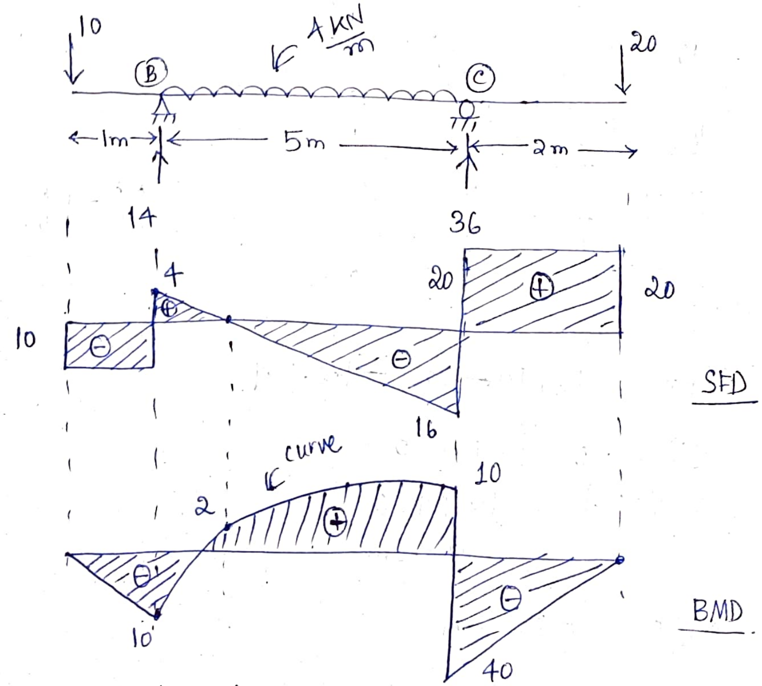
$\Sigma M_B = 0$

$\Rightarrow \; 5R_C + 10\times1 = 20(5+2) + 20\times2.5$

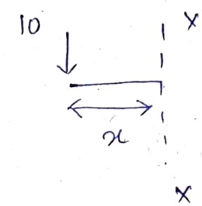
$= 190$

$\rightarrow \; R_C = \frac{180}{5} = 36$ KN

$R_B = 14$ KN



Take a section X-X at a distance x from extreme left.



$\therefore SF_x = -10$

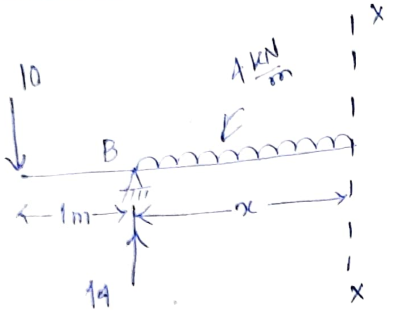
$BM_x = -10\times x$

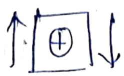
for $0 \le x \le 1$

$SF_0 = -10, \; SF_1 = -10$

$BM_0 = 0, \; BM_1 = -10\times1 = -10$

Take a section X-X at a distance x from Support 'B'.





$SF_{xx} = -10+14 \cancel{+} 4x = -4x+4$

~~$SF_{xx}=0$~~ $\quad SF_{xx}=0$

~~$4x+4$~~ $\quad -4x+4=0$

$\Rightarrow x = 1m$

$\therefore$ SF $\quad 0 \leq x \leq 5$

$\therefore \; SF_0 = -4\times0+4 = +4$

$SF_5 = -4\times5+4 = -16$

$SF_1 = -4\times1+4 = 0$

$BM_{xx} = -10\times1 + 14\times x - 4x\times\frac{x}{2}$

$= \cancel{-10+14x-2x^2} \; -10+14x-2x^2$

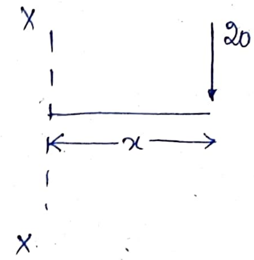
$BM_0 = -10$

$BM_1 = -10+14-2\times1^2 = \cancel{+8} \; +2$ kNm

$BM_5 = -10+14\times5 - 2\times5^2 = +10$ kNm

~~$BM_1 = -10+14-2 = 2$ kNm~~

Take a section at a distance x from extreme right.



$SF_x = +20$

$BM_x = -20x$

for $\quad 0 \leq x \leq 2$

$SF_0 = 20$ kN

$SF_2 = +20$ kN

$BM_0 = -20\times0 = 0$

$BM_2 = -20\times2 = -40$ kNm

Position of maximum BM

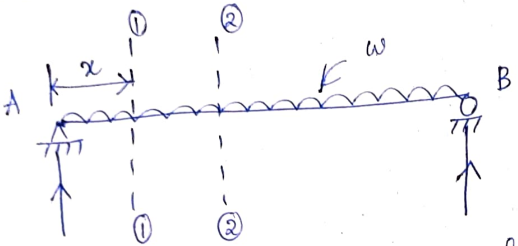
Maximum BM will occur where SF is 0.

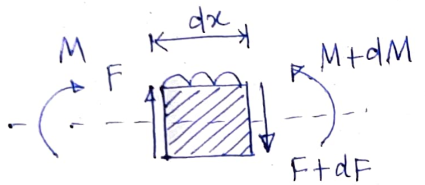
$\therefore$ $(BM)_{max}$ at x where

$$\boxed{V_x = 0}$$

Relation between intensity of Load, SF & BM

Suppose a beam carrying a udl w per unit length. Consider the equilibrium portion of beam between section 1-1 & 2-2.





where F & F+dM = SF at 1-1 & 2-2

M & M+dM = BM at 1-1 & 2-2.

The portion of beam of length dx is in equilibrium.

Hence resolving the forces acting on this part vertically

$$F - w(dx) - (F + dF) = 0$$

$$\Rightarrow F - w dx - F - dF = 0$$

$$\Rightarrow \boxed{\frac{dF}{dx} = -w}$$

Taking the moments of forces & couples about section 2-2.

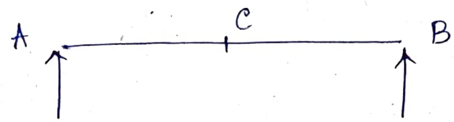
$$F \times dx + M - (M + dM) = 0$$

$$\Rightarrow \boxed{F = \frac{dM}{dx}}$$

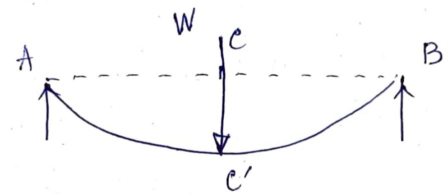
6. Slope & Deflection

Shape & nature of elastic curve (deflection curve)

Elastic curve - the curve along which the axis of a beam is bent under the action of a load.



(beam position before loading)

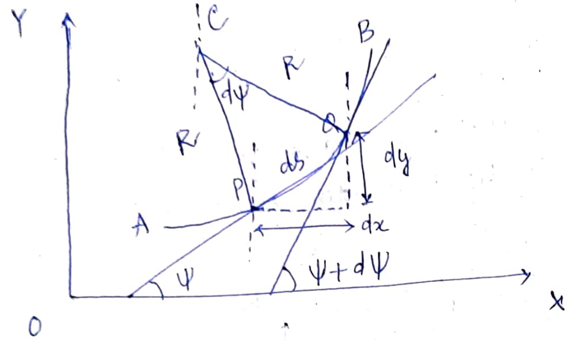


(beam position after loading with load W)

→ If the equation for the elastic curve is known, the differential equations of the theory of bending can be used to determine the amount of deflection for any section of the beam, as well as angle of rotation the BM & the SF.

Relation between slope, deflection & curvature

(no derivation) $\psi = psi$



AB = deflection curve.

P & Q are two points on AB.

ψ & $\psi + d\psi$ are angles made by tangents at P & Q with X-axis.

Normals at P & Q meet at C.

$$\boxed{PC = QC = R}$$

$$\therefore PQ = R \times d\psi$$

$$\Rightarrow R = \frac{PQ}{d\psi} = \frac{ds}{d\psi}$$

$$\tan\psi = \frac{dy}{dx}$$

$$\sin\psi = \frac{dy}{ds}$$

$$\cos\psi = \frac{dx}{ds}$$

$$\therefore R = \frac{ds}{d\psi} = \frac{ds/dx}{d\psi/dx} = \frac{\frac{1}{\cos\psi}}{d\psi/dx}$$

$$\Rightarrow R = \frac{\sec\psi}{(d\psi/dx)}$$

differentiating ~~we will get~~ $\frac{dy}{dx} = \tan\psi$

$$\sec^2\psi \frac{d\psi}{dx} = \frac{d^2y}{dx^2}$$

$$\Rightarrow \frac{d\psi}{dx} = \frac{1}{\sec^2\psi} \frac{d^2y}{dx^2}$$

we know $R = (\sec\psi) \times \frac{1}{(d\psi/dx)}$

$$= \frac{\sec\psi}{\cancel{\sec^2\psi}} \times \frac{1}{(d^2y/dx^2)}$$

$$\Rightarrow R = \frac{\sec\psi \times \sec^2\psi}{(d^2y/dx^2)} = \frac{\sec^3\psi}{(d^2y/dx^2)}$$

$$\Rightarrow \frac{1}{R} = \frac{(d^2y/dx^2)}{(1+\tan^2\psi)^{3/2}}$$

$$\therefore \boxed{\frac{1}{R} = \frac{d^2y}{dx^2}} \quad \text{as } \tan^2\psi \approx 0$$

but we know $\frac{M}{I} = \frac{E}{R}$

$$\Rightarrow \boxed{\frac{1}{R} = \frac{M}{EI}}$$

$$\therefore \frac{M}{EI} = \frac{d^2y}{dx^2}$$

$$\Rightarrow \boxed{M = EI\frac{d^2y}{dx^2}}$$

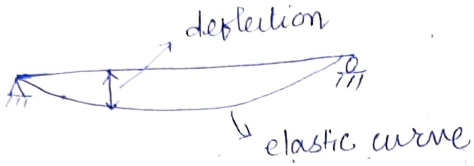
$$F = \frac{dM}{dx} = EI\frac{d^3y}{dx^3}$$

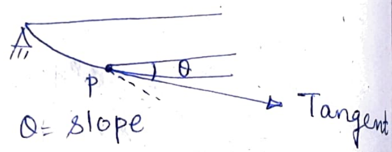
$$w = \frac{dF}{dx} = EI\frac{d^4y}{dx^4}$$

slope = $\frac{dy}{dx}$, deflection = y

R = curvature

Importance of slope and deflection





→ deflection in beam is calculated to determine the vertical depth of its sag from initial horizontal axis of beam.

→ Slope is calculated as it defines the angle of beam axis between initial position & final position after deflection.

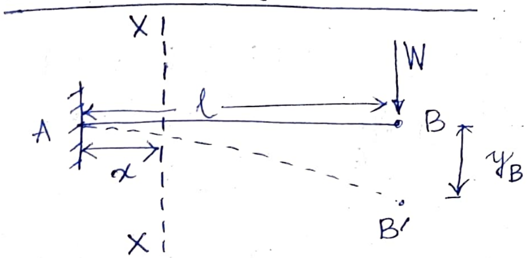
→ deflection of beam controls the effective length to depth ratio of beam.

→ due to excessive deflection minor cracks may develop in structural finishes. So to avoid cracks in finishes deflection should be permissible limit.

→ Slope & deflection → how the beam will bend & to which extent.

Slope & deflection of a cantilever under concentrated load

by Double Integration Method



Consider a section X–X at a distance 'x' from fixed end.

$$M_{x-x} = -W(l-x)$$

$$\Rightarrow EI\frac{d^2y}{dx^3} = -W(l-x)$$

$$\Rightarrow EI\frac{d^2y}{dx^2} = -Wl + Wx$$

$$\Rightarrow EI\frac{dy}{dx} = -Wlx + \frac{Wx^2}{2} + C_1 \quad (1 \text{ integration})$$

at $x=0$, $y=0$

at $x=0$, $\frac{dy}{dx}=0$

$$\therefore \boxed{C_1 = 0}$$

Again integrating we will get

$$EIxy = -\frac{Wlx^2}{2} + \frac{Wx^3}{6} + C_1x + C_2 \quad (2 \text{ integration})$$

$$\boxed{C_2 = 0}$$

$$EI\frac{d^2y}{dx^2} = -Wlx + \frac{Wx^2}{2}$$

$\therefore$ So $EI\dfrac{dy}{dx} = -Wlx + \dfrac{Wx^2}{2}$ $\quad (\because C_1 = 0, C_2 = 0)$

$\Rightarrow EIy = -\dfrac{Wlx^2}{2} + \dfrac{Wx^3}{6}$

$\therefore$ at $x = l$, $y = y_B$

$\therefore EIy_B = -\dfrac{Wl l^2}{2} + \dfrac{Wl^3}{6} = \dfrac{-3Wl^3}{6} + \dfrac{Wl^3}{6} = -\dfrac{Wl^3}{3}$

$\Rightarrow \boxed{y_B = \dfrac{-Wl^3}{3EI}}$ (negative sign shows deflection is in downward direction)

$\therefore$ Again $EI\dfrac{dy}{dx} = -Wlx + \dfrac{Wx^2}{2}$

$x = 0$, $\dfrac{dy}{dx} = 0$

$x = l$, $\dfrac{dy}{dx} = \theta_B$

$EI\theta_B = -Wl^2 + \dfrac{Wl^2}{2} = -\dfrac{Wl^2}{2}$

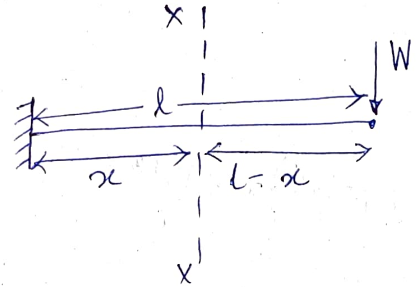
$\Rightarrow \boxed{\theta_B = -\dfrac{Wl^2}{2EI}}$ (negative sign shows that tangent at B makes an angle in anticlockwise dirn)

by Macaulay's Method

(SINGLE EQ^N IS WRITTEN TO SOLVE THE PROBLEM)

Rule

1. Always take origin on the extreme left of the beam ~~(and extreme right for cantilever)~~
2. Take ↻ moment as '+' & ↺ moment as '-'.
3. For any term when $(x-a) < 0$, the term is neglected.
4. The quantity $(x-a)$ should be integrated as $\dfrac{(x-a)^2}{2}$ and not as $\dfrac{x^2}{2} - ax$.



$M_{x-x} = -W(l-x)$

$\Rightarrow EI\dfrac{d^2y}{dx^2} = -W(l-x)$

$\Rightarrow EI\dfrac{dy}{dx} = +\dfrac{W(l-x)^2}{2} + C_1$

$\Rightarrow EIy = -\dfrac{W(l-x)^3}{6} + C_1x + C_2$

at $x = 0$, $\dfrac{dy}{dx} = 0$

$\Rightarrow EI \times 0 = \dfrac{Wl^2}{2} + C_1 \Rightarrow \boxed{C_1 = \dfrac{-Wl^2}{2}}$

$\therefore EI\frac{dy}{dx} = \frac{W(l-x)^2}{2} - \frac{Wl^2}{2}$

at $x = l$, $\frac{dy}{dx} = \theta_B$.

$\therefore EI \times \theta_B = -\frac{Wl^2}{2}$

$\Rightarrow \boxed{\theta_B = -\frac{Wl^2}{2EI}}$

Again we know $x=0$, $y=0$

$EI \times y = -\frac{W(l-x)^3}{6} + C_1 x + C_2$

$\Rightarrow C_2 = \frac{Wl^3}{6}$

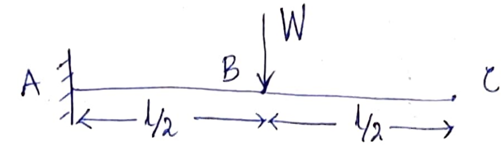
$\therefore EIy = -\frac{W(l-x)^3}{6} + \left(-\frac{Wl^2}{2}\right)x + \frac{Wl^3}{6}$

at $x = l$, $y = y_B$

$\therefore EI \times y_B = -\frac{Wl^3}{2} + \frac{Wl^3}{6} = -\frac{Wl^3}{3}$

$\Rightarrow \boxed{y_B = \frac{-Wl^3}{3EI}}$

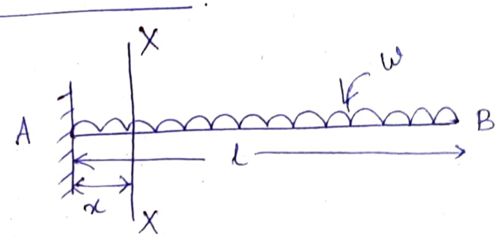
Case-II



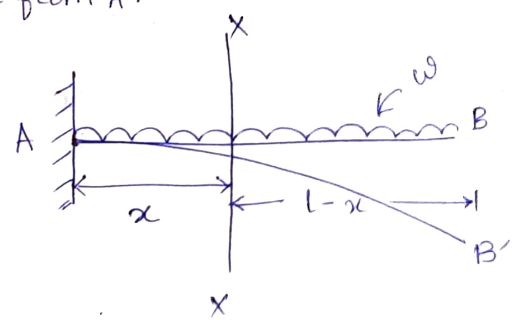
Find out slope & deflection under B using DI & Macaulay's Method.

Slope & deflection ~~under~~ of a cantilever under udl

~~by DI Method~~ Macauly by Macauly's Method



Take a section X-X at a distance of x from A.



$M_x = -\frac{w(l-x) \times (l-x)}{2}$

$= -\frac{w(l-x)^2}{2}$

$$EI\frac{d^2y}{dx^2} = -M = \frac{w(l-x)^2}{2}$$

$$\Rightarrow EI\frac{dy}{dx} = +\frac{w(l-x)^3}{6} + C_1$$

at $x=0$, $\frac{dy}{dx}=0$

$$\therefore \boxed{C_1 = -\frac{wl^3}{6}}$$

$$\therefore EI\frac{dy}{dx} = \frac{w(l-x)^3}{6} - \frac{wl^3}{6}$$

$$\Rightarrow EI\times y = -\frac{w(l-x)^4}{24} - \frac{wl^3}{6}x + C_2$$

at $x=0$, $y=0$

$$\boxed{C_2 = \frac{wl^4}{24}}$$

$$\therefore EI\frac{dy}{dx} = +\frac{w(l-x)^3}{6} - \frac{wl^3}{6}$$

at $x=l$, $\cancel{y = y_B}$ $\theta = \theta_B$

$$EI\times\theta_B = -\frac{wl^3}{6}$$

$$\Rightarrow \boxed{\theta_B = -\frac{wl^3}{6EI}}$$

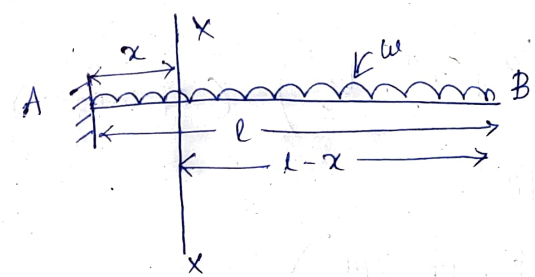
$$EIy = -\frac{w(l-x)^4}{24} - \frac{wl^3}{6}x + \frac{wl^4}{24}$$

at $x=l$, $y=y_B$

$$EI\times y_B = -\frac{wl^4}{6} + \frac{wl^4}{24} = -\frac{wl^4}{8}$$

$$\Rightarrow \boxed{y_B = -\frac{wl^4}{8EI}}$$

by DI Method



$$M_x = -\frac{w(l-x)^2}{2} = -\frac{w}{2}(l^2+x^2-2lx)$$

$$EI\frac{d^2y}{dx^2} = \cancel{-wl} - \frac{w}{2}(l^2+x^2-2lx)$$

$$\Rightarrow EI\frac{dy}{dx} = -\frac{w}{2}\left(l^2x+\frac{x^3}{3}-lx^2\right) + C_1$$

$x=0$, $\frac{dy}{dx}=0$ $\Rightarrow \boxed{C_1=0}$

$$\therefore EIy = -\frac{w}{2}\left(\frac{l^2x^2}{2}+\frac{x^4}{12}-\frac{lx^3}{3}\right) + C_1x + C_2$$

$x=0, y=0$ $\Rightarrow \boxed{C_2=0}$

$$\therefore EI\frac{dy}{dx} = -\frac{w}{2}\left(l^2x+\frac{x^3}{3}-lx^2\right)$$

$x=l$, $\frac{dy}{dx}=\theta_B$

$$\therefore EI\times\theta_B = -\frac{w}{2}\left(l^3+\frac{l^3}{3}-l^3\right) \quad (c_1=0)$$

$$\Rightarrow \boxed{\theta_B = -\frac{wl^3}{6EI}}$$

Again we know

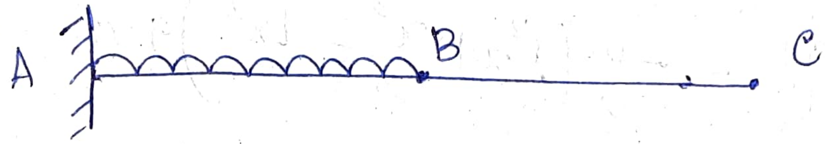
$$EI\times y = -\frac{w}{2}\left(\frac{l^2x^2}{2}+\frac{x^4}{12}-\frac{lx^3}{3}\right) \quad (c_1=0, c_2=0)$$

$x=l$, $y=y_B$

$$EIy_B = -\frac{w}{2}\left(\frac{l^4}{2}+\frac{l^4}{12}-\frac{l^4}{3}\right)$$

$$\Rightarrow \boxed{y_B = -\frac{wl^4}{8EI}}$$

Case-II

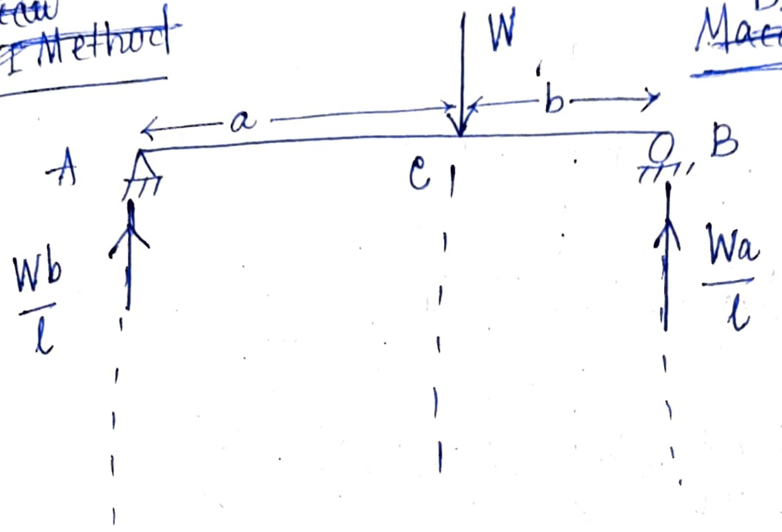


find slope & deflection at B using DI & Macaulay's Method.

Slope & Deflection of a s/s beam under concentrated load

~~Macau~~ ~~DI Method~~

DI ~~Macaulay's~~ Method



for segment AC $(0 \le x \le a)$ $M = \frac{Wbx}{l}$

for segment BC $(a \le x \le b)$ $M = \frac{Wbx}{l} - W(x-a)$

Segment AC

$$M = \frac{Wbx}{l}$$

$$\Rightarrow EI\frac{d^2y}{dx^2} = \frac{Wbx}{l}$$

$$\Rightarrow EI\frac{dy}{dx} = \frac{Wbx^2}{2l} + C_1$$

$$\Rightarrow EIy = \frac{Wbx^3}{6l} + C_1x + C_2$$

$x = 0,\ y = 0$

$\therefore$ $\boxed{C_2 = 0}$

$$\cancel{\frac{Wba^3}{6l} + C_1 a + \overset{0}{C_2} = \frac{Wba^3}{6l} - \frac{Wa^3}{6} + \frac{Wa^3}{2} + C_3 a + C_4}$$

Segment BC

$$M_x = \frac{Wbx}{l} - W(x-a)$$

$$\Rightarrow EI\frac{d^2y}{dx^2} = \frac{Wbx}{l} - W(x-a)$$

$$\Rightarrow EI\frac{dy}{dx} = \frac{Wbx^2}{2l} + \frac{W(x-a)^2}{2} + C_3$$

$$\Rightarrow EI\,y = \frac{Wbx^3}{6l} + \frac{W(x-a)^3}{6} + C_3 x + C_4$$

$x = l,\ y = 0$

$$\Rightarrow \frac{Wbl^3}{6l} + \frac{Wb^3}{6} + C_3 l + C_4 = 0$$

$$\Rightarrow \frac{Wbl^2}{6} + \frac{Wb^3}{6} + C_3 l + C_4 = 0$$

when $x = a$, θ_c = same for both eqⁿ.

$$\frac{Wba^2}{2l} + C_1 = \frac{Wba^2}{2l} + C_3$$

$$\Rightarrow \boxed{C_1 = C_3}$$

when $x = a$, y_c = same for both eqⁿ.

$$\frac{Wba^3}{6l} + C_1 a = \frac{Wba^3}{6l} + C_3 a + C_4$$

$$\Rightarrow \boxed{C_4 = 0}$$

We know

$$\frac{Wbl^2}{8} + \frac{Wb^3}{6} + C_3 l = 0$$

$$\Rightarrow \boxed{C_3 = -\frac{Wb(l^2-b^2)}{6l} = C_1}$$

$$C_2 = 0, \quad C_4 = 0$$

Segment AC

Slope at A, $x = 0$, $\frac{dy}{dx} = \theta_A$

$$EI\frac{dy}{dx} = \frac{Wbx^2}{2l} + C_1$$

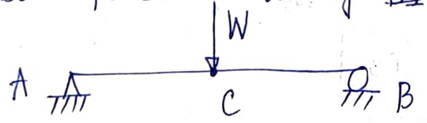
$$\Rightarrow EI \times \theta_A = \cancel{0} + \cancel{\frac{Wb(b^2-l^2)}{6l}} - \frac{Wb(l^2-b^2)}{6l}$$

$$\Rightarrow \boxed{\theta_A = -\frac{Wb(l^2-b^2)}{6EIl}}$$

Similarly slope & deflection at other points can be found out.

* Solve above problem using ~~DI~~ Macaulay's Method.

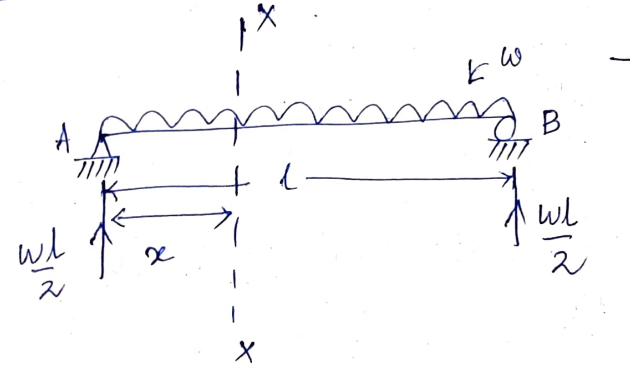
Task



find θ_c, y_c using DI & Macaulay's Method.

Slope & deflection of a s/s beam under udl

DI Method



$$M_x = +\frac{wl}{2}x - \frac{wx^2}{2}$$

$$\Rightarrow EI\frac{d^2y}{dx^2} = \frac{wlx}{2} - \frac{wx^2}{2}$$

$$\Rightarrow EI\frac{dy}{dx} = \frac{wlx^2}{4} - \frac{wx^3}{6} + C_1$$

$$\Rightarrow EIy = \frac{wlx^3}{12} - \frac{wx^4}{24} + C_1x + C_2$$

$x = 0$, $y = 0$

$$\Rightarrow \boxed{C_2 = 0}$$

$x = l$, $y = 0$

$$\Rightarrow \frac{wl^4}{12} - \frac{wl^4}{24} + C_1 l = 0$$

$$\Rightarrow \boxed{C_1 = -\frac{wl^3}{24}}$$

$\therefore x = 0$, $\frac{dy}{dx} = \theta_A$

$$\therefore EI\theta_A = -\frac{wl^3}{24} \Rightarrow \boxed{\theta_A = \frac{-wl^3}{24EI}}$$

We know

$$EIy = \frac{wlx^3}{12} - \frac{wx^4}{24} + c_1x + c_2$$

$$\Rightarrow EIy = \frac{wlx^3}{12} - \frac{wx^4}{24} - \frac{wl^3x}{24}$$

So deflection at the centre of beam

$x = l/2$

$$EI \times y_{centre} = \frac{wl(l/2)^3}{12} - \frac{w(l/2)^4}{24} - \frac{wl^3 l}{48}$$

$$= \frac{wl^4}{96} - \frac{wl^4}{384} - \frac{wl^4}{48}$$

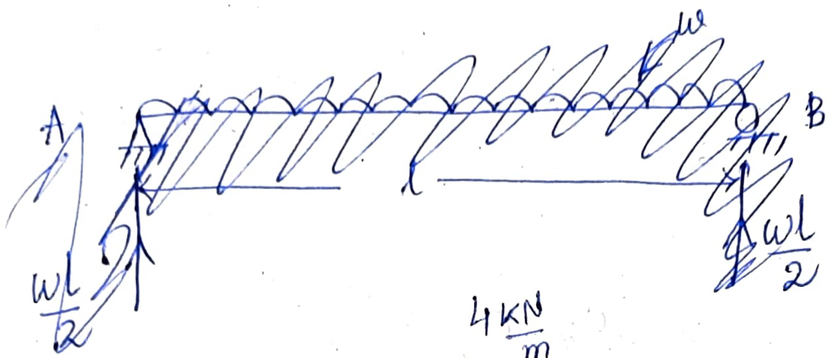
$$= \frac{4wl^4 - wl^4 - 8wl^4}{384}$$

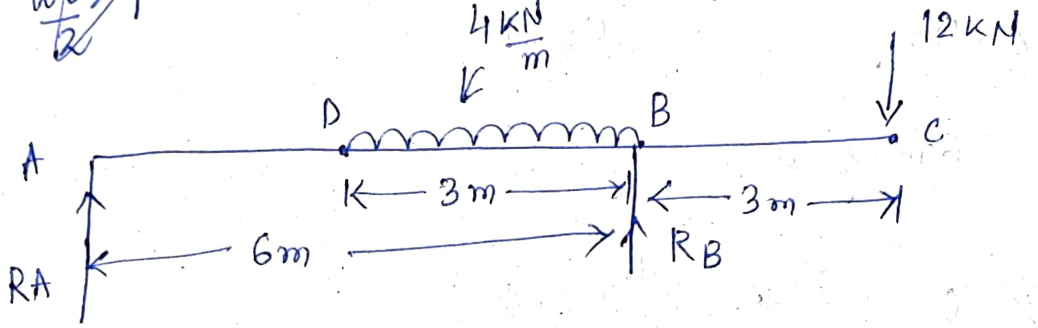
$$= -\frac{5}{384} wl^4$$

$$\Rightarrow \boxed{\cancel{y_{centre} = -\frac{5wl^4}{384EI}}}$$

$$\boxed{y_{centre} = \frac{-5}{384} \frac{wl^4}{EI}}$$

Macaulay's Method to calculate slope and deflection of a beam with udl (w)





∴ We know $RA + RB = 12 + 12 = 24\,KN$.

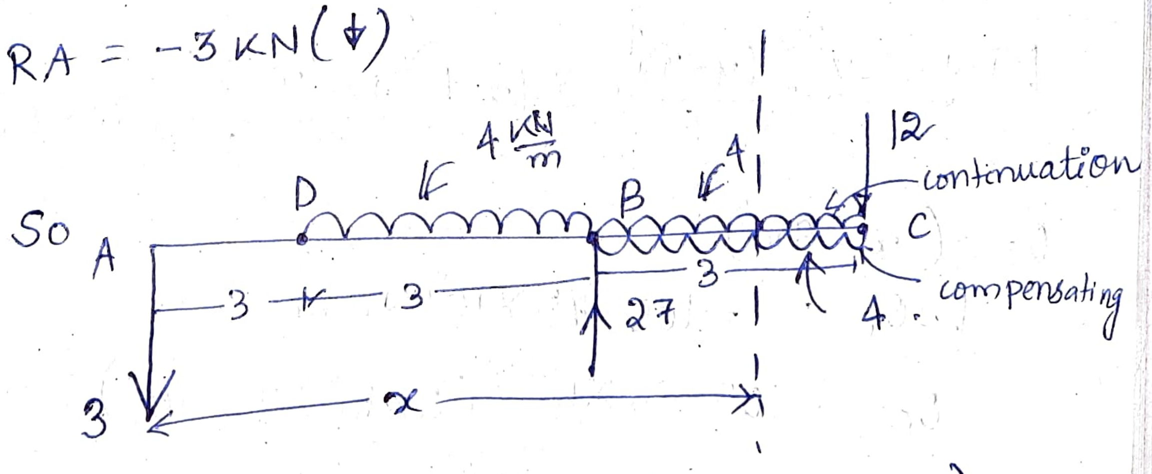
$\Sigma MA = 0$

$\Rightarrow 12 \times 9 + (12 \times 4.5) = 6 \times RB$

$\Rightarrow \quad RB = \frac{162}{6} = 27\,KN\,(\uparrow)$

$RA = -3\,KN\,(\downarrow)$

So



$$M_x = -3 \times x - 4(x-3)\frac{(x-3)}{2} + 27(x-6) + 4(x-6)\frac{(x-6)}{2}$$

$EI\frac{d^2y}{dx^2}$

$M_x = -3x - 2(x-3)^2 + 27(x-6) + 2(x-6)^2$

$\Rightarrow EI\frac{dy}{dx} = -\frac{3x^2}{2} - \frac{2(x-3)^3}{3} + \frac{27(x-6)^2}{2} + \frac{2}{3}(x-6)^3 + C_1$

$\Rightarrow EIy = -\frac{x^3}{2} - \frac{(x-3)^4}{6} + \frac{9}{2}(x-6)^3 + \frac{(x-6)^4}{6} + C_1x + C_2$

$x = 0, y = 0 \Rightarrow C_2 = 0$

$x = 6m, y = 0$

$\Rightarrow 0 = -\frac{6^3}{2} - \frac{3^4}{6} + C_1 6$

$\Rightarrow C_1 = 20.25$

$\therefore EI\frac{dy}{dx} = -\frac{3x^2}{2} - \frac{2}{3}(x-3)^3 + \frac{27}{2}(x-6)^2 + \frac{2}{3}(x-6)^3 + 20.25$

$EIy = -\frac{x^3}{2} - \frac{(x-3)^4}{6} + \frac{9}{2}(x-6)^3 + \frac{(x-6)^4}{6} + 20.25x + 0$

θ_c $(x = 9m)$

$EI\theta_c = -105.75 \times 10^9 \, Nmm^2$

$\theta_c = -105.75 \times 10^9 / (2\times10^5 \times 5\times10^8)$

$= -0.0010575$ radians

y_c $(x = 9m)$

$\therefore EIy_c = -263.25 \times 10^{12} \, Nmm^3$

$\Rightarrow y_c = -2.6325$ mm

(2019)

Numerical Problem Tips

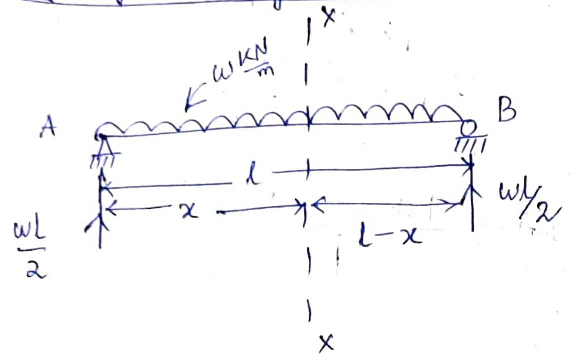
→ While solving numerical problems for DI method just follow the procedure as discussed & put the numerical values as & when required to get the answer.

→ while solving numericals for Macaulay's Method just follow the rules of Macaulay's Method as discussed & put the numerical values as & when required to get the answer.

→ Solve numerical problems & to find out slope and deflection of a cantilever/SS beam using DI & Macaulay's Method from standard text book as mentioned in the syllabus.

—x—

Slope & deflection of a s/s beam with udl using Macaulay's Method



Take a section at a distance x from support A.

$$M_x = +\frac{wl}{2}(l-x) - w(l-x)\frac{(l-x)}{2}$$

$$M_x = +\frac{wl}{2}(l-x) - \frac{w(l-x)^2}{2}$$

$$\Rightarrow EI\frac{d^2y}{dx^2} = +\frac{wl}{2}(l-x) - \frac{w(l-x)^2}{2}$$

$$\Rightarrow EI\frac{dy}{dx} = -\frac{wl}{2}\frac{(l-x)^2}{2} + \frac{w}{2}\frac{(l-x)^3}{3} + C_1 \quad \text{— (1)}$$

$$\Rightarrow EIy = +\frac{wl}{4}\frac{(l-x)^3}{3} - \frac{w}{6}\times\frac{(l-x)^4}{4} + C_1x + C_2 \quad \text{— (2)}$$

We know the boundary condition as

$x=0, y=0$ & $x=l, y=0$

using boundary conditions in eqn (2) we will get

$$0 = \frac{wl}{12}\times l^3 - \frac{w}{24}l^4 + C_2$$

$$\Rightarrow \boxed{C_2 = -\frac{wl^4}{24}}$$

Similarly $0 = C_1 l + C_2 \Rightarrow \boxed{C_1 = -\frac{C_2}{l} = \frac{wl^3}{24}}$

So our slope eqn

$$EI\frac{dy}{dx} = -\frac{wl}{4}(l-x)^2 + \frac{w}{6}(l-x)^3 + \frac{wl^3}{24}$$

deflection eqn

$$EI\times y = \frac{wl}{12}(l-x)^3 - \frac{w}{24}(l-x)^4 + \frac{wl^3}{24}x - \frac{wl^4}{24}$$

using any value of 'x' we can find slope & deflection at any point of beam.

∴ Deflection at the centre of beam

$x = l/2$

$$\therefore EI\times y_c = \frac{wl}{12}\left(l/2\right)^3 - \frac{w}{24}\left(l/2\right)^4 + \frac{wl^3}{24}\times l/2 - \frac{wl^4}{24}$$

$$= \frac{wl^4}{96} - \frac{wl^4}{384} + \frac{wl^4}{48} - \frac{wl^4}{24}$$

$$= \frac{4wl^4 - wl^4 + 8wl^4 - 16wl^4}{384}$$

$$= \frac{-5wl^4}{384}$$

$$\Rightarrow \boxed{y_c = \frac{-5}{384}\frac{wl^4}{EI}}$$

7. Indeterminate Beams

Indeterminacy in beams

→ The beam is known by determinate beam if it can be fully analyzed using equilibrium equations.

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M_A = 0$

→ The beam is known as indeterminate beam if it can't be fully analyzed using equilibrium equations.

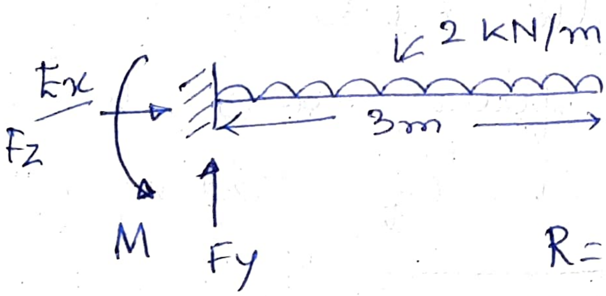
if $R = 3n$ (Determinate beam)

$R > 3n$ (indeterminate beam)

$R < 3n$ (not stable)

R = unknown reaction

n = no. of beam segment



$R = 3$

$n = 1$.

$R = 3n \Rightarrow$ determinate

$R = 5$, $n = 1$

here $R > 3n$

(indeterminate)

Principle of ~~cone~~ consistent deformation / compatibility

Compatibility conditions means requirement of continuity such as in joints where 02 or more members meet.

1/ The members meeting at a joint will continue to meet at the same joint even after deformation takes place.

2/ At rigid joints, the angle between any two members remains the same even after deformation takes place. The compatibility conditions will help in formulating additional equations.

Principle of Superposition

→ The structure can be analysed for different loads separately and the results be superposed to get the final results due to different combinations of loadings.

$\cancel{\Sigma A\bar{x} = 0}$

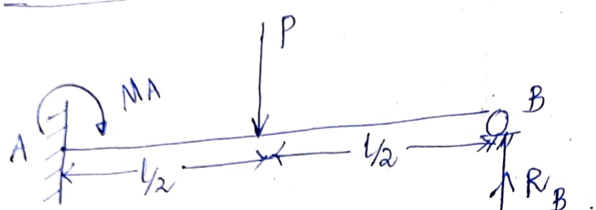
$\cancel{\Sigma A \neq 0}$

$$\boxed{A_1\bar{x}_1 = A_2\bar{x}_2}$$

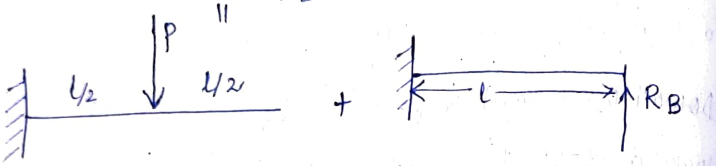
$$A_1 = A_2$$

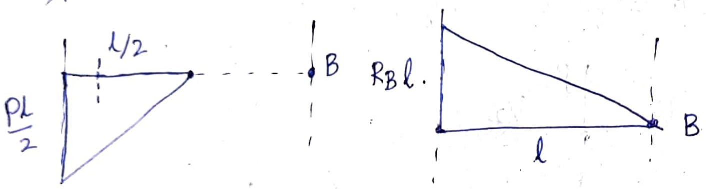
$$\bar{x}_1 = \bar{x}_2$$

Analysis of propped cantilever



→ Total deflection at B due to load P & R_B is '0'.





$A_1 = \frac{1}{2} \times \frac{l}{2} \times \frac{Pl}{2} = \frac{Pl^2}{8}$

$\bar{x}_1 = \frac{l}{2} + \frac{l}{3} = \frac{5}{6} l.$

$A_1 \bar{x}_1 = \frac{Pl^2}{8} \times \frac{5}{6} l = \frac{5Pl^3}{48}$

$A_2 = \frac{1}{2} \times l \times R_B l = \frac{R_B l^2}{2}$

$\bar{x}_2 = \frac{2}{3} l.$

$A_2 \bar{x}_2 = \frac{R_B l^2}{2} \times \frac{2}{3} l = \frac{R_B l^3}{3}$

~~As supposed~~ We know $A_1 \bar{x}_1 = A_2 \bar{x}_2$

$\Rightarrow \frac{5Pl^3}{48} = \frac{R_B l^3}{3}$

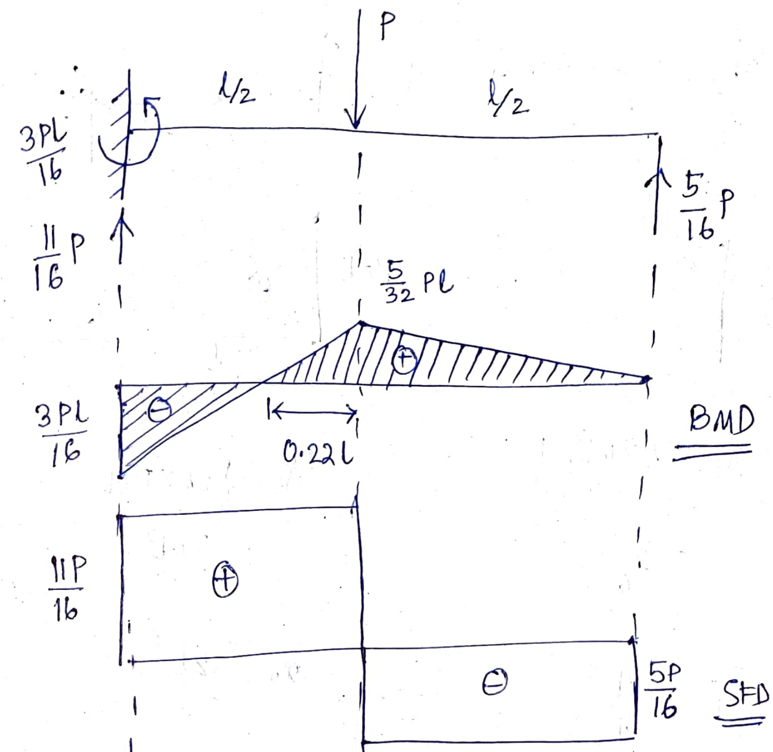
$\Rightarrow \boxed{R_B = \frac{5}{16} P}$

$\therefore R_A = P - \frac{5}{16} P = \frac{11}{16} P$

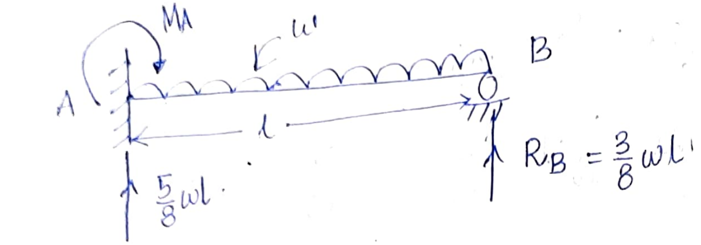
$R_B l = P \times \frac{l}{2} + M_A$

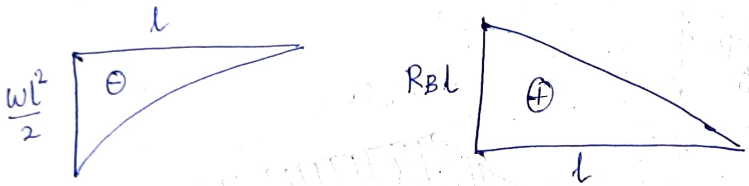
$\Rightarrow \frac{5}{16} Pl - \frac{Pl}{2} = M_A$

$\Rightarrow M_A = \frac{5}{16} Pl - \frac{8Pl}{16} = -\frac{3Pl}{16}$ (↺)



Case-II





$A_1 = l \times \frac{wl^2}{2} \times \frac{1}{3}$

$= \frac{wl^3}{6}$

$A_2 = \frac{1}{2} \times l \times R_B l$

$= \frac{R_B l^2}{2}$

$\bar{x}_1 = \frac{3L}{4}$

$\bar{x}_2 = \frac{2l}{3}$

$A_1\bar{x}_1 = \frac{wl^3}{6} \cdot \frac{3l}{4}$

$= \frac{wl^4}{8}$

$A_2\bar{x}_2 = \frac{R_B l^3}{3}$

As total deflection at $B \approx 0$

$\therefore A_1\bar{x}_1 = A_2\bar{x}_2$

$\Rightarrow \frac{wl^4}{8} = \frac{R_B l^3}{3}$

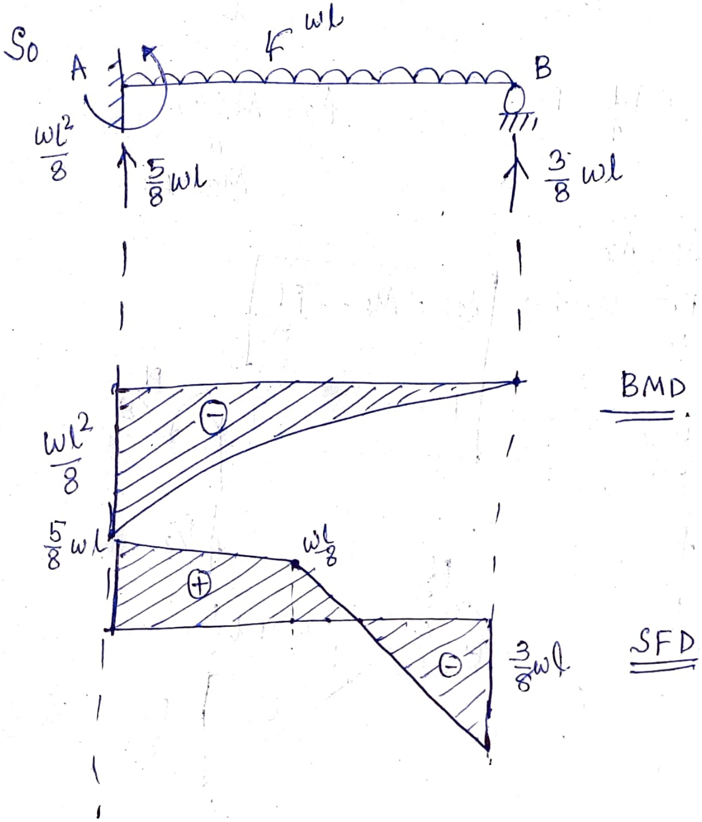
$\Rightarrow \boxed{R_B = \frac{3}{8}wl}$

$RA = wl - \frac{3}{8}wl = \frac{5}{8}wl$

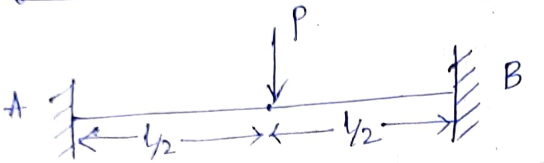
$MA + wl \times \frac{l}{2} = \frac{3}{8}wl \times l$

$\Rightarrow MA = \frac{3}{8}wl^2 - \frac{wl^2}{2} = \frac{3wl^2 - 4wl^2}{8} = -\frac{wl^2}{8}$

So



Analysis of fixed beam with point load



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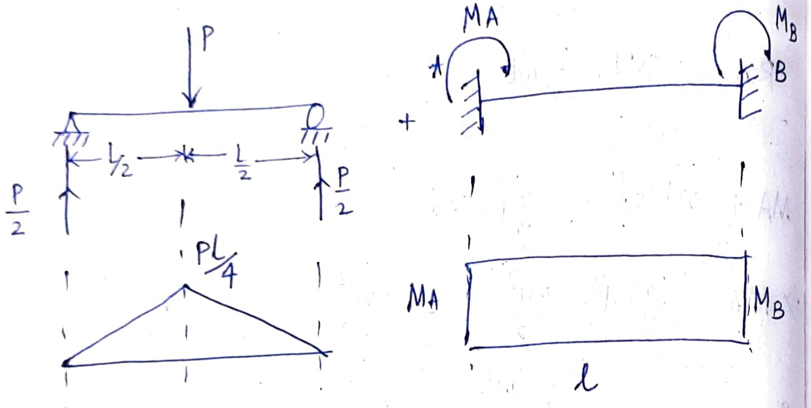
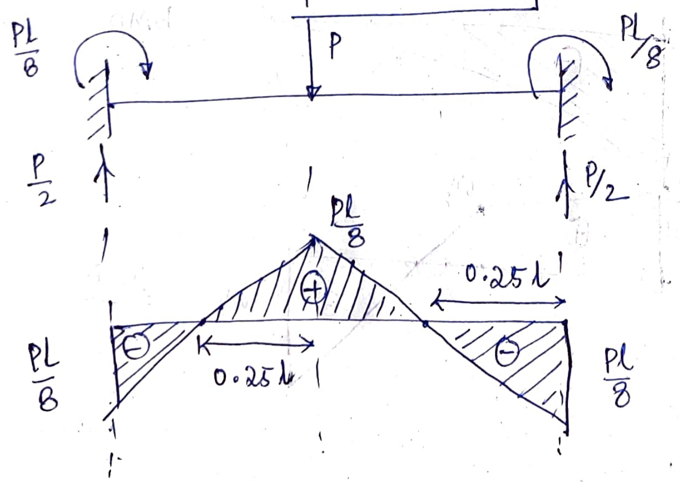


$A_1 = \frac{1}{2} \cdot l \cdot \frac{Pl}{4} = \frac{Pl^2}{8}$

$A_2 = M_A \times l$ or $M_B \times l$

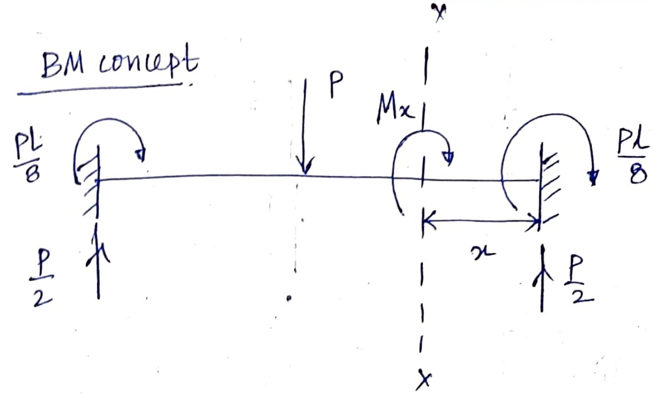
$A_1 = A_2$

$\Rightarrow \frac{Pl^2}{8} = M_A \times l \Rightarrow \boxed{M_A = M_B = \frac{Pl}{8}}$



BMD

BM concept



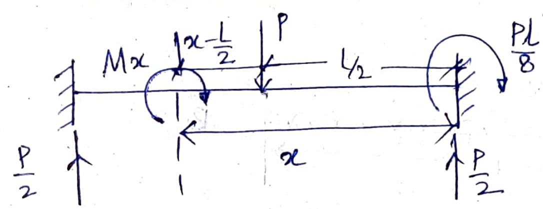
$0 \le x \le \frac{l}{2}$

$M_x + \frac{Pl}{8} = \frac{P}{2}x$

$\Rightarrow M_x = \frac{Px}{2} - \frac{Pl}{8}$ (at $x = \frac{l}{4}, M_x = 0$)

$M_0 = -\frac{Pl}{8}, \quad M_{l/2} = \frac{Pl}{8}$

for $l/2 \le x \le l$.



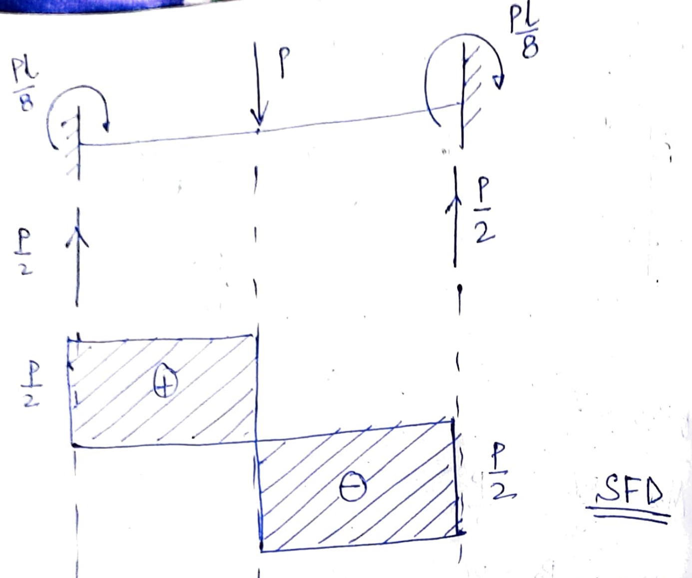
$M_x + \frac{Pl}{8} + P\left(x - \frac{l}{2}\right) = \frac{P}{2}x$

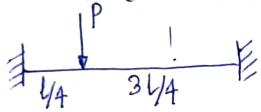
$\Rightarrow M_x = \frac{Px}{2} - Px - \frac{Pl}{8} + \frac{Pl}{2}$

$= \frac{4Px - 8Px - Pl + 4Pl}{8}$

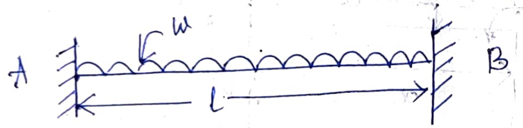
$= \frac{3Pl}{8} - \frac{4Px}{8} = \frac{3Pl}{8} - \frac{Px}{2}$

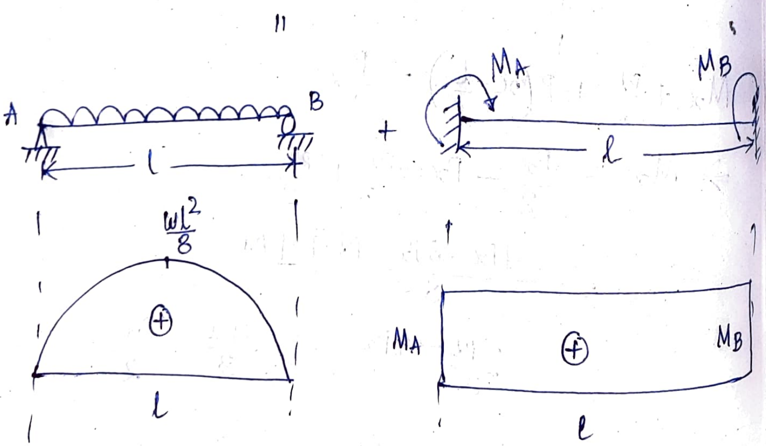
$M_l = \frac{3Pl}{8} - \frac{Pl}{2} = -\frac{Pl}{8}$



Task:  Draw SFD & BMD.

Analysis of fixed beam with udl





$A_1 = \frac{2}{3} \cdot l \cdot \frac{wl^2}{8}$

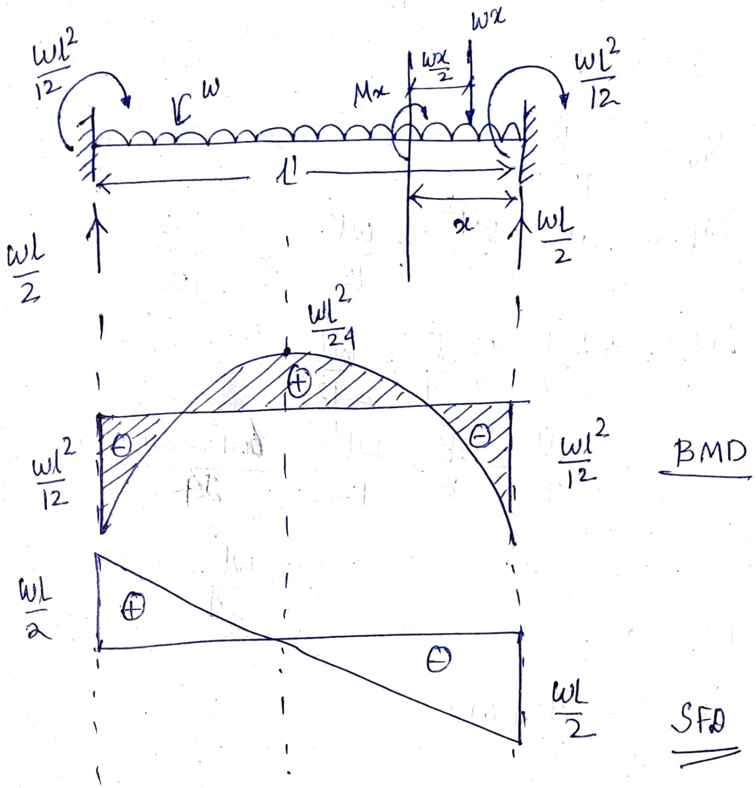
$= \frac{wl^3}{12}$

$A_2 = M_A \times l$ or $M_B \times l$.

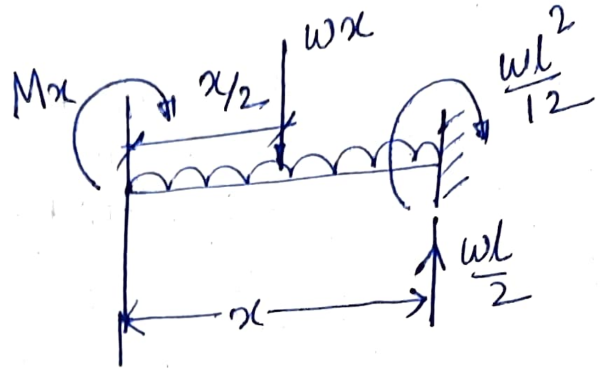
$A_1 = A_2$

$\Rightarrow \frac{wl^3}{12} = M_A l$

$\Rightarrow \boxed{M_A = M_B = \frac{wl^2}{12}}$



Take a section at a distance 'x'



$$M_x + \frac{wl^2}{12} + \frac{wx^2}{2} = \frac{wl}{2}x$$

$$\Rightarrow M_x = \frac{wl}{2}x - \frac{wx^2}{2} - \frac{wl^2}{12}$$

$0 \leq x \leq l$

$$M_0 = -\frac{wl^2}{12}$$

$$M_l = \cancel{\frac{wl^2}{2}} - \cancel{\frac{wl^2}{2}} - \frac{wl^2}{12} = -\frac{wl^2}{12}$$

$$M_{\frac{l}{2}} = \frac{wl}{2}\cdot\frac{l}{2} - \frac{w}{2}\cdot\frac{l^2}{4} - \frac{wl^2}{12}$$

$$= \frac{wl^2}{4} - \frac{wl^2}{8} - \frac{wl^2}{12} = \frac{6wl^2 - 3wl^2 - 2wl^2}{24}$$

$$= \frac{wl^2}{24}$$

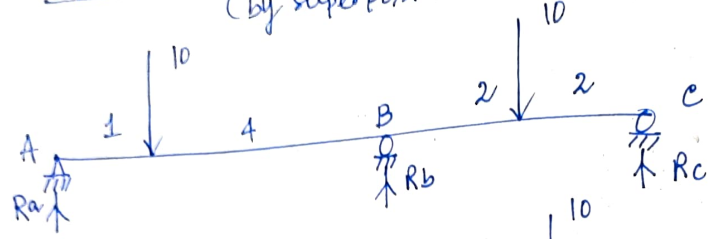
$0 \leq x \leq l$

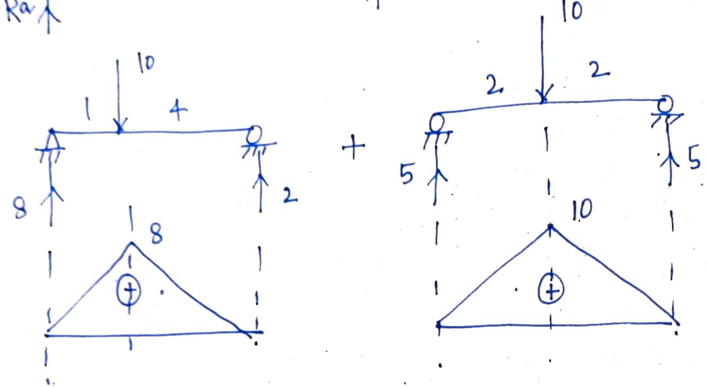
$$S_F = -\frac{wl}{2} + wx$$

$$S_0 = -\frac{wl}{2}, \quad S_l = \frac{wl}{2}$$

— o —

Two span continuous beam with point load (by superposition theorem)





We know $R_a + R_b + R_c = 20$

$\therefore \Sigma M_A = 0$ (as end of girder)

$\Sigma M_C = 0$

99

$\therefore \Sigma M_A = 0$

$\Rightarrow 9R_c + 5R_b = 70 + 10 = 80$ —— (i)

$\Sigma M_C = 0$

$\Rightarrow 9R_a + 4R_c = 80 + 20 = 100$ —— (ii)

$\therefore R_a = 8$ kN

$R_b = 7$ kN

$R_c = 5$ kN

Take a section at a distance x from A

$$M_{x-x} = R_a x - 10(x-1) + R_b(x-5) - 10(x-7)$$

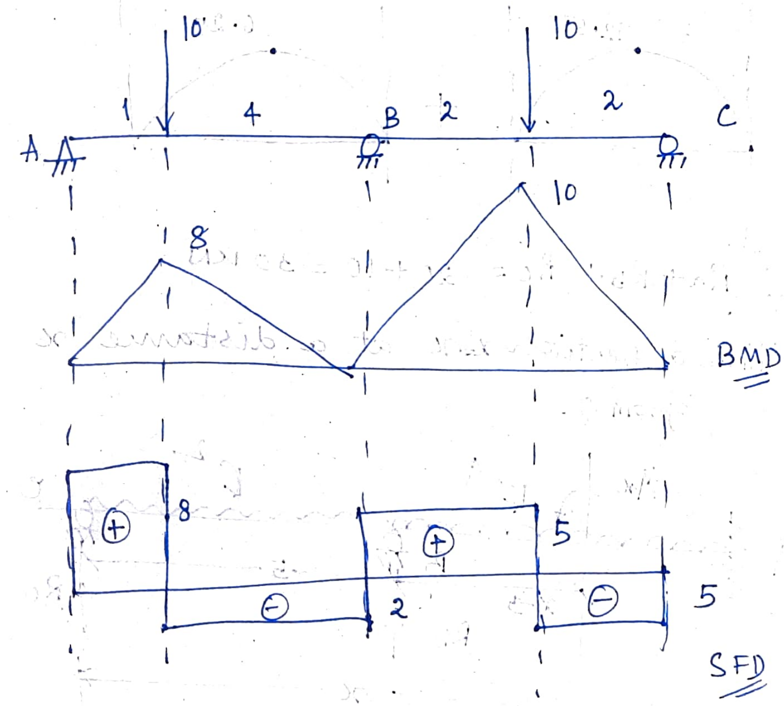
(Macaulay's Method)

at $x = 0$, $M_0 = 0$.

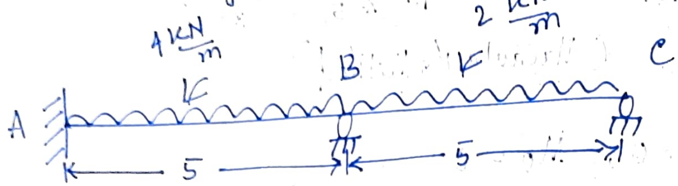
$x = 9$, $M_9 = 72 - 80 + 28 - 20 = 0$.

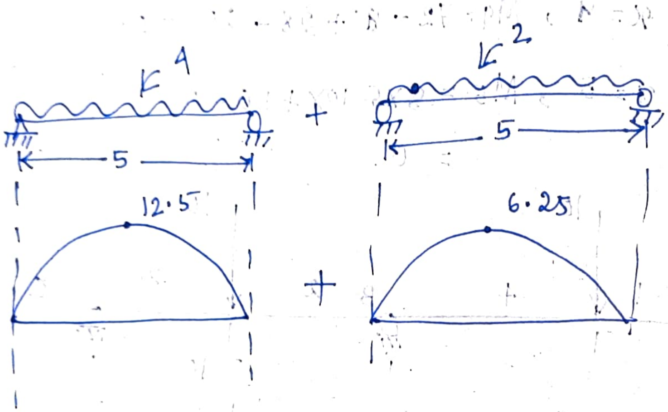
$x = 5$, $M_5 = 8 \times 5 - 10 \times 4 + 0 - 0$

$= 0$.



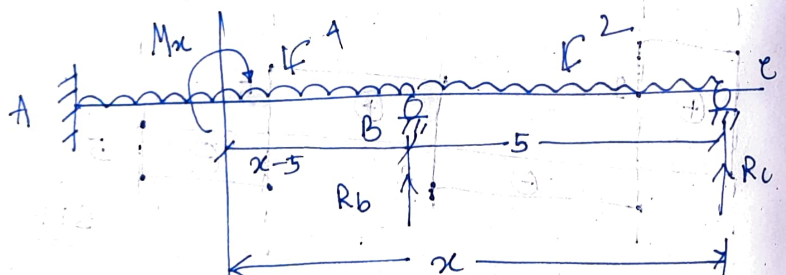
Two span continuous beam with udl (by superposition theorem)





$\therefore\ R_a + R_b + R_c = 20 + 10 = 30$ kN

Take a section x-x at a distance x from c.



Using Macaulay's Method

$$M_x = R_c x + R_b(x-5) - 10(x-2.5) - \frac{4(x-5)(x-5)}{2}$$

$$= R_c x + R_b(x-5) - 10(x-2.5) - 2(x-5)^2$$

$$EI\frac{d^2y}{dx^2} = R_c x + R_b(x-5) - 10(x-2.5) - 2(x-5)^2$$

$$\Rightarrow EI\frac{dy}{dx} = \frac{R_c x^2}{2} + \frac{R_b}{2}(x-5)^2 - 5(x-2.5)^2 - \frac{2}{3}(x-5)^3 + C_1$$

$$EIy = \frac{R_c}{6}x^3 + \frac{R_b}{6}(x-5)^3 - \frac{5}{3}(x-2.5)^3 - \frac{2}{12}(x-5)^4 + C_1 x + C_2$$

at $x=0, y=0 \Rightarrow C_2 = 0$

$x=10, y=0 \Rightarrow \frac{1000}{6}R_c + \frac{125}{6}R_b - \frac{5}{3}(7.5)^3 - \frac{5^4}{6} + 10C_1 = 0$

$x=5, y=0 \Rightarrow \frac{125}{6}R_c - \frac{(2.5)^3 \times 5}{3} + 5C_1 = 0$

$x=10, \frac{dy}{dx}=0 \Rightarrow R_c \times 50 + \frac{25}{2}R_b - 7.5^2 \times 5 - \frac{2}{3}\times 125 + 10C_1 = 0$

$166.67R_c + 20.83R_b + 10C_1 = 703.125 + 104.17 = 807.295$ — (i)

$20.83R_c + 5C_1 = 26.042$ — (ii)

$50R_c + 12.5R_b + 10C_1 = 365$ — (iii)

$R_c = 2.1$

$R_b = 23.6$

$C_1 = -3.55$

$\therefore R_a = 4.8$

$$M_x = R_c x + R_b(x-5) - 10(x-2.5) - 2(x-5)^2$$

$$= 2.1x + 23.6(x-5) - 10(x-2.5) - 2(x-5)^2$$

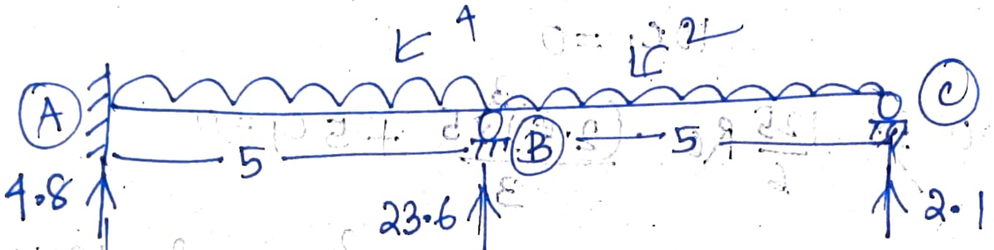
at $x = 5$

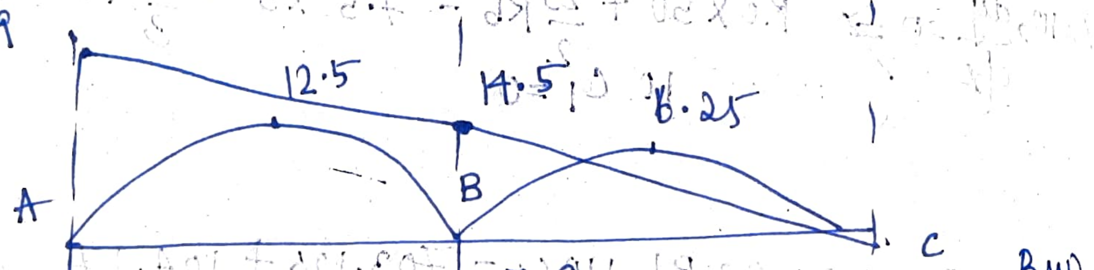
$M_B = -14.5$

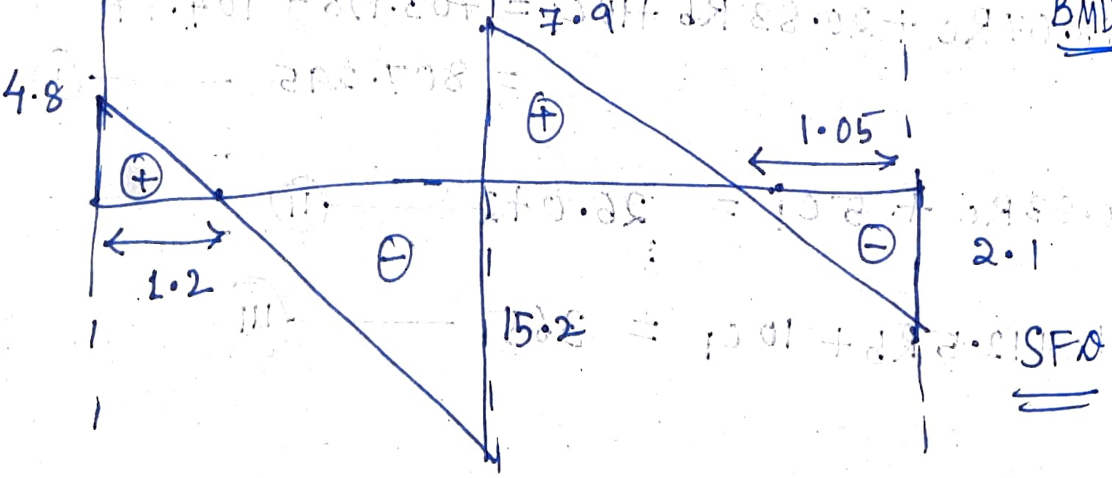
at $x = 10$

$M_A = 21 + (23.6 \times 5) - 10 \times 7.5 - 50$

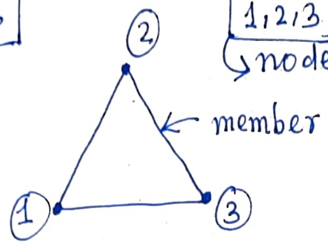
$= 19$







8. Trusses



1,2,3
↳ nodes

Types of trusses

Truss

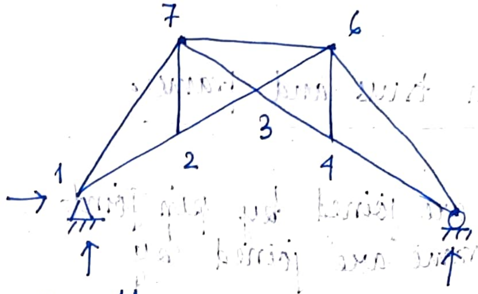
Truss is defined as the triangulation of members to make the stabilized structure.

Types of trusses

1. Simple truss → single Δ truss
2. Planar truss → 2-dimensional truss (node & member in same plane)
3. Space frame truss
   ↳ nodes and members are located in 3-dimensional space.

Statically determinate & indeterminate trusses

A truss is considered statically determinate if all of its support reactions and member forces can be calculated using only the equations of static equilibrium.



$m = 11$

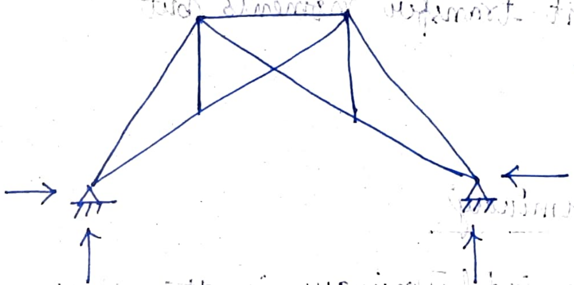
$j = 7$

$R = 3$

~~m+r~~ $\boxed{m+R = 2j}$

$11+3 = 2 \times 7$

⇒ Truss is statically determinate.



$m = 11$

$j = 7$

$R = 4$

$\boxed{m+R > 2j}$

⇒ statically indeterminate truss with 1 redundant support reaction.

for space truss

determinate $m+6 = 3j$

indeterminate $m+6 > 3j$

Extra

Difference between truss and frame.

→ Members of trusses are joined by pin joint while members of frame are joined by rigid joint.

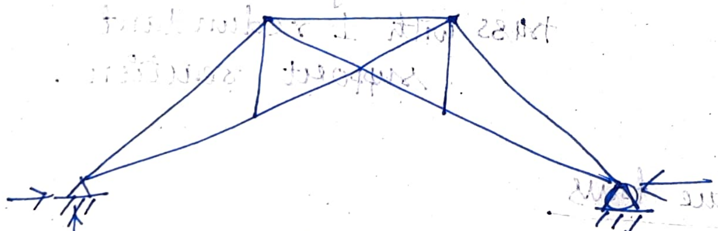
→ In truss members are free to rotate about joint but not in case of frames.

→ So truss can't transfer moments but frames can.

Degree of Indeterminacy

The degree of indeterminacy is the no. of redundant forces in the structure.

for 2D truss



$m = 11$ $\quad R = 4$
$j = 7$

$\therefore m + R > 2j$

$(15) > 14$

$\therefore$ Degree of Indeterminacy $15 - 14 = 01$

Stable & unstable truss

→ A truss is said to be stable if it is externally & internally stable.

→ A truss is said to be unstable if it is externally & internally not stable.

~~for plane truss~~

external stability:

A truss is externally stable if at
- all the reactions are not parallel to each other.
- all the reactions are not concurrent. i.e. passing through same point.

Internal stability

for plane truss

$m + r = 2j$ (stable)

$m + r < 2j$ (unstable)

$m + r > 2j$ (indeterminate)

for space truss

$m + 6 = 3j$ (stable)

$m + 6 < 3j$ (unstable)

$m + 6 > 3j$ (indeterminate)

Advantages of Truss

→ long span bridge can be designed using truss element rather than deep beam.

→ bridges using truss elements become easier even economic than that of beam.

→ The shape of the bridge is using truss element can be customized more easily.

→ Fabrication, transportation, erection of bridges using truss elements are easier than using beam elements.

→ carries heavy loads.

→ trusses are lot lighter than a beam of same flexural strength.

→ Trusses have high strength to weight ratio (specific strength = strength/weight)

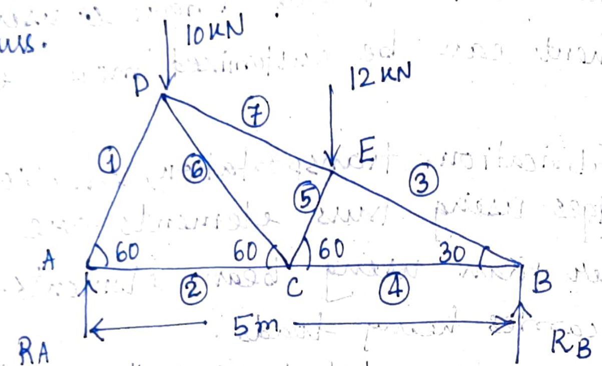
→ Truss allows the max. free/clear space between supporting frames unlike beams where intermediate supports would have needed.

→ less deflection.

5.2 Analysis of Trusses

1. Method of Joints

1. A truss of span 5m is loaded as shown. Find the reactions and forces in the members of truss.

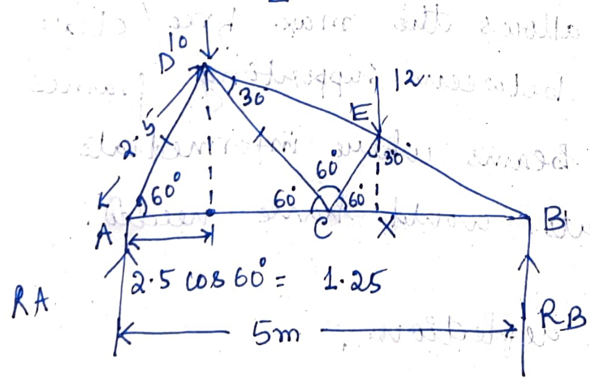


Soln

Δ ABD is a right angled Δ.

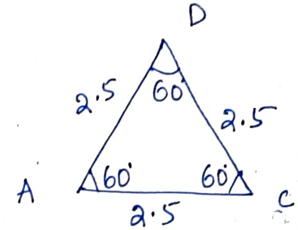
$AD = AB \cos 60°$

$= 5 \times \frac{1}{2} = 2.5$ m



$CE = CD \cos 60° = 2.5 \times \frac{1}{2} = 1.25$ m

$CX = CE \cos 60° = 1.25 \times \frac{1}{2} = 0.625$ m



$\therefore AX = AC + CX = 2.5 + 0.625 = 3.125$ m.

We know $R_A + R_B = 10 + 12 = 22$ kN

$\Sigma M_A = 0$

$\Rightarrow 5R_B = (10 \times 1.25) + (12 \times 3.125)$

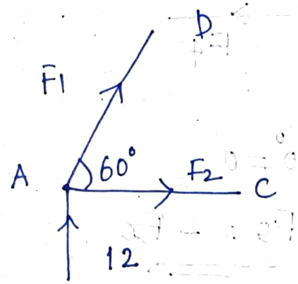
$= 50$

$\Rightarrow$ ~~RB = 10 kN ∴ RA = 12 kN~~

$\therefore R_B = 10$ kN

$R_A = 12$ kN.

~~Take~~ $\boxed{\text{Joint A}}$



$\therefore F_1 \cos 60° + F_2 = 0$

$\therefore F_1 \sin 60° + 12 = 0$

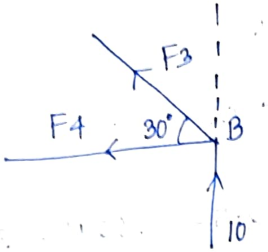
$\Rightarrow F_1 = -12 \times \frac{2}{\sqrt{3}} =$

-13.86 kN (compressive)

$\therefore F_2 = -F_1 \cos 60°$

$= -(-13.86) \cos 60° = 6.93$ kN (tensile)

Joint B

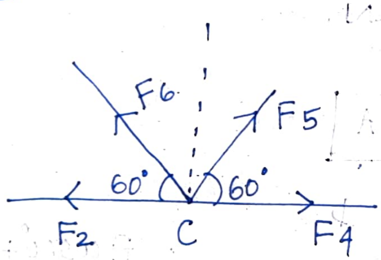


$F_3 \sin 30^\circ + 10 = 0 \Rightarrow F_3 = -20\ kN$ (compressive)

$F_3 \cos 30^\circ + F_4 = 0$

$\Rightarrow F_4 = -F_3 \times \frac{\sqrt{3}}{2} = 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3} = 17.32\ kN$ (tensile)

Joint C



$F_5 \sin 60^\circ + F_6 \sin 60^\circ = 0$

$\Rightarrow \boxed{F_5 + F_6 = 0} \Rightarrow F_5 = -F_6$

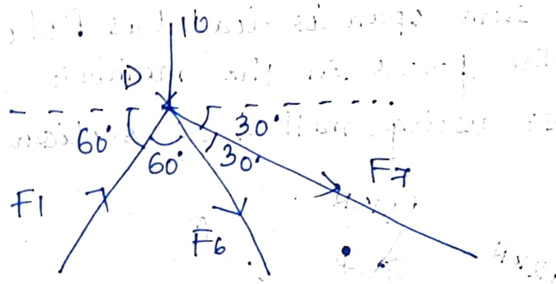
$F_5 \cos 60^\circ + F_4 = F_6 \cos 60^\circ + F_2$

$\Rightarrow \frac{F_5}{2} + 17.32 = \frac{F_6}{2} + 6.93$

$\Rightarrow \frac{F_6 - F_5}{\cancel{2}} = 20.78 \Rightarrow F_6 = 10.39\ kN$ (tension)

$\therefore F_5 = -10.39\ kN$ (compression)

Joint D

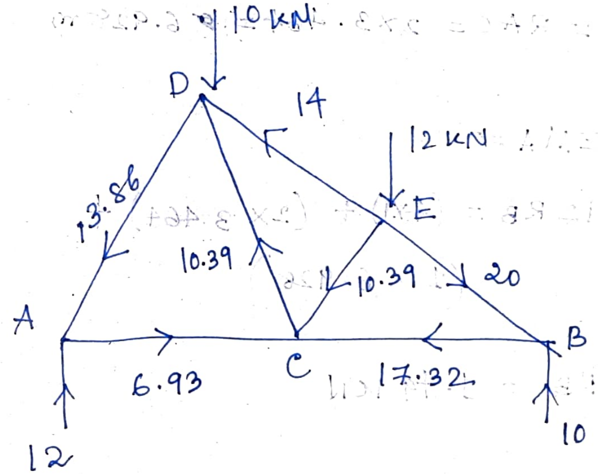


$F_1 \sin 60^\circ = 10 + F_7 \sin 30^\circ + F_6 \sin 60^\circ$

$\Rightarrow 13.86 \frac{\sqrt{3}}{2} = 10 + \frac{F_7}{2} + 10.39 \times \frac{\sqrt{3}}{2}$

$\Rightarrow 12 = 10 + \frac{F_7}{2} + 9$

$\Rightarrow -7 = F_7/2 \Rightarrow F_7 = -14\ kN$ (compressive)



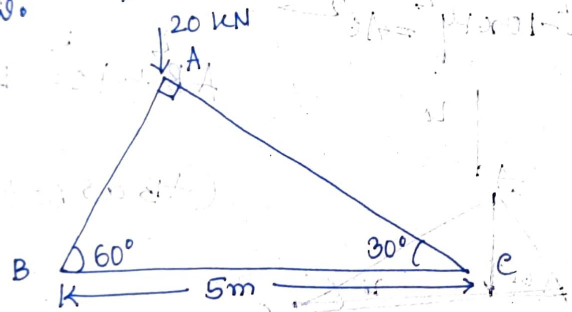
(Ans)

Method of Section

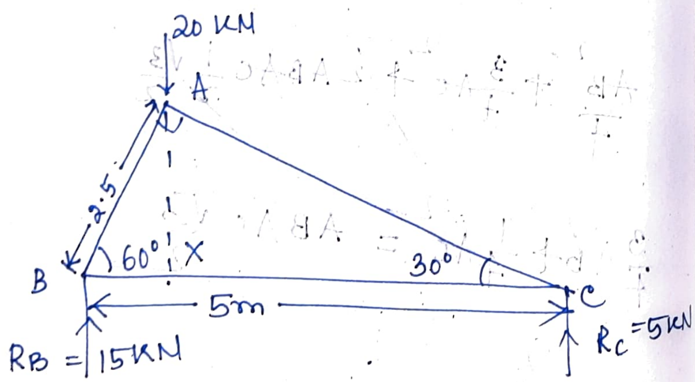
Rules

- → Section shouldn't cut more than 03 members.
- → The part of the truss on any side of the section may be treated as free body in equilibrium.

Q/ Use method of section to analyse the truss below.



Soln



$\therefore R_B + R_C = 20\text{ KN}$

$AX = AB \sin 60^\circ = AC \sin 30^\circ$

$\therefore AB \times \frac{\sqrt{3}}{2} = \frac{AC}{2}$

$\Rightarrow AC = \sqrt{3}\, AB$

Again we know $AB^2 + AC^2 = 5^2$

$\Rightarrow 4AB^2 = 5^2$

$\Rightarrow 2AB = 5$

$\Rightarrow AB = 2.5\text{ m}$

$\therefore AC = 2.5\sqrt{3}\text{ m}$

$\therefore \Sigma M_B = 0$

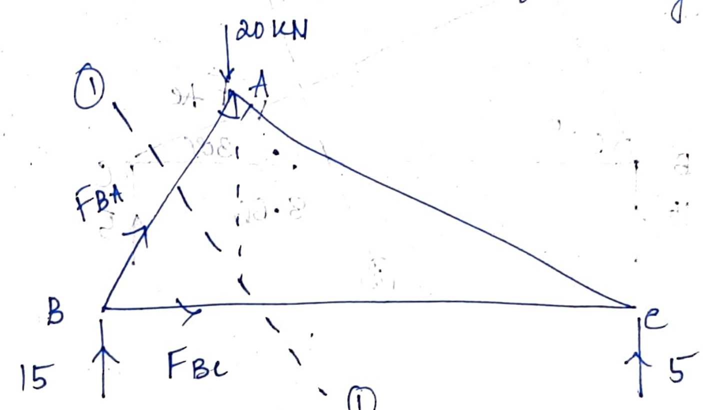
$\Rightarrow 5R_C = 20 \times BX$

$= 20 \times (2.5 \cos 60^\circ)$

$\Rightarrow R_C = \frac{20 \times 1.25}{5} = 5\text{ KN}$

$\therefore R_B = 15\text{ KN}$

Take a section ① - ① cutting AB & BC. Take left portion of section for analysis.



$\Sigma M_A = 0$

$\Rightarrow 15 \times BX = F_{BC} \times AX$

$\Rightarrow 15(2.5 \cos 60°) = F_{BC} \times (2.5 \sin 60°)$

$\Rightarrow \frac{15}{2} = F_{BC} \times \frac{\sqrt{3}}{2}$

$\Rightarrow F_{BC} = 5\sqrt{3}$ kN (tensile)

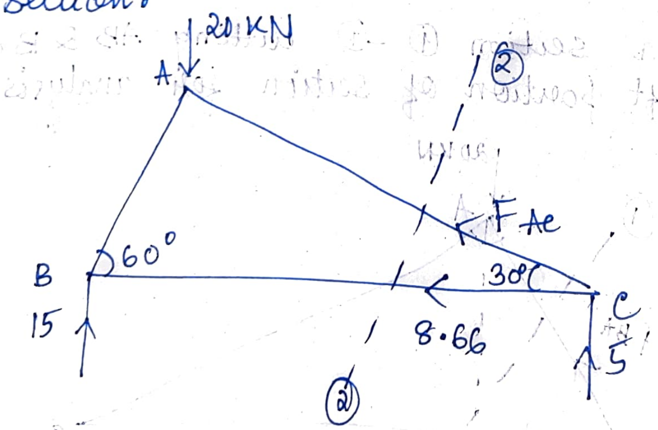
$= 8.66$ kN

$\Sigma M_C = 0$

$\Rightarrow 15 \times 5 + (F_{BA} \sin 60°) \times 5 = 0$

$\Rightarrow F_{BA} = -\frac{15 \times 2}{\sqrt{3}} = -17.32$ kN (compressive)

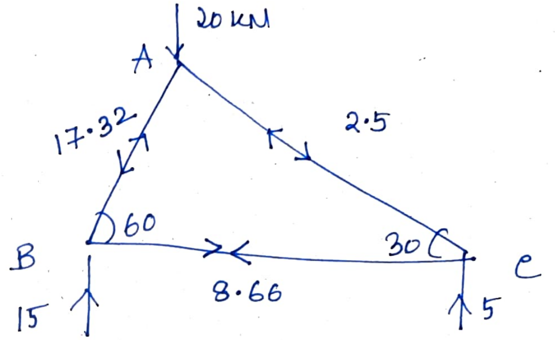
Again take a section ② — ② cutting AC & BC. Take right portion of the section.



$\Sigma M_B = 0.$

$\Rightarrow 5 \times 5 + (F_{AC} \times \sin 30°) \times 5 = 0$

$\Rightarrow F_{AC} = \frac{-5}{2} = -2.5$ kN (compressive)

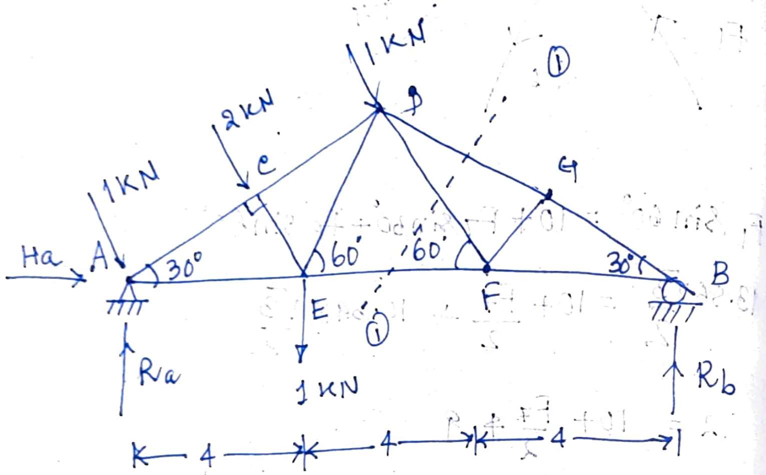


(solⁿ)

———— x ————

2. Method of Section

Q/ A truss of 12m span is loaded as shown. Determine the forces in the member DG, GF & EF using method of section.



$AC = AE\cos 30° = 4\frac{\sqrt{3}}{2} = 3.464\ m$

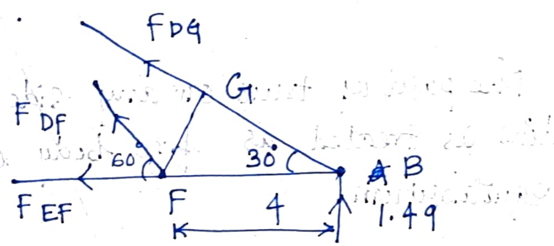
$AD = 2AC = 2\times 3.464 = 6.928\ m$

Take $\Sigma M_A = 0$

$\Rightarrow 12R_B = (4\times 1) + (2\times 3.464) + (1\times 6.928)$

$\Rightarrow R_B = 1.49\ kN$

As we required forces in the member DG, GF & EF, so take a section 1-1 passing through DG, GF & EF.



$\Sigma M_F = 0$

$\Rightarrow (1.49)\times 4 + (F_{DG}\times FG) = 0$

$\Rightarrow (1.49\times 4) + (F_{DG}\times 4\times \sin 30°) = 0$

$\Rightarrow \boxed{F_{DG} = -2.98\ kN\ \text{(compressive)}}$

$\Sigma M_D = 0$

$\Rightarrow R_B\times BD\cos 30° - F_{FE}\times BD\times \sin 30°$

$\Rightarrow \boxed{F_{EF} = 2.58\ kN\ \text{(tensile)}}$

$\Sigma M_B = 0$

$\Rightarrow F_{DF}\times \sin 60°\times (BF) = 0$

$\Rightarrow \boxed{F_{DF} = 0}$