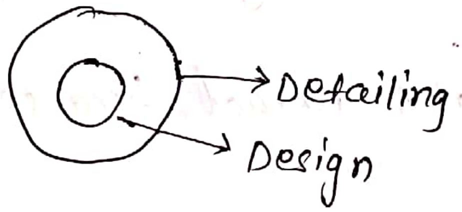
Working stress method (WSM)

Date - 21.12.19

Structural design

Structural design is the methodological investication of stability, strength, rigidity of the structures under the action of a predicted load to study.

What is the difference between design and detailing?



* Design is the part of detailing.

Objectives of design and detailing :-

(i) Stability :- prevent, overturning, sliding of structure under the action of load.

(ii) Strength :- To resist safely the stresses induce by the loads.

(iii) Servicibility :- To ensure satisfactory performance under service load condition. (To prevent deflection, cracks with vibration).

(iv) economy

(v) aesthetics

Different methods of design of concrete structures

There are 3 methods of design of concrete structures

(i) Working stress method (WSM)

(ii) Limit state method (LSM)

(iii) Ultimate load method (ULM)

(i) Working stress Method :-

(i) It is a traditional method of design

(ii) In this method assume that concrete is to be elastic

(iii) steel and concrete behaves elastically.

(iv) In this method hook's law is obeyed

(v) In this method working load is greater then permissible load.

(vi) It is outdated method because of uneconomical design.

(vii) This method is uneconomical design

(ii) Ultimate load method (ULM)

(i) working loads are increased by suitable load factor to obtain ultimate load.

(ii) Non-linear behaviour of concrete is assume

(iii) structure is design to resist ultimate load.

(iii) Limit state Method (LSM)

(i) Limit state is the state at which structure become unfit for use.

(ii) It is of two types

(i) Limit state of collapse

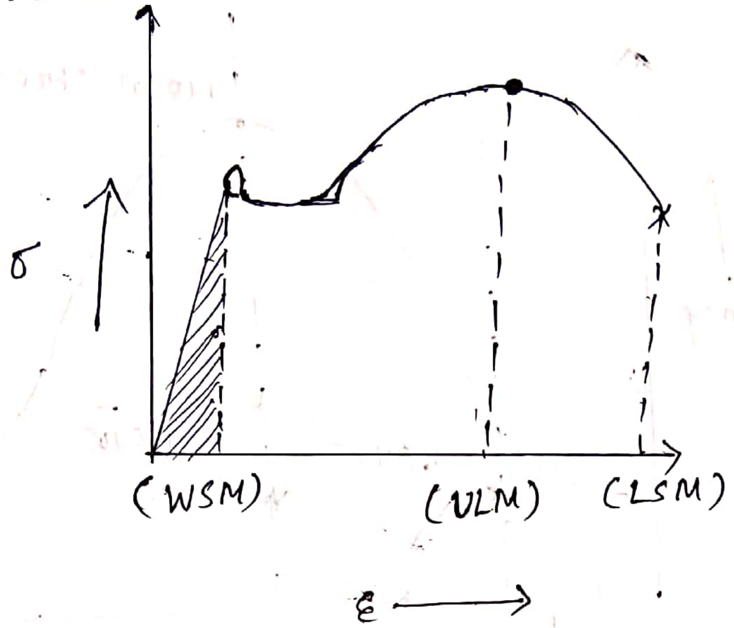
(ii) Limit state of servicibility.

(i) Limit state of collapse

It deals with the strength and stability of the structure under maximum design load.

(ii) Limit state of servicibility :-

It deals with the deflection and cracking under service load.



Reinforced concrete :-

cement concrete = cement + sand + aggregate + water

→ cement concrete is prepared by two method

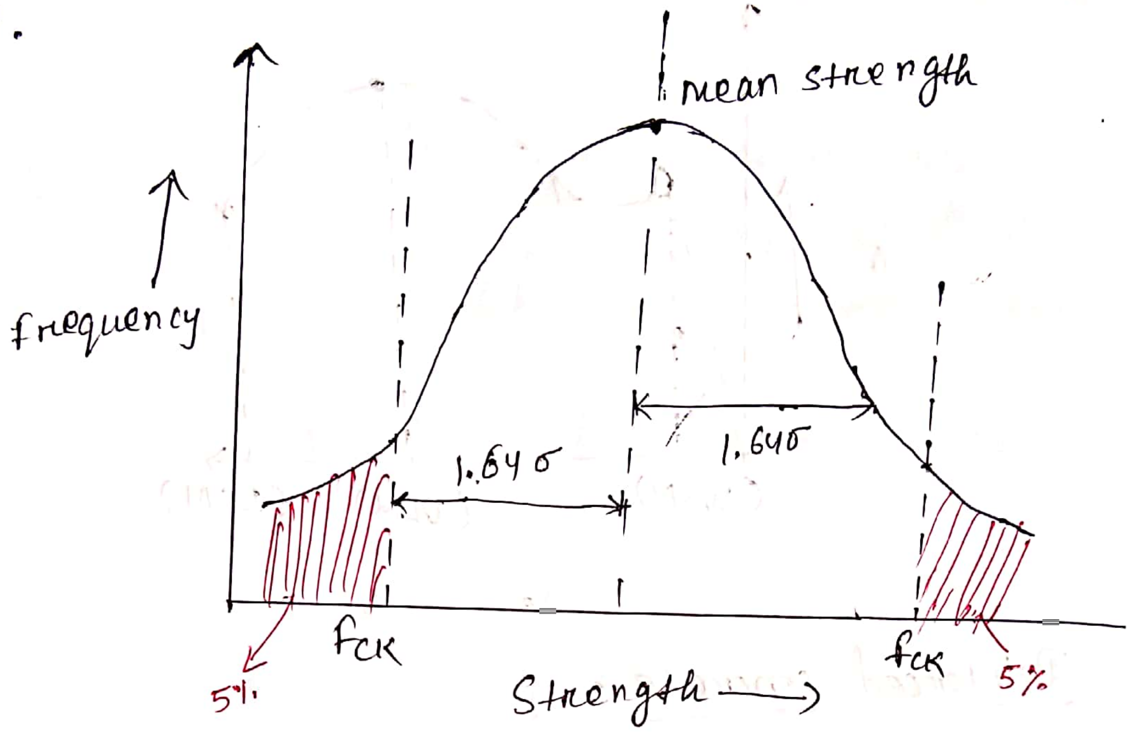
(i) Nominal mix

(ii) Design mix.

Nominal mix	Design mix
(i) Quality control is secrifice (ii) materials are not placed under Ideal condition. (iii) skill workmenship are not required.	(i) This is Ideal mix for Ideal strength. (ii) skill workmenship is used to achieve Ideal proportion of mix (iii) concrete is manufacture at the batching plant and transported to the site. (iv) plastisizers and super-plastsizers are added which does not alter the strength but delayed the setting time ex - Gypsum

characteristic compressive strength (f_{ck})

It is defined as the strength below which not more than 5% of test results expected to fail.

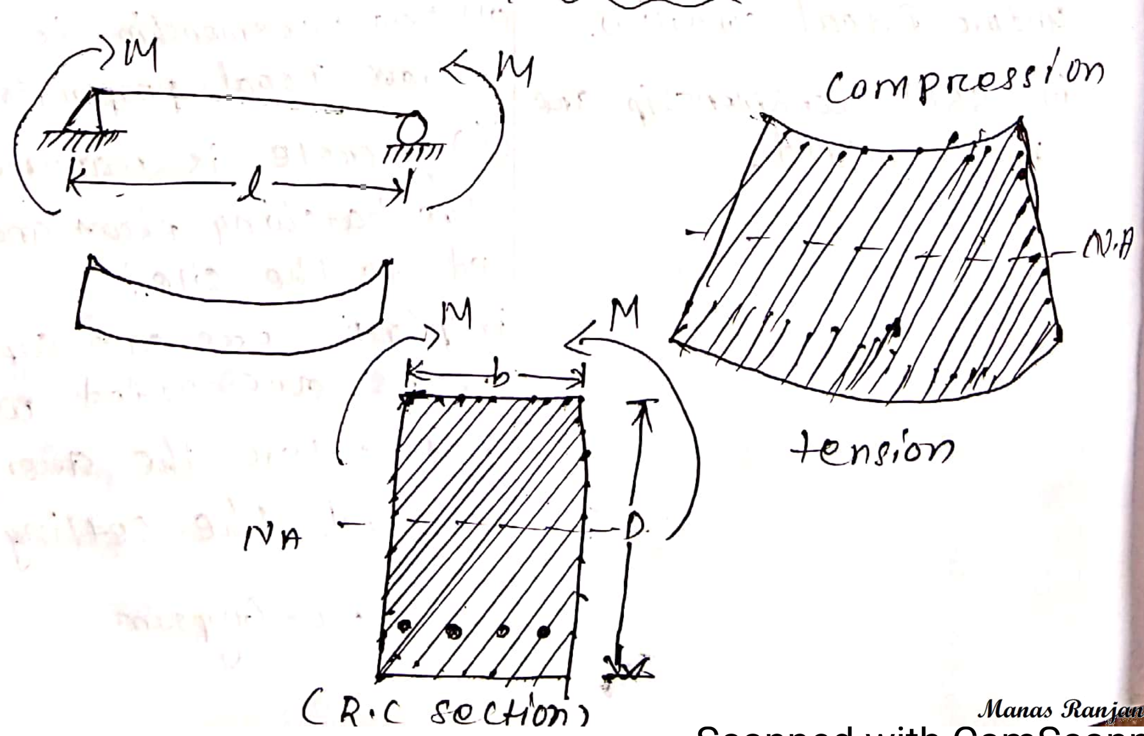


σ = standard deviation

Young's modulous of elasticity (E_c) = $5000\sqrt{f_{ck}}$

Tensile strength of concrete in flexure (f_{ct}) = $0.7\sqrt{f_{ck}}$

R.C sections and their behaviours



Grades of concrete and steel

for concrete

Grades	Proportion
M10	1:3:6
M15	1:2:4
M20	1:1.5:3

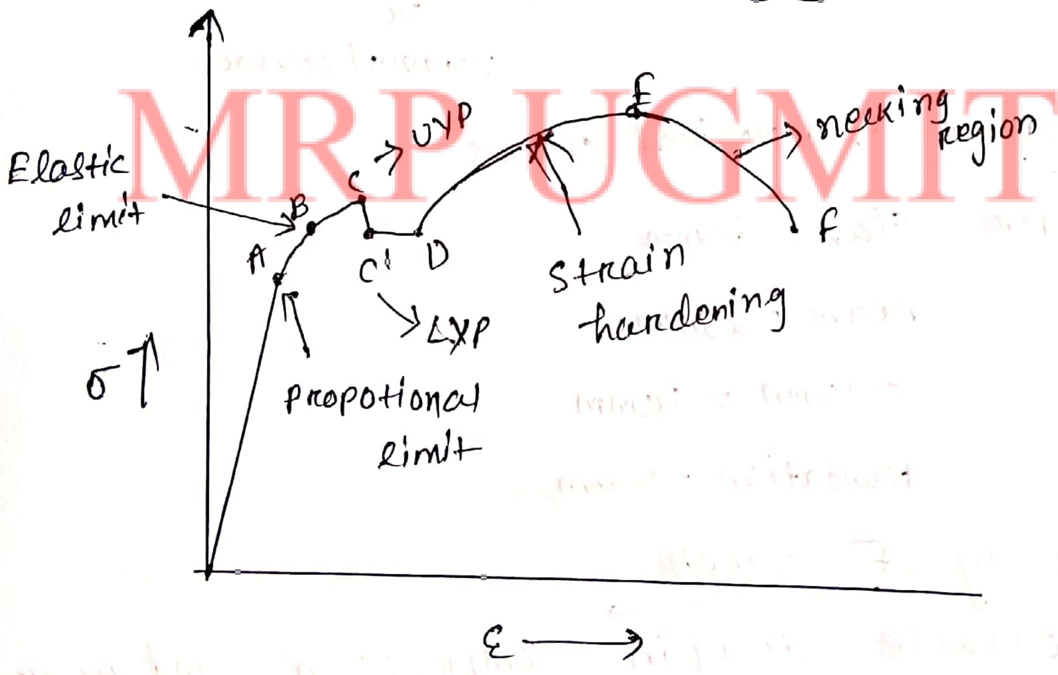
→ M20 mix, M = mix, 20 = characteristic strength at 28 days.

steel Grade

fe 250, fe 415, fe 500, fe 550

→ fe 250, fe = ferrous iron, 250 - yield stress.

Stress-Strain curve for mild steel:



Mild steel fe 250

fe → Iron

250 → yield stress/tensile strength

σ_{st} = 140 N/mm² (20 φ)

= 130 N/mm² (> 20φ)

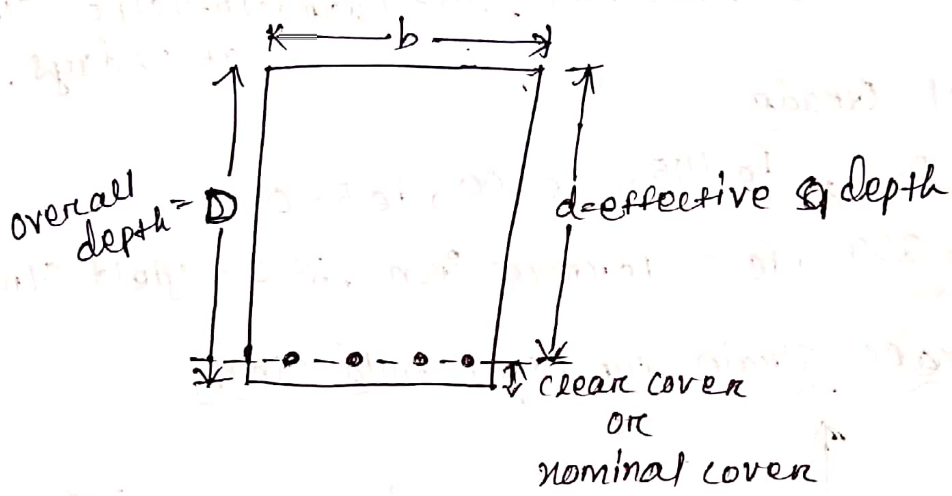
σ_{sc} = 130 N/mm²

HYSD Steel :- manufactured by hot rolled process.

fe 415, fe 500

$\sigma_{st} = 230\ N/mm^2$, $\sigma_{sc} = 190\ N/mm^2$

TMT Bars (Thermo mechanically treated



value of the Nominal covers

for Slab = 20mm

Beam = 25mm

column = 40mm

foundation = 50mm

Property of concrete

Concrete strong in compression and weak in tension.

Assumption in WSM

(i) Steel and concrete behave as linear elastic material.

(ii) Bond between steel and concrete is perfect with in elastic limit of steel.

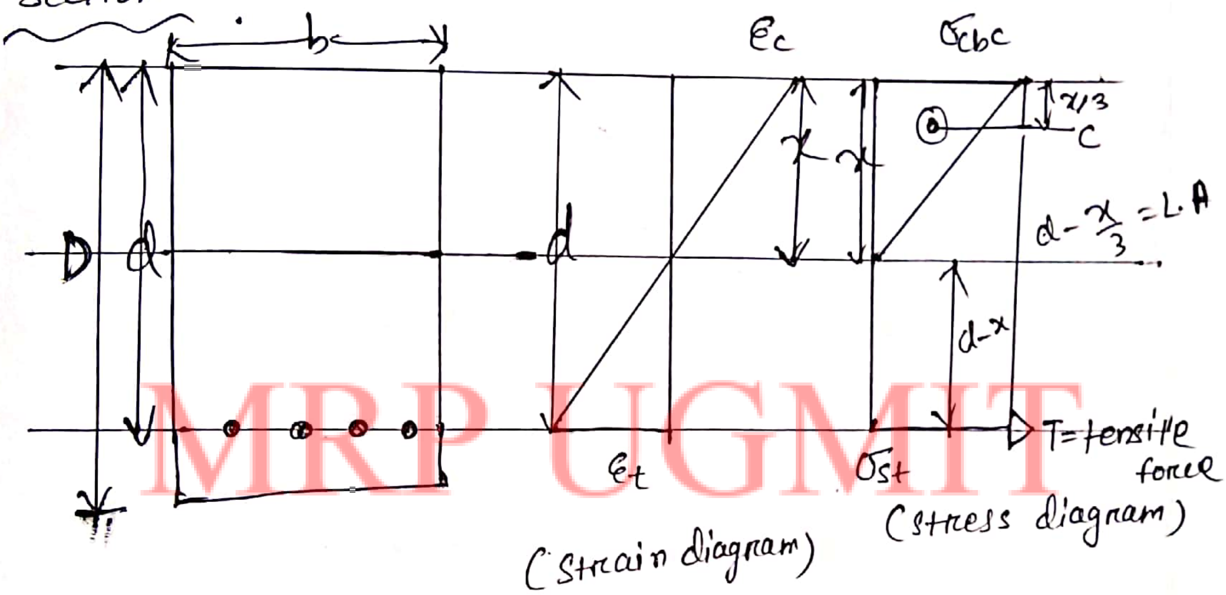
(iii) The stress in the concrete and steel is related by modular ratio.

$$\text{Modular ratio } (m) = \frac{280}{3\sigma_{cbc}}$$

(iv) The strain stress relationship between steel and concrete under working load.

(v) The tensile strength of concrete is negligible.

Flexural design and analysis of single reinforced section



(Strain diagram) (stress diagram)

Here,

D = overall depth

d = effective depth

x = Neutral axis distance from extreme compression fibre

σ_{cbc} = compressive stress at outermost compression fibre

σ_{st} = tensile stress of reinforcement.

Neutral axis

Neutral axis is the axis where either stress or strain is zero and it devides the section into two parts (compression and tension)

For stress diagram

$$\frac{\sigma_{cbc}}{x} = \frac{\sigma_{st}/m}{d-x}$$

$$\Rightarrow \frac{m\sigma_{cbc}}{\sigma_{st}} = \frac{x}{d-x}$$

$$\Rightarrow \frac{\sigma_{st}}{m\sigma_{cbc}} = \frac{d-x}{x}$$

$$\Rightarrow \sigma_{st} \times x = m\sigma_{cbc} \times d - m\sigma_{cbc} \times x$$

$$\Rightarrow (\sigma_{st} + m\sigma_{cbc})x = m\sigma_{cbc} \times d$$

$$\Rightarrow x = \frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}} \times d$$

Or $$x = \left(\frac{mc}{t+mc}\right)d$$

$$K = \frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}} = \frac{mc}{t+mc}$$

$$K = \frac{\frac{280}{3\sigma_{cbc}} \times \sigma_{cbc}}{\sigma_{st} + \frac{280}{3\sigma_{cbc}} \times \sigma_{cbc}}$$

$$= \frac{280/3}{\sigma_{st} + \frac{280}{3}}$$

$$\boxed{x = K \times d} \quad \left(K = \frac{mc}{t+mc}\right)$$

K = Neutral axis constant

m = modular ratio = $\frac{280}{3\sigma_{cbc}}$

Q Find out the neutral axis constant of a section having compressive stress 200 N/mm2 and tensile strength of 180 N/mm2.

$$K = \frac{mc}{t+mc}$$

$$= \frac{\frac{280}{3 \times 200} \times 200}{180 + \frac{280}{3 \times 200} \times 200}$$

$$= 0.341$$

* Moment of compressive area

= area in compression × distance between C.G of compressive area and Neutral axis.

$$= bx \times \frac{x}{2} = \frac{bx^2}{2} \longrightarrow (i)$$ [compressive area = b×x, CG = $\frac{x}{2}$]

* Moment of tensile area

= Distance of centroid of steel reinforcement of Neutral axis.

$$= m \times A_{st} \times (d-x) \longrightarrow (ii)$$

From equation ① & ② we get.

$$\therefore \boxed{\frac{bx^2}{2} = mA_{st}(d-x)}$$

Lever arm (jd) :-

$$d - \frac{x}{3}$$

$$\Rightarrow d - \frac{Kd}{3} \quad (x = Kd)$$

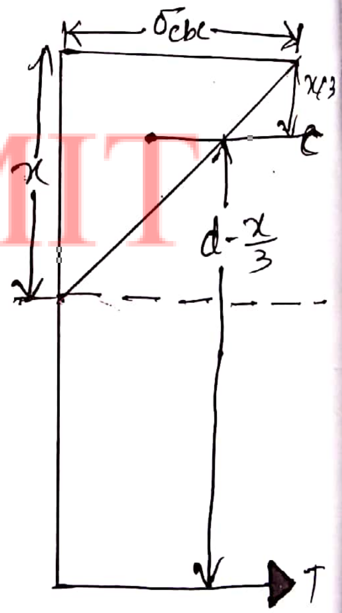
$$= d\left(1 - \frac{K}{3}\right)$$

$$= dj \quad \left(1 - \frac{K}{3} = j\right)$$

where j = Lever arm depth factor or Lever arm constant.

Moment of Resistance (MOR)

Resistance = How much moment stress can resist.



C = Compression force

= Average compressive $\times$ Area stress

T = tensile force

Moment of Resistance = C $\times$ L.A
(MOR) or
T $\times$ L.A

$$MOR = \frac{1}{2} \times \sigma_{cbc} \times bx\left(d - \frac{x}{3}\right)$$

or

$$\sigma_{st} \times A_{st}\left(d - \frac{x}{3}\right)$$

$$\therefore MOR = \frac{1}{2} c \times b \times Kd \times d\left(1 - \frac{K}{3}\right)$$

$$= \frac{1}{2} cbKd^2 \times j$$

$$\boxed{M = Q \times b \times d^2}$$

$$\boxed{Q = \frac{1}{2} cKj}$$

Q = resisting moment factor

Q

Q. A R.C beam of 200mm width and 350mm depth is acted by compressive stress of 5N/mm2 and tensile stress of 140N/mm2 find the depth of the Neutral axis, area of the steel, % of steel and moment of Resistance.

Given data,

$b = 200mm$

$d = 350mm$

$\sigma_{cbc} = 5 N/mm^2$

$\sigma_{st} = 140 N/mm^2$

$x = ?$, $A_{st} = ?$, $P_t = ?$, $MOR = ?$

$$m = \frac{280}{3 \times \sigma_{cbc}} = \frac{280}{3 \times 5} = 18.66$$

$$x = k \times d = \frac{m\sigma_{cbc}}{\sigma_{st} + m \cdot \sigma_{cbc}} \times d$$

$$= \frac{18.66 \times 5}{140 + 18.66 \times 5} \times 350$$

$$x = 139.96$$

$$\frac{bx^2}{2} = mA_{st}(d - x)$$

$$\Rightarrow \frac{200 \times (139.96)^2}{2} = 18.66 \times A_{st} (350 - 139.96)$$

$$\Rightarrow \frac{200 \times (139.96)^2}{2 \times 18.66 \times (350 - 139.96)} = A_{st}$$

$$A_{st} = 499.79 mm^2$$

P_t = % of steel

$$P_t = \frac{A_{st}}{bd} \times 100$$

$$P_t = 0.713\%$$

$MOR = Qbd^2$

$= \left(\frac{1}{2} \times \sigma_{cbc} \times k \times j\right) \times bd^2$

$= \left(\frac{1}{2} \times \sigma_{cbc} \times \frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}} \times \left(1 - \frac{k}{3}\right)\right) \times bd^2$

$= \left(\frac{1}{2} \times 5 \times \frac{18.66 \times 5}{140 + 18.66 \times 5} \times \left(1 - \frac{\frac{18.66 \times 5}{140 + 18.66 \times 5}}{3}\right)\right) \times bd^2$

$= 0.866 \times 200 \times 350^2$

$= 21217000$

* Under reinforced section

Rc beam in which steel reaches yield strain at loads lower than the loads at which concrete reaches failure strain.

→ Steel fails earlier.

→ Gives enough working before failure.

Over reinforced section

→ Opposite of under reinforced section.

→ Concrete fails earlier

→ Sudden failure

Advantages and disadvantages of WSM

Advantages

→ It is a simple method

→ Due to simplicity it is used for design of some structure such as over-head water tank.

→ Gives large section, so less serviceability.

less deflection

Disadvantages.

→ Actual factor of safety is not known.

→ Different types of load acting simultaneously have different. degrees of uncertaincies.

→ Large section so uneconomical.

→ It assumes $\sigma \alpha \varepsilon$ (linear) which is not true.

Balanced section.

RC beams sections in which tension steel reaches yield strain simultaneously as concrete reaches failure strain is known as balanced section

Chp-2 Philosophy of limit state method (LSM)

Defination of LSM

It referes to the method which considers the ultimate strength of the material at failure and also assumes that structures is serviceable for intended period of design.

Advantage of LSM over WSM

In WSM design service load is consider in design but in LSM the structures are designed to withstand the load at which failure occurs.

(Safety + servicibility)

design load → Safety; servicibility → service load

→ The structure is economical.

* Types of limit state

① Limit state of collapse

② Limit state of servicibility.

① Limit state of collapse.

It deals about strength and stability of the structure under maximum design load.

② Limit state of servicibility.

It deals with the deflection and cracking under service load.

Partial safety factor for material strength.

$$\underset{(WSM)}{\text{factor OF safety}} = \frac{\text{yield stress}}{\text{permissible stress}}$$

partial safety factor :- Loads are multiplied
(LSM)
to partial safety factor to obtain design load.

Two types

* partial factor of safety for load (0.6 - 1.5)
* partial factor of safety for material strength (1.1 - 1.5)

Characteristic strength.

The term 'characteristic strength' means the value of the strength of material which not more than 5 percent of the test results are expected to fall.

The characteristic strength, The characteristic value shall be assumed as the minimum yield stress/0.2 percent proof stress specified in the relevant Indian standard specifications.

Characteristic load.

The term 'characteristic load' means that value of load which has a 95 percent probability of not being exceeded during the life of the structure.

Design load

The load is assumed for design of a structure.
ex - suppose a crane can lift max 50kg so it is designed for 50kg or less.

* **Various other IS specification.**

$$\frac{\text{minimum reinforcement}}{\text{(slab)}} = 0.12\% \text{ of gross area } (B \times D) \text{ of slab}$$

(If HYSD)

$= 0.15\%$ of BD

(mild steel)

for Column = 0.8% of gross section

* **maximum spacing of main bars in slab**

min $\begin{cases} 3d \\ 300mm \end{cases}$ (d = effective diameter)

* **maximum spacing of distribution bars in slab**

min $\begin{cases} 5d \\ 450mm \end{cases}$

→ Diameter of reinforcing bar $\ngtr \frac{1}{8} \times D$

$$\boxed{d \ngtr \frac{D}{8}}$$

→ minimum amount of reinforcement steel (A_{st}) in beam

$$\frac{(A_{st})_{min}}{BD} = \frac{0.87}{f_y}$$

$$\frac{(A_{st})_{min}}{Bd} = \frac{0.85}{f_y} \quad (IS-456)$$

Bending

Shear

$$\frac{A_{st}}{bS_v} \geq \frac{0.4}{0.87 f_y}$$

Cover to reinforcement in slab. - 25mm

Beam - 30mm

Column - 40mm

footing — 50-70mm

Lapping, anchorage and effective span of beam and slabs.

Lapping - It should be avoided in the tensile zone of member

→ Two piece of reinforcement bars are overlapped

→ Transmit the load from one bar to another bar as well as retain continuity.

Anchorage.

Anchorage bonds are provided to avoid slipping of reinforcement from the concrete.

effective span

It is c-c distance between two supports and depends upon end conditions of supports.

* clause 22.2 of IS-456

Chapter (3) Analysis & Design of Single & Double Reinforced Section (LSM)

LSM → collapse
→ serviceability

limit state of collapse (flexure)

→ Limit state is a condition just before collapse.

→ A structure designed by limit state should give proper strength & serviceability throughout its life.

→ The limit state of collapse deals with the safety of the structure & limit state of serviceability deals with the durability of the structures.

Assumptions

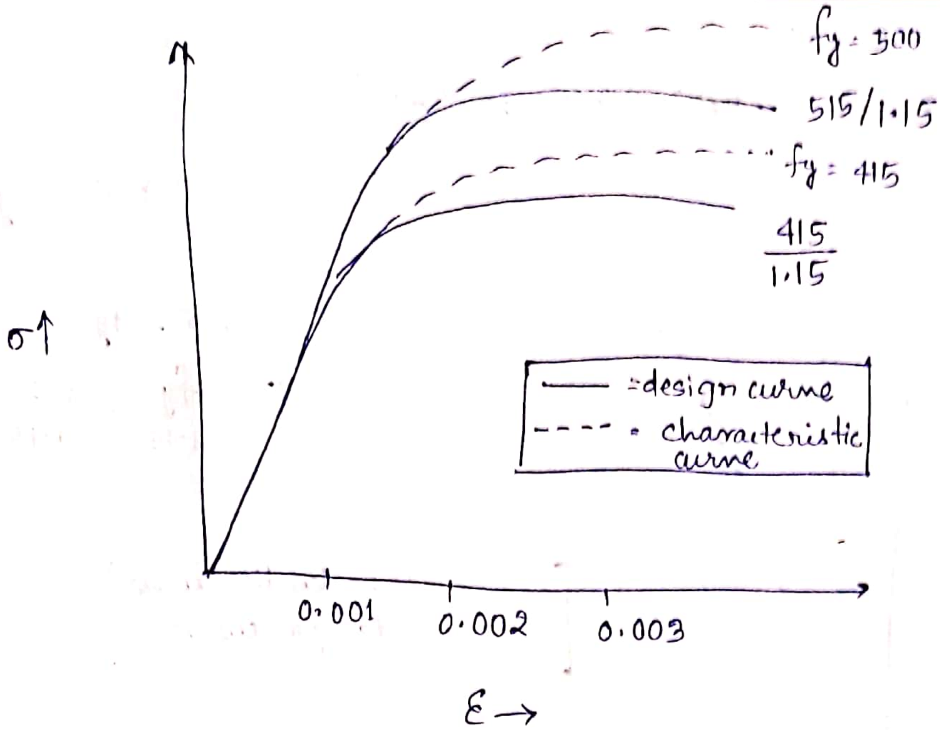
1 - Plane section normal to the axis remain plain after bending. (strain $\propto$ distance from N.A.)

2 - The max. strain at outermost compression fibre is taken as 0.0035 in bending.

3 - The relationship between stress-strain distribution in concrete is assumed to be parabolic.

4 - The tensile strength of concrete is neglible.

5 - The stress in the reinforcement are taken from the stress-strain curve for the type of the steel used.

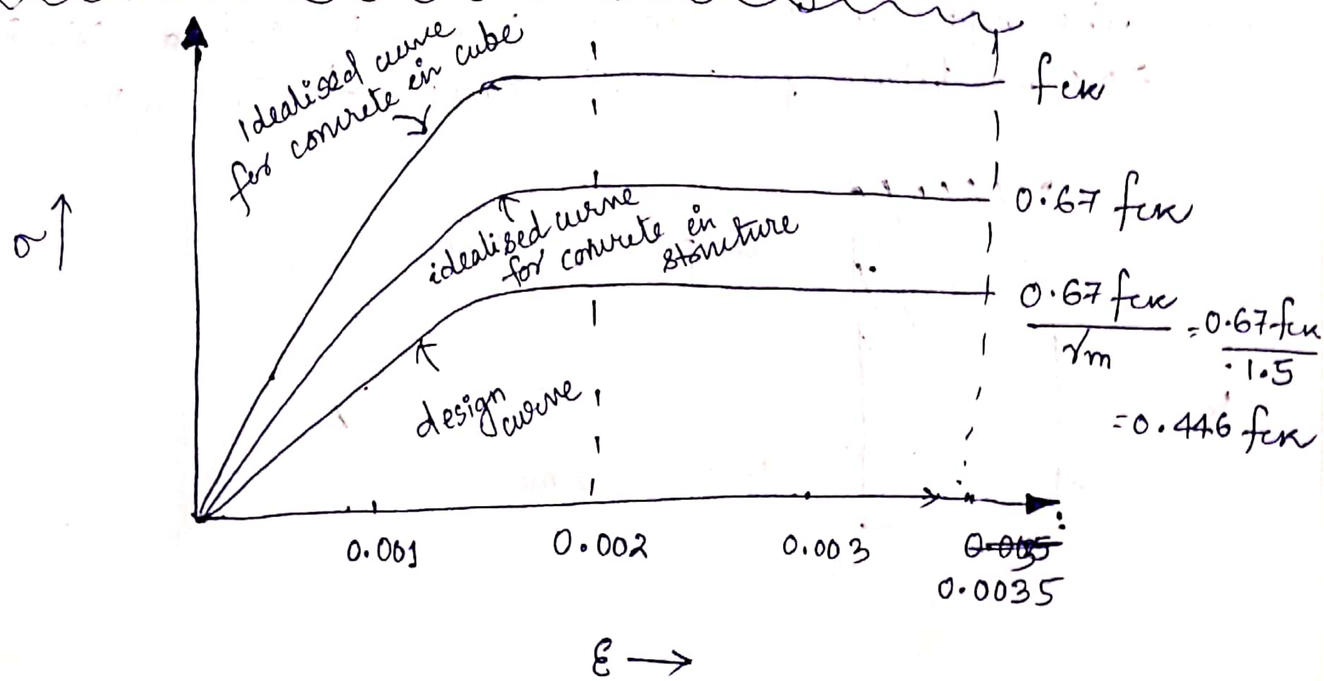


(Stress - strain curve for high strength deformed bars)

6 - The max. strain in the tension reinforcement in the section at failure $\not<$ $\frac{f_y}{1.15\,E_s} + 0.002$

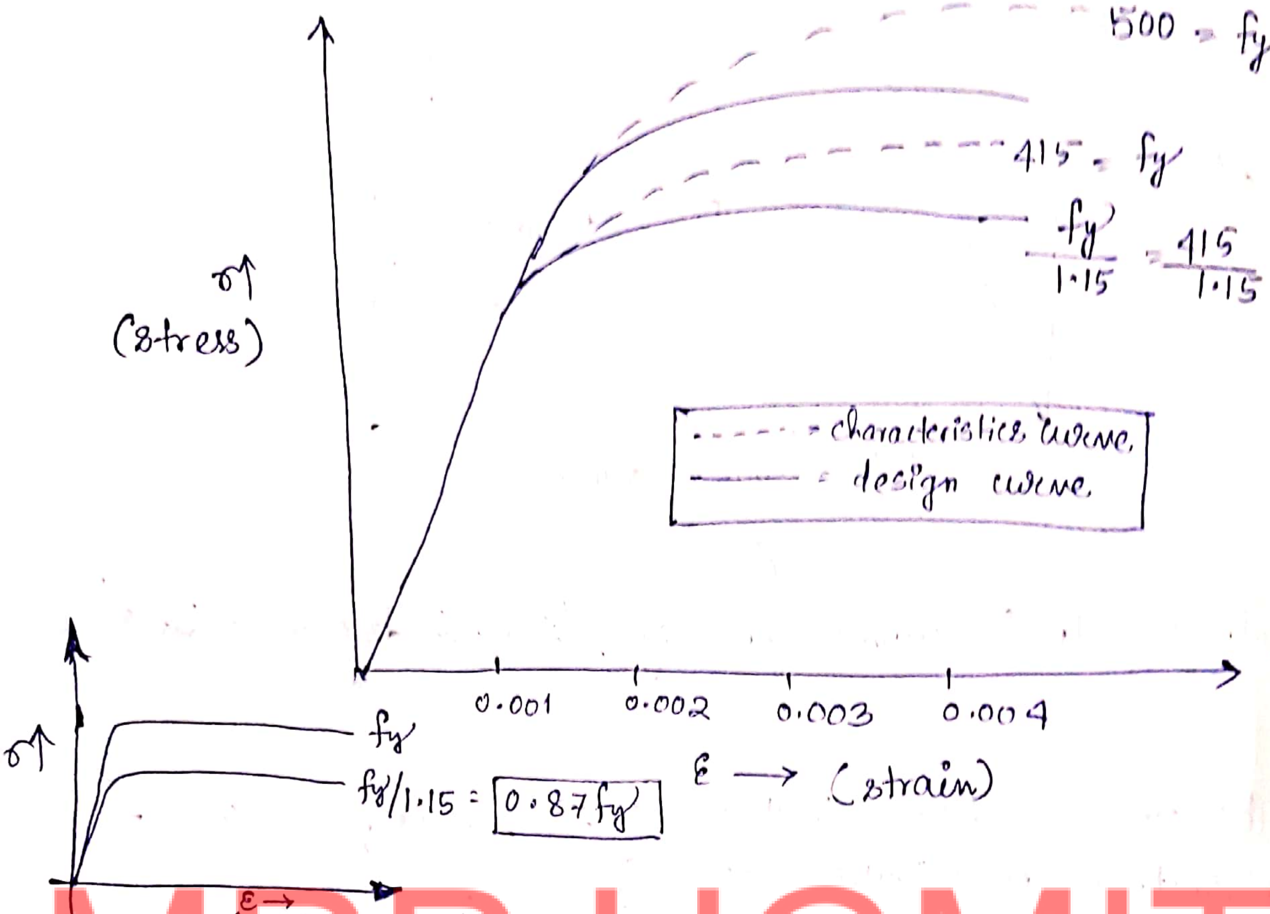
(f_y = characteristic strength, E_s = modulus of Elasticity)

Stress - Strain Relationship for concrete & steel



(σ - ε relationship for concrete)

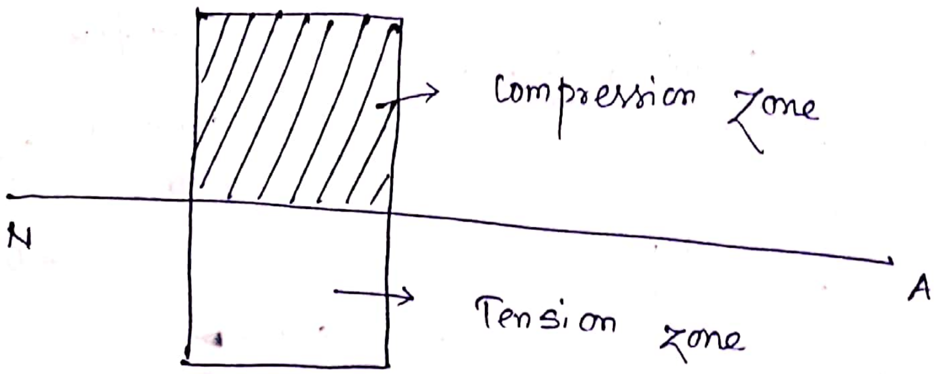
Stress - strain relationship for Steel



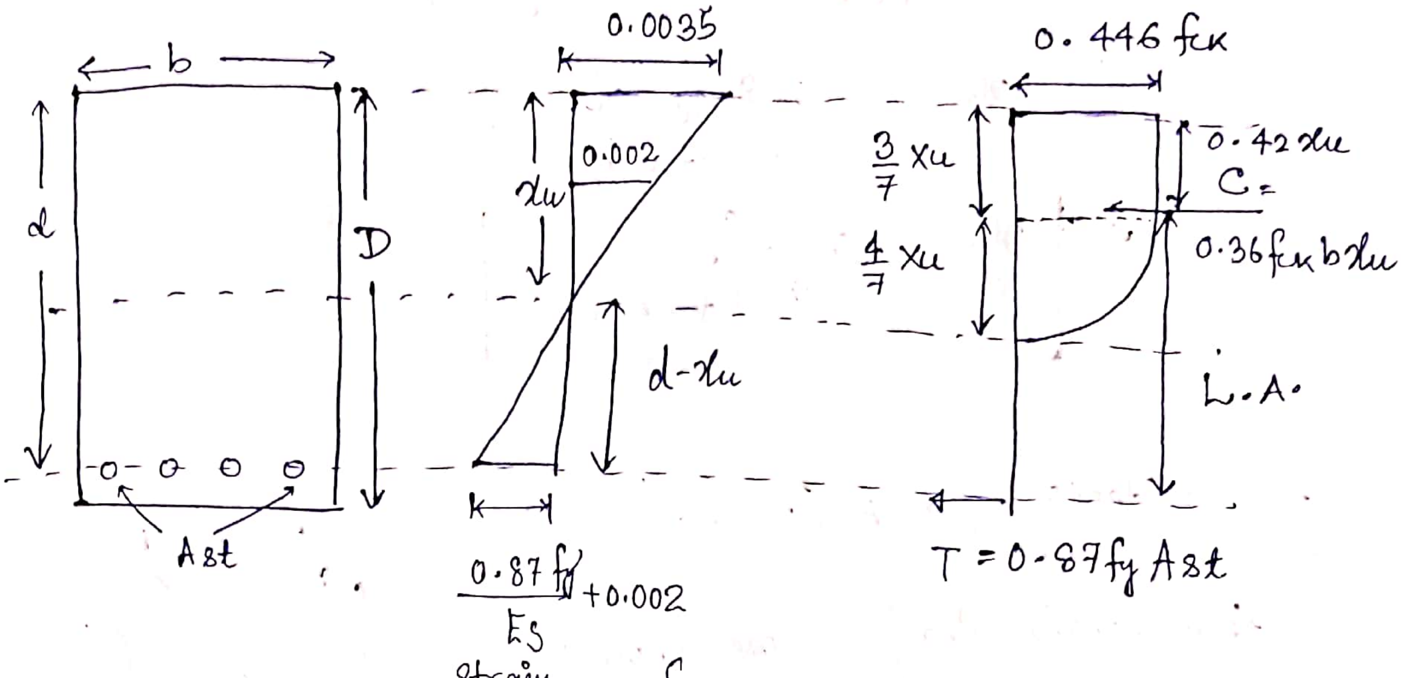
Neutral Axis (NA)

→ It is the axis where stresses are zero & it is situated at the C.G. of the section.

→ The upper portion above N.A. is called compression zone & below portion is called tension zone.



Stress & Strain block diagram of Singly Reinforced Section



Section | Strain ~~Stress~~ diagram | Stress diagram

where b = width of section

d = effective depth

D = overall depth

A_{st} = area of steel in tension zone $= n \times \frac{\pi}{4} \times \phi^2$ (ϕ = dia. of bar)

x_u = N.A. distance from compression zone

C = force of compression $= 0.36 f_{ck} b x_u$

T = force of tension $= 0.87 f_y A_{st}$

L.A. = lever arm = $d - 0.42 x_u$

→ The depth of N.A. can be calculated by taking equilibrium of tensile (T) & compressive (C) forces.

i.e. $C = T$

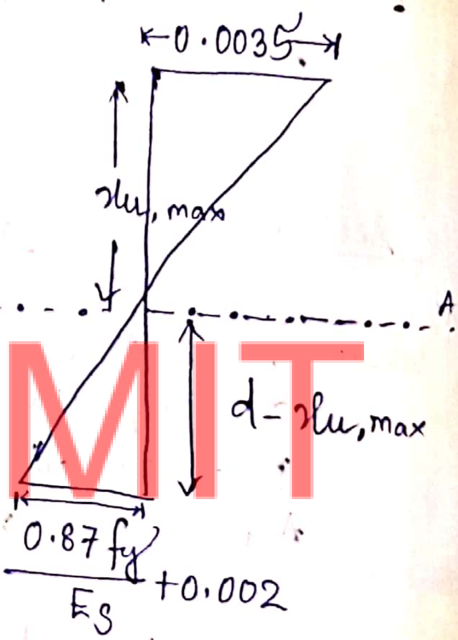
$$\Rightarrow 0.36 f_{ck} b x_u = 0.87 f_y A_{st}$$

$$\Rightarrow x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$\Rightarrow \boxed{\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b d}}$$

Limit depth of N.A. ($x_{u,max}$)

from strain diagram we know



$$\frac{0.0035}{x_{u,max}} = \frac{(0.87 f_y/E_s) + 0.002}{d - x_{u,max}}$$

$$\Rightarrow \boxed{\frac{x_{u,max}}{d} = \frac{0.0035}{\frac{0.87 f_y}{E_s} + 0.0055}}$$

for		
	$f_y = 250$	$\frac{x_{u,max}}{d} = 0.53$
	$f_y = 415$	$\frac{x_{u,max}}{d} = 0.48$
	$f_y = 500$	$\frac{x_{u,max}}{d} = 0.46$

Moment of Resistance

It is equal to the moment of couple formed by two equal & opposite forces (C & T)

M_u = ultimate MOR = C × L.A.

or

T × L.A.

$$\boxed{M_u = 0.36 f_{ck}\, b\, x_u (d - 0.42 x_u) \quad \text{or} \quad M_u = 0.87 f_y A_{st} (d - 0.42 x_u)}$$

limiting value for moment of resistance :-

$$\boxed{M_{u,lim} = 0.36 f_{ck}\, b \frac{x_{u,max}}{d}\left(1 - 0.42 \frac{x_{u,max}}{d}\right) b\, d^2}$$

$$M_{u,lim} = 0.36 f_{ck}\, b \frac{x_{u,max}}{d}\left(1 - 0.42 \frac{x_{u,max}}{d}\right) d^2$$

$\Rightarrow \boxed{M_{u,lim} = Q b d^2}$ { Q = limiting ~~value of moment~~ coefficient }

where $Q = 0.36 f_{ck} \frac{x_{u,max}}{d}\left(1 - 0.42 \frac{x_{u,max}}{d}\right)$

Percentage of Steel (P_t)

$$\boxed{p_t = \frac{A_{st}}{b \times d} \times 100}$$

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Balanced Section

→ In balanced Section the steel reinforcement reaches the yield strain $\left(\frac{0.87 f_y}{E_s} + 0.002\right)$ at the same time the concrete reaches ultimate strain (0.0035).

Under Reinforced Section

→ In this case the steel fails first by reaching its yield strain value.

→ As steel fails first it gives sufficient warning before failure, so, under reinforced section is preferred by designers.

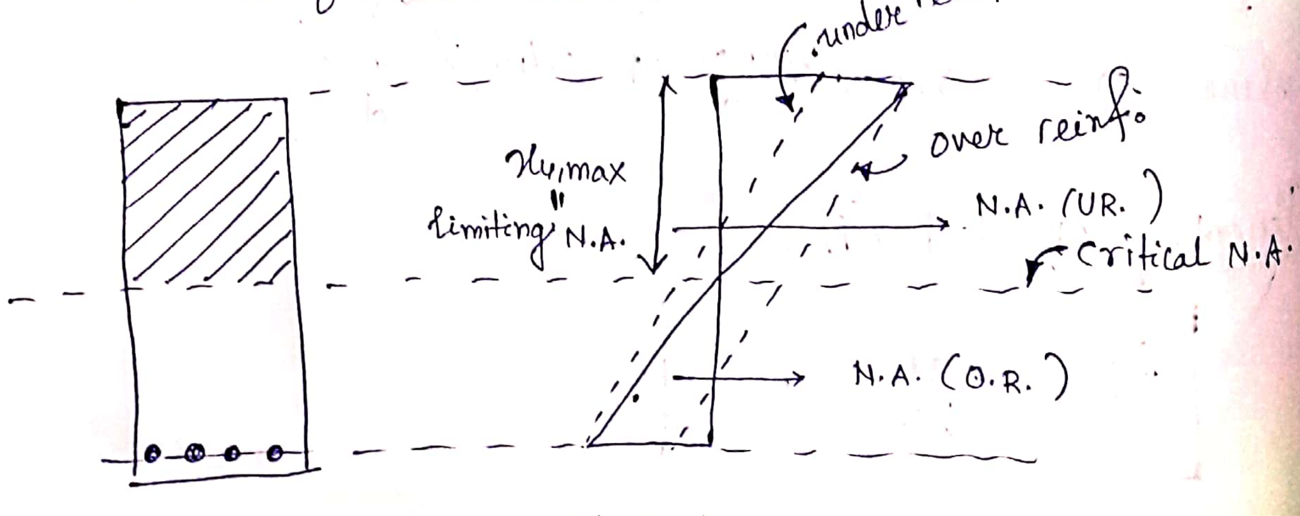
→ N.A. shifts upward.

Over Reinforced Section

→ In this case concrete fails first by reaching its ultimate strain.

→ So structure fails by ~~cr~~ crushing failure of concrete, hence not preferred.

→ N.A. shifts downward.



Strain diagram

Analysis of Design

→ for the given f_{ck} & f_y, find the N.A.

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

→ find the limiting N.A. ($x_{u,max}$)

→ compare x_u & $x_{u,max}$.

→ $x_u < x_{u,max}$ ⇒ beam is under reinforced.
$x_u \approx x_{u,max}$ ⇒ " balanced.
$x_u > x_{u,max}$ ⇒ " over reinforced.

Problem 1

Analyse a rectangular beam 300 mm × 500 mm (d) to determine the M_u for the tension reinforcement of 4-16 ϕ bars. Consider M20 concrete & Fe415 steel.

Problem 2

Determine the moment of resistance of a beam of dimension 250 mm × 350 mm. The area of steel consists of 3 bars of 12 mm diameter placed at a distance of 40 mm from bottom of beam. Use M20 & Fe415.

Problem 3

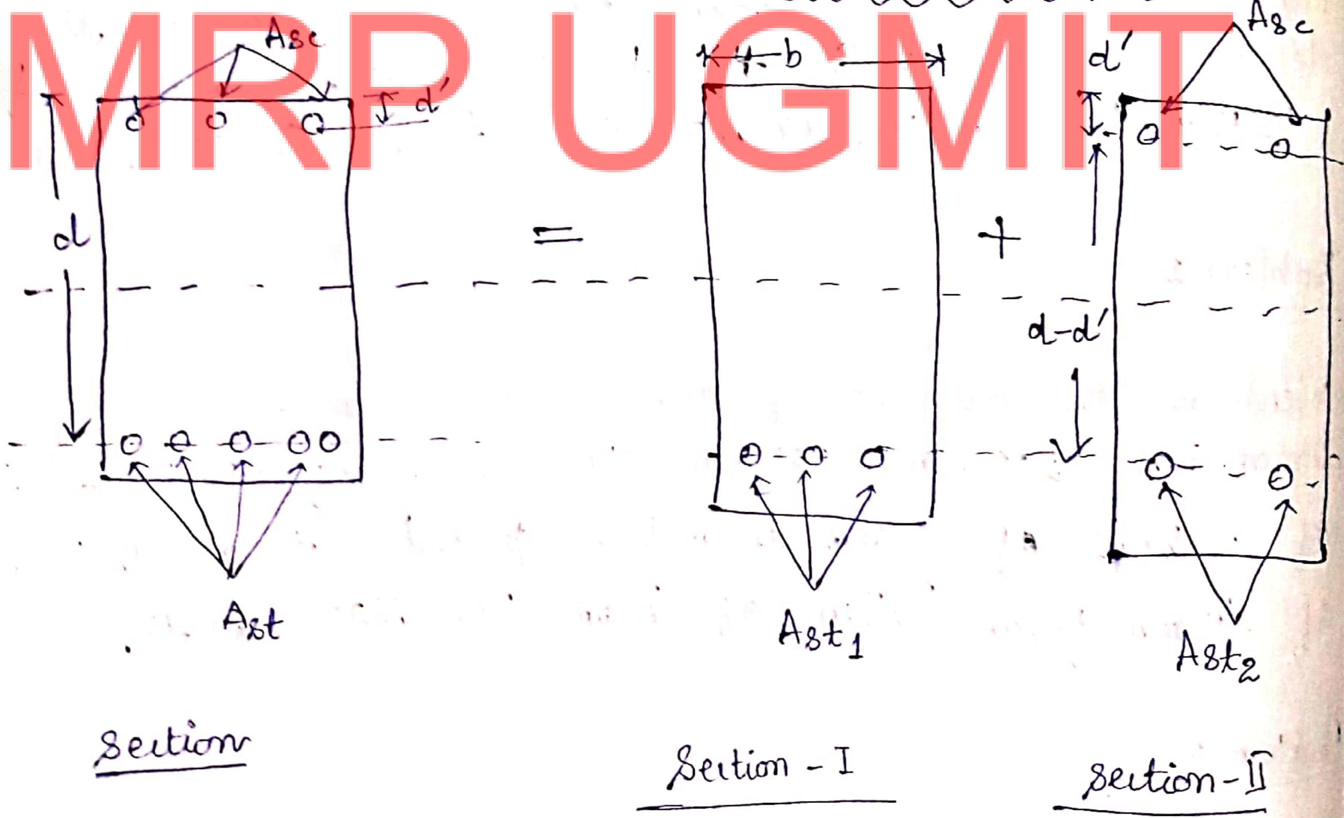
A rectangular beam is 20 cm wide & 40 cm deep upto the centre of reinforcement. find the area of reinforcement required if it has to resist a moment of 25 kNm. Use M20, Fe415.

Necessity of doubly reinforced Section

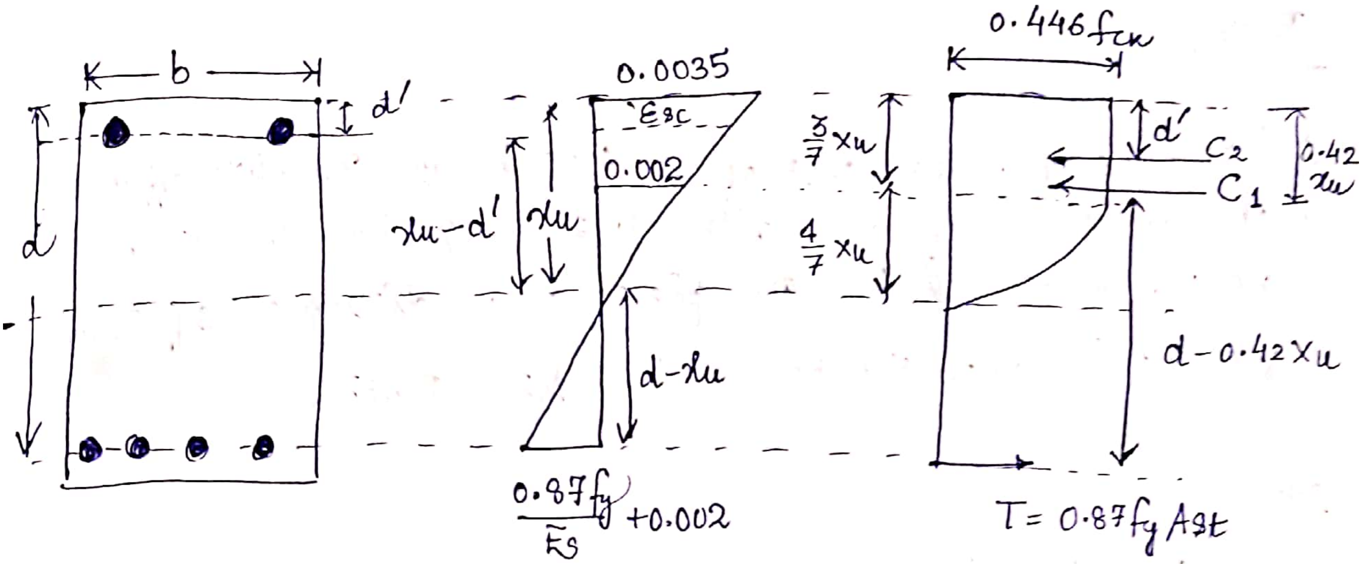
→ Double reinforced ~~se~~ beams/sections are provided to increase the moment carrying capacity / moment of resistance of the section.

→ The moment carrying capacity of beam can be increased by increasing the depth of section but it is not always possible to increase the depth of the beam because of architectural / aesthetics restrictions.

Design of doubly reinforced rectangular Section :-



$$\boxed{M_u = M_{u,lim} + M_{u2}}$$



Total compression $(C) = C_1 + C_2$

$$= 0.36 f_{ck} b x_u + (f_{sc} - f_{cc}) A_{sc}$$

Total tension $(T) = 0.87 f_y A_{st}$

As $C = T$

$$\Rightarrow 0.36 f_{ck} b x_u + (f_{sc} - f_{cc}) A_{sc} = 0.87 f_y A_{st}$$

$$\Rightarrow \boxed{x_u = \frac{0.87 f_y A_{st} - (f_{sc} - f_{cc}) A_{sc}}{0.36 f_{ck} b}}$$

Moment of Resistance for double reinforced section

$$M_u = M_{u,lim} + M_{u2}$$

$$\boxed{M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u) + (f_{sc} - f_{cc}) A_{sc} (d - d')}$$

f_{sc} = stress at compression steel level (ε_{sc})

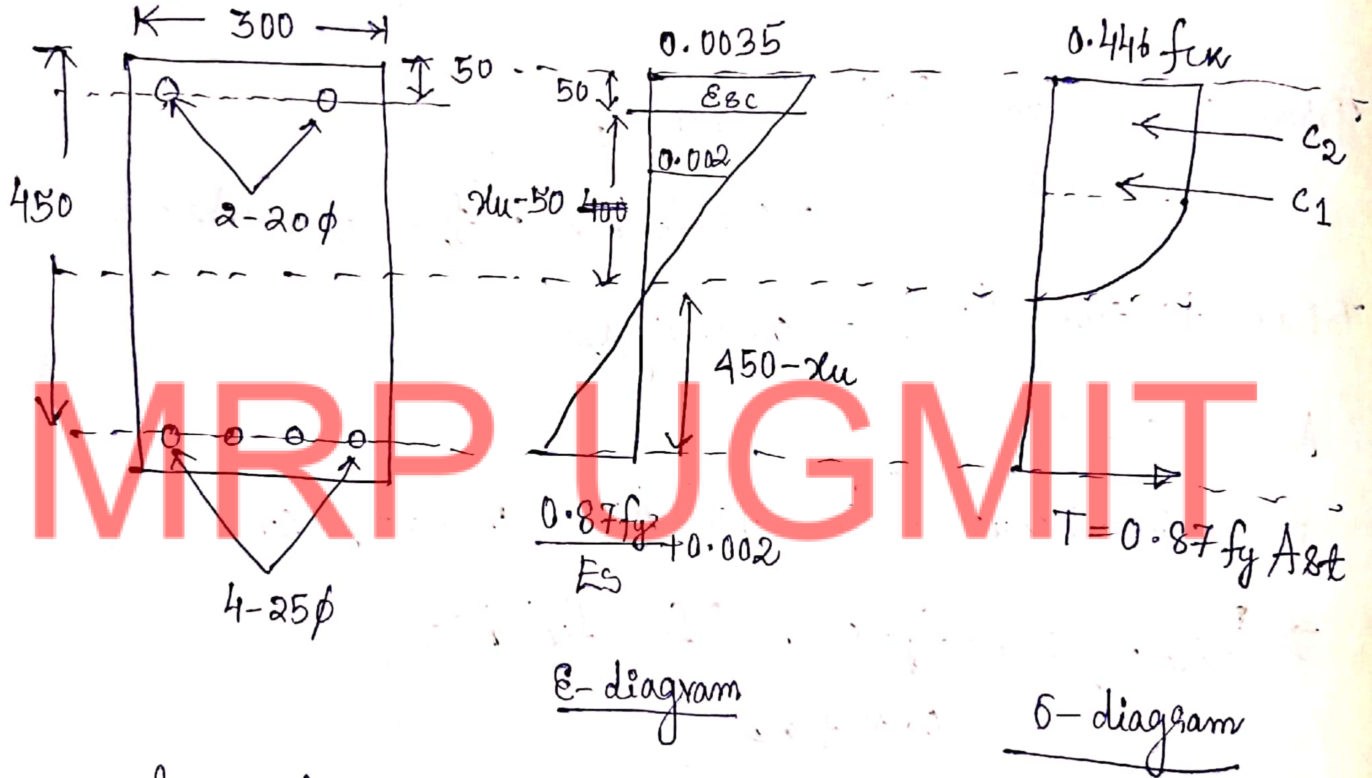
$$\frac{0.0035}{x_u} = \frac{\varepsilon_{sc}}{x_u - d'}$$

$$\Rightarrow \varepsilon_{sc} = 0.0035 \left(\frac{x_u - d'}{x_u} \right)$$

Problem

Q/ Find the factored MOR of an RCC beam 300 X 450 (effective). The beam is reinforced with 4-25 mm ϕ bar in tension zone. 2-20 mm ϕ bar are placed at a distance of 50 mm from top in compression zone. Use M20, Fe415.

Soln



ε- diagram

σ- diagram

where $b = 300$ mm

$d = 450$ mm

$A_{sc} = 2\times\frac{\pi}{4}\times20^2 = 628 \text{ mm}^2$

$A_{st} = 4\times\frac{\pi}{4}\times25^2 = 1964 \text{ mm}^2$

$d' = 50$ mm

$f_{ck} = 20 \frac{N}{mm^2}$

$f_y = 415 \frac{N}{mm^2}$

Manas Ranjan Pradhan

$$\frac{0.0035}{x_u} = \frac{\varepsilon_{sc}}{x_u - 50}$$

$$\Rightarrow \varepsilon_{sc} = \left(\frac{x_u - 50}{x_u}\right) 0.0035$$

As we know value of $\frac{f_{sc}}{d'/d}$:-

f_y	0.05	0.10	0.15	0.20
250	217	217	217	217
415	355	353	342	329
500	424	412	395	370

In our case $\frac{d'}{d} = \frac{50}{450} = 0.11 \simeq 0.15$

$\Rightarrow f_{sc} = 342\ N/mm^2$ (from table)

$$\therefore x_u = \frac{0.87 f_y A_{st} - f_{sc} A_{sc}}{0.36 f_{ck} b} \quad [f_{cc} \approx 0]$$

$\Rightarrow x_u = 228.85\ mm$

$x_{u,max} = 0.48 d = 216\ mm$

$x_u > x_{u,max}$, Hence section is over-reinforced.

$\therefore M_u = 0.36 f_{ck} b x_{u,max} (d - 0.42 x_{u,max}) + f_{sc} A_{sc} (d - d')$

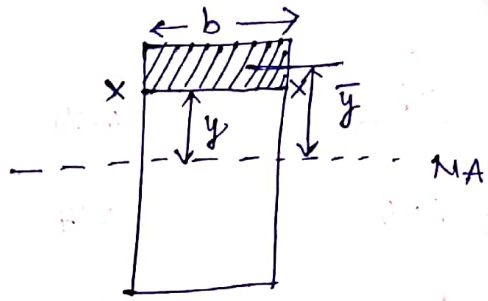
$= 253.53\ kNm$ (Ans)

CHAPTER - 4

Shear, bond & development length

→ A beam subjected to transverse load is subjected to shear force & bending moment.

$$\boxed{q = \cancel{Fa}\ \frac{F A\bar{y}}{Ib}}$$



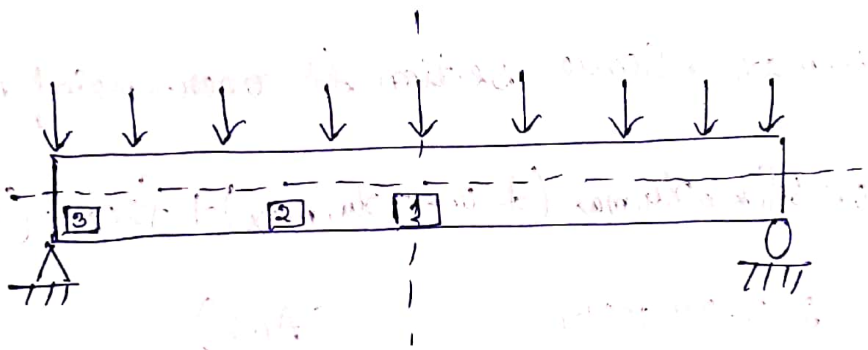
where q = shear stress
F = Shear force at section x - x
I = moment of inertia of section at C.G.
b = width of section

$A\bar{y}$ = first moment of the area above the section about the neutral axis.

As per IS - 456

$$\boxed{\tau_v = \frac{V_u}{bd}}$$

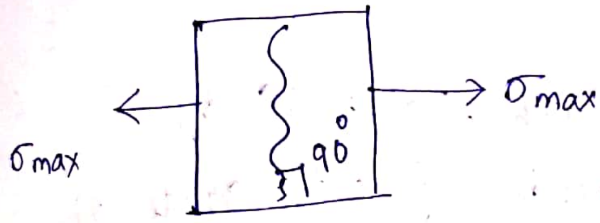
where
τ_v = nominal shear stress
b = width of beam
d = effective depth
V_u = factored shear force at the section.



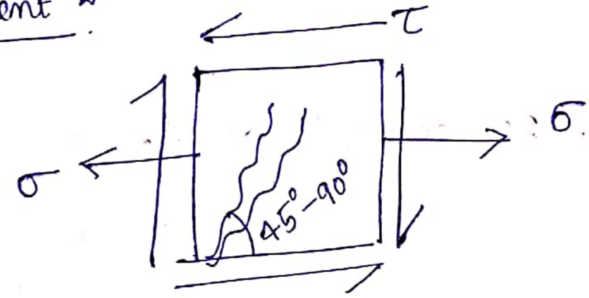
Element 1

Max B.M

S.F. = 0



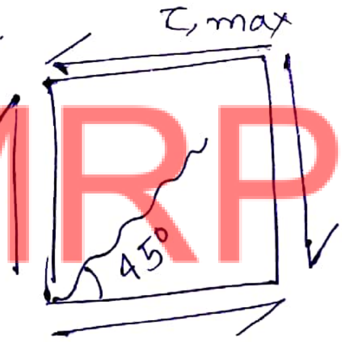
Element 2



Element 3

B.M. = 0

S.F. = max.



Nominal shear stress in R-C section

→ It is the shear force generated on the structure due to force imposed on given c/s area.

$$\boxed{\tau_v = \frac{V_u}{bd}}$$ as per IS-456

→ It is based on the geometry of the structure.

Design shear strength of concrete (τ_c, N/mm^2)

→ Design shear is the actual shear strength of the structure which it can resist.

→ τ_c can be found out from IS-456, Table-19 by calculating $p_t \left(\frac{A_{st}}{bd} \times 100\right)$ & comparing with grade of concrete (M20, M25 ....).

Example for $p_t = 0.50$ & M20 concrete grade

$$\Rightarrow \tau_c = 0.48 \ \frac{N}{mm^2}$$

Maximum shear stress ($\tau_{c,max}$)

→ IS-456, Table 20.

Grade	$\tau_{c,max}$ (N/mm^2)
M15	2.5
M20	2.8
M25	3.1
M30	3.5
M35	3.7
M40 & above	4.0

$$\boxed{\tau_v \text{ or } \tau_c < \tau_{c,max}}$$

check it Always

Design of shear reinforcement

→ calculate nominal shear stress (τ_v).

$$\tau_v = \frac{V_u}{bd}$$

→ Design shear strength of concrete (τ_c)

~~Table 19~~ (IS-456)

→ Max. shear stress in concrete ($\tau_{c,max}$) (IS-456)

→ Minimum shear reinforcement.

$$\boxed{\frac{A_{sv}}{b S_v} \geq \frac{0.4}{0.87 f_y}}$$

where A_{sv} = total c/s area of stirrup leg effective in shear

S_v = spacing of stirrup

b = width

f_y = yield stress

→ Max. spacing of stirrup

* (max. spacing of vertical stirrup ≯ 0.75d or 300 mm whichever is less).

* In case of inclined stirrup at 45°, max. spacing ≯ d or 300mm which is less.

→ Case-I Design of shear reinforcement –

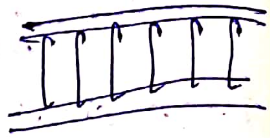
when $\tau_v > \tau_c$ (shear reinforcement is to be designed)

$V_s = V - \tau_c b d$

V = S.F. due to design load

* Vertical stirrup

$$\boxed{V_s = \frac{\sigma_{sv} \cdot A_{sv} \cdot d}{S_v}}$$



↳ where

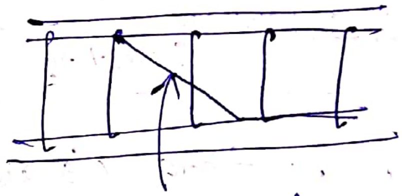
σ_{sv} = permissible tensile stress $\not> 230\ \frac{N}{mm^2}$

A_{sv} = c/s of stirrup legs

S_v = Spacing of stirrup

* if bent up bars are used

↳ $\boxed{V_s' = \sigma_{sv} \cdot A_{sv} \cdot \sin\alpha}$



where $V_s' \not> \frac{V_s}{2}$

α = angle between bent up bars & the member axis

↳ for the balance $(V_s - V_s')$ design for vertical stirrup.

* for inclined stirrup : $\boxed{V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}(\sin\alpha + \cos\alpha)}$

Case-I $(\tau_v < \tau_c)$

↳ Nominal shear reinforcement is to be provided in form of vertical stirrup.

$$\boxed{\frac{A_{sv}}{b\, S_v} \geq \frac{0.4}{0.87 f_y}}$$

Numerical problem

Q/ A reinforced concrete beam 400×600 mm effective is simply supported & carries a udl of 60 kN/m including self weight over a span of 6m. The section is reinforced with 5-20φ bars. Use M20 & mild steel (Fe250). design shear reinforcement for the beam

↳ vertical stirrups are used.

↳ Two bars are bent up at 45° at supports.

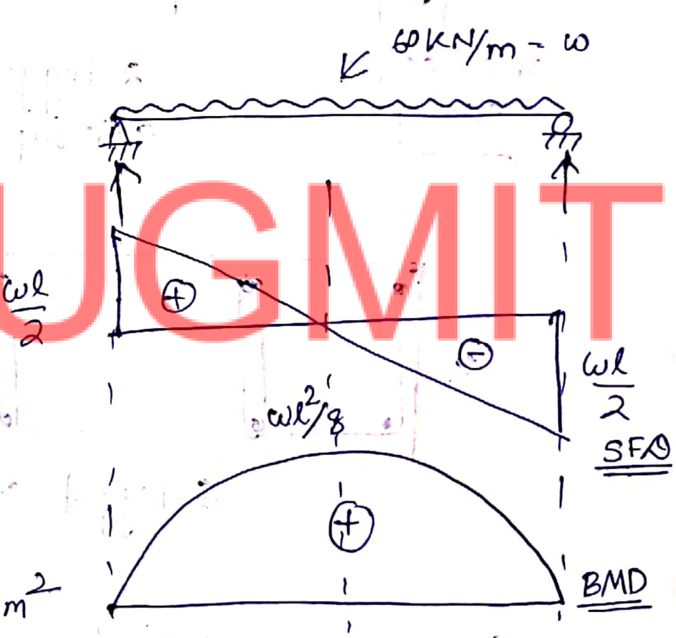
Vertical stirrups design

Soln

given $b = 400$ mm, $d = 600$mm, $l = 6$m, $w = 60$ kN/m

S.F. $V = \frac{wl}{2} = \frac{60 \times 6}{2} = 180$ kN

$= 180000$ N



$\therefore \tau_v = \frac{V_u}{bd} = 0.75$ N/mm²

$\tau_{c,max} = 1.8$ N/mm²

$\therefore \tau_v < \tau_{c\,max}$ (OK)

design shear strength (τ_c)

$A_{st} = 5 \times \frac{\pi}{4} \times 20^2 = 1570.7$ mm²

$p_t = \frac{A_{st}}{bd} \times 100 = 0.65\%$

from IS-456 code (M20, $p_t = 0.65$)

$$\tau_c = 0.3 + \frac{(0.35 - 0.3)}{0.75 - 0.5} \times (0.75 - 0.65)$$

$= 0.32$ N/mm²

$\therefore \boxed{\tau_v > \tau_c} \Rightarrow$ Shear reinforcement is required.

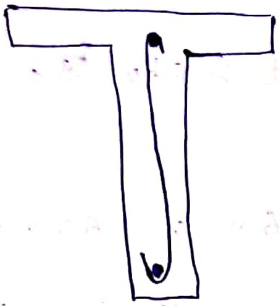
$\therefore$ Shear force $(V_s) = V - \tau_c bd$

$= \cancel{180 kN - 0.} = 180000 - 0.32 \times 400 \times 600$

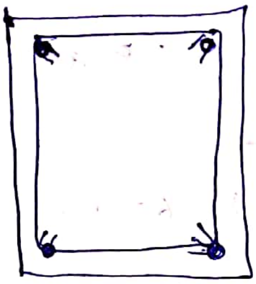
$= 103.2\ kN$

<u>Vertical stirrup</u>

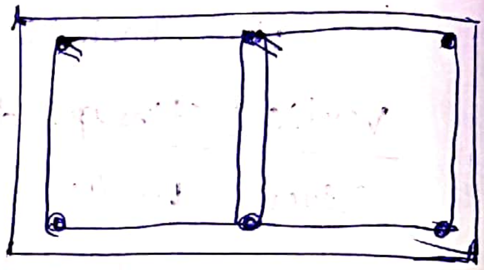
Let us provide 10 mm ϕ 2 legged stirrup



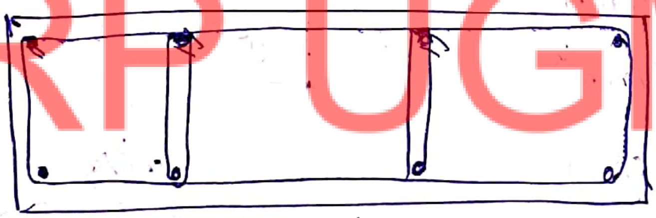
one legged stirrup



2-legged stirrup



4-legged stirrup



6-legged stirrup

$$\therefore S_v = \frac{\sigma_{sv} \cdot A_{sv} \cdot d}{V_s}$$

$$A_{sv} = 2 \times \frac{\pi}{4} \times 10^2 = 157.1\ mm^2$$

$$S_v = \frac{140 \times 157.1 \times 600}{103200} = 127.8 \simeq 120\ mm$$

<u>Check</u>

$$S_v < \begin{cases} 0.75d = 0.75 \times 600 = 450\ mm \\ 300\ mm \end{cases}$$

$\therefore$ Hence OK.

Contd →

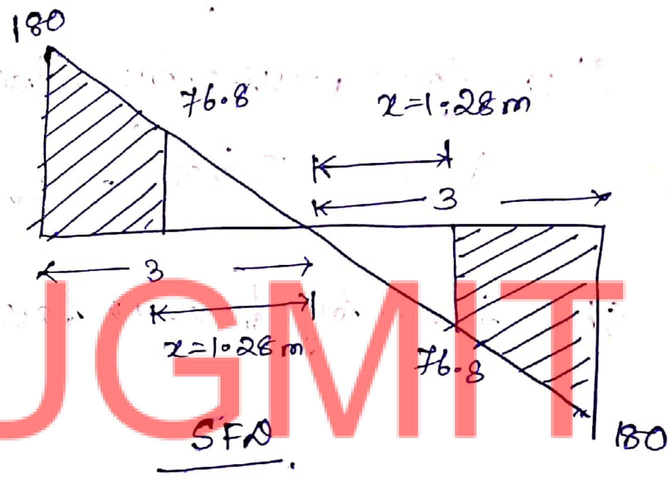
Maximum spacing as per minimum shear reinforcement —

$$S_v = \frac{0.87 f_y A_{sv}}{0.4 \times b} = 213.5\ mm \simeq 210\ mm$$

Zone of max. spacing —

$V_c = \tau_c bd.$

$= 0.32 \times 400 \times 600 = 76.8\ KN$



$\frac{76.8}{180} = \frac{x}{3}$

$\Rightarrow x = 1.28\ m$

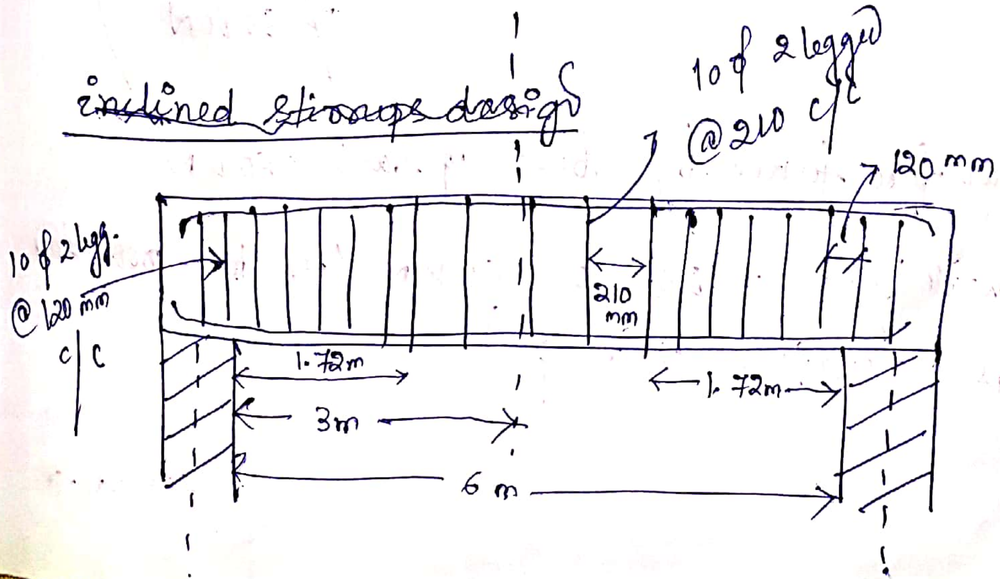
$\therefore$ Design zone length

$= 3 - 1.28 = 1.072\ m$

$\therefore$ Provide 10φ - 2legged stirrups @ 120 mm c/c upto 1.72 m from supports & 10φ - 2legged @ 210 mm c/c in the remaing portion (middle zone).

(Ans)

~~inclined stirrups design~~



contd →

using 2-no. of bent up bars {provided near support to resist diagonal tension; equal no. of bent up bars are provided to maintain symmetry)

Remaing main steel no = 05-02 = 03 (Google pic of bent up bars for more info)

Area of steel available near the support $= 3 \times \frac{\pi}{4} \times 20^2 = 94.2\ mm^2$

$p_t = \frac{A_{st}}{bd} = \frac{94.2}{400 \times 600} \times 100 = 0.39$

$\therefore \tau_c = 0.22 + \frac{0.3 - 0.22}{0.5 - 0.25}(0.5 - 0.39) = 0.25\ N/mm^2$

Shear resistance of concrete $= \tau_c bd = 0.25 \times 400 \times 600 = 60\ kN$

Shear force resisted by shear reinforcement

$V_s = V - \tau_c bd = 180000 - 60000 = 120\ kN$

Shear taken by bent up bars $V_{s1} = \sigma_{sv} \cdot A_{sv} \sin\alpha$

$= 140 \times 2 \times \frac{\pi}{4} \times 20^2 \times \sin 45°$

$= 62.191\ kN$

(Check: shear force taken by bent up bars $\ngtr \frac{V_s}{2}$

$\ngtr \cancel{16}\ \frac{120}{2}$

$\ngtr 60\ kN$)

$\therefore$ So shear force taken by bent up bars = 60 kN

$\therefore$ Balance shear force = 120 − 60 = 60 kN (To be provided as vertical stirrups)

contd →

provide
Spacing of 8 mm ϕ 2 legged vertical stirrups

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100.53 \text{ mm}^2$$

$$S_v = \frac{\sigma_{sv} \cdot A_{sv} \cdot d}{V_{s1}}$$

$$= \frac{140 \times 100.53 \times 600}{60000} = 140.7 \text{ mm} \simeq 140 \text{ mm}$$

∴ Maximum spacing as per nominal reinforcement

$$S_v = \frac{0.87 f_y A_{sv}}{0.4 b} = 136.6 \text{ mm} \simeq 130 \text{ mm}$$

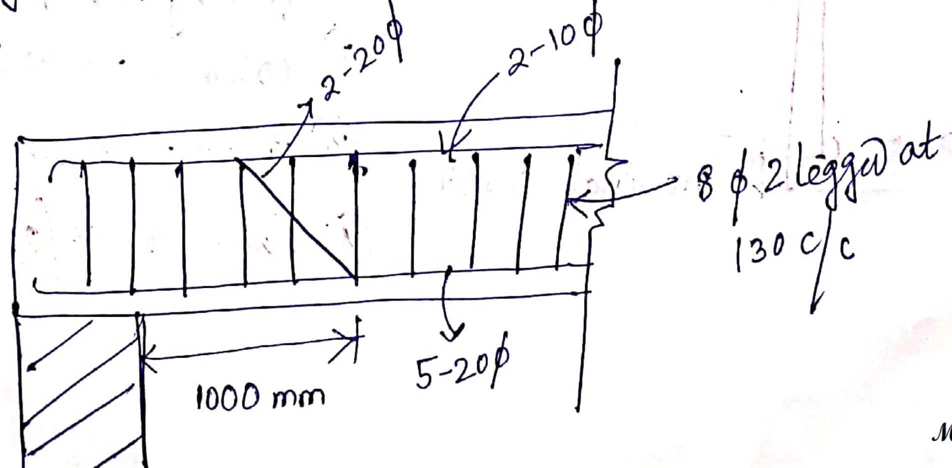
Max. spacing should be least of following —

0.75 d = 0.75 × 600 = 450 mm

300 mm

∴ So Spacing = 130 mm.

∴ Provide 8 ϕ 2 legged vertical stirrups @ 130 mm c/c throughout the length of beam alongwith 2 bent-up bars.



(Solved)

Extra

Assumption (Limit state of collapse : Compression)

→ The ~~main co~~ max. compressive strain in concrete is in axial compression is taken as 0.002.

→ The max. compressive strain at the highly compressed extreme fibre in concrete subjected to axial compression & bending and when there is no tension on the section shall be 0.0035 − 0.75 × strain at least compressed extreme fibre.

* curtailment ~~of~~ & bent-up bars

→ Bent up bars ($\frac{L}{4}$ from support)

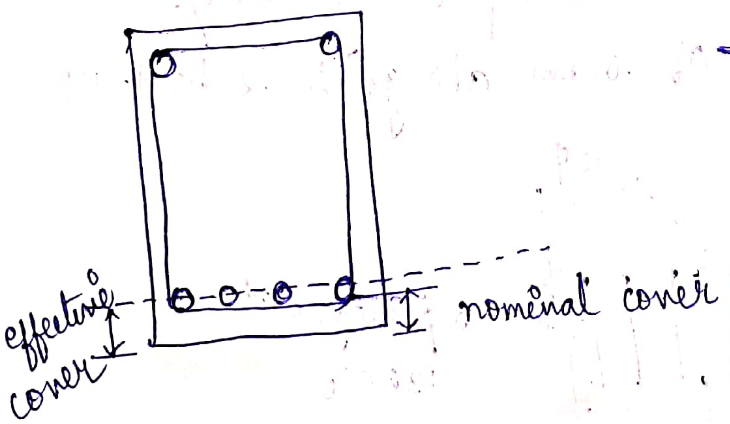
→ effective depth of member (d)

→ $12 \times \phi$ where ϕ = dia. of bar

(except s/s or end of cantilever) whichever is greater.

(curtailment) reinf. shall extent beyond the point at which is no longer req. to resist flexure

* Nominal Cover / clear cover



Nominal Cover

1. footing → 50 mm
2. column → 40 mm
3. beam → 25 mm
4. slab → 15/25 mm

Bond & types of bond

→ Bond is defined as the adhesion between concrete & steel which resists the slipping of steel bars from concrete.

Types

1. Anchorage bond / Development Length
2. Flexural bond

Bond stress (τ_{bd})

→ The stress generated due to bond between concrete & steel is bond stress.

→

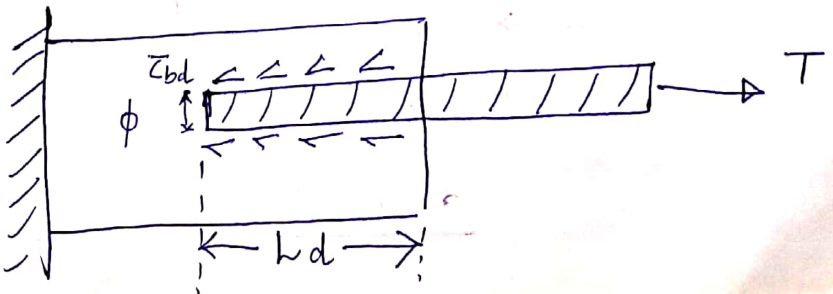
Concrete grade	bond stress (τ_{bd})
M20	1.2
M25	1.4
M30	1.5
M35	1.7
M40 & above	1.9

Check for bond stress

→ for deformed bars (bars with surface projection for better bonding) the τ_{bd} value is increased by 60%.

→ for bars in compression; the value of τ_{bd} ~~in~~ tension should be increased by 25% as compared to tension.

Development Length (L_d)



where T = force in tension
L_d = development length
τ_{bd} = bond stress
ϕ = dia. of bar

To avoid slipping $\quad T \le \tau_{bd} \times 2\pi \frac{\phi}{2} \times L_d$

$$\therefore \quad T = \sigma_{st} \times \frac{\pi}{4} \times \phi^2$$

$$\therefore \sigma_{st} \times \frac{\pi}{4} \times \phi^2 = \tau_{bd} \times 2\pi \times \frac{\phi}{2} \times L_d$$

$$\Rightarrow \boxed{L_d = \frac{\phi\,\sigma_{st}}{4\tau_{bd}}} \Rightarrow \boxed{L_d = \frac{0.87 f_y \phi}{4\tau_{bd}}}$$

As discussed

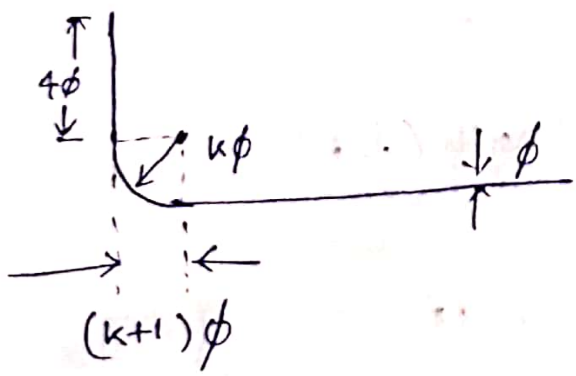
→ for deformed bars $L_d = \frac{\phi\,\sigma_{st}}{4(1.6\tau_{bd})} = \frac{\phi\,\sigma_{st}}{6.4\,\tau_{bd}}$

→ for bars in tension $L_d = \frac{\phi\,\sigma_{st}}{4\tau_{bd}}$

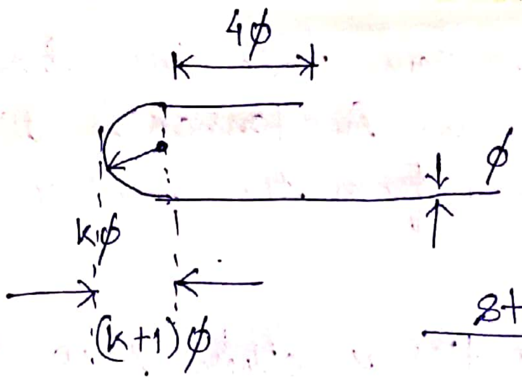
→ for bars in compression $L_d = \frac{\phi\,\sigma_{st}}{4(1.25\,\tau_{bd})} = \frac{\phi\,\sigma_{st}}{5\tau_{bd}}$

Anchorage values for hooks 90° bend & 45° bend of standard lapping of bars :—

Bars in tension



90° bend



Standard hook

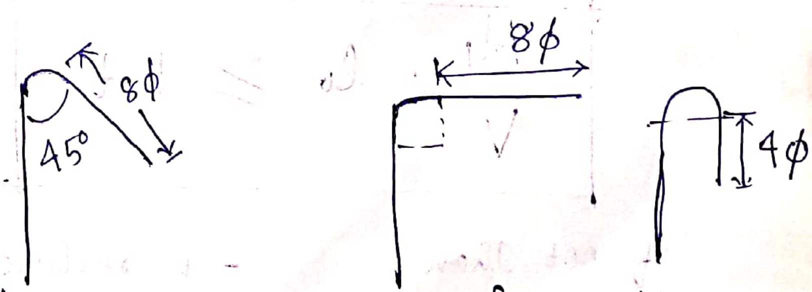
Minimum k for mild steel = 2
cold worked steel = 4

→ deformed bars mayn't need anchorage.
→ Hooks should be provided for plain bars in tension.
→ The anchorage value of standard bend shall be considered as 4 times the diameter of bar for each 45° bend subject to max. value of 16 times the diameter of bar.
→ The anchorage value of standard U-type hook ~~shall~~ bar shall be 16 times the diameter of bar.

Bars in compression

→ The anchorage length of straight compression bars shall be equal to its development length.
→ The development length shall include the projected length of hooks, bends & straight length beyond bends.

Bars in shear



→ Inclined bars in tension zone will have the Ld ~~as~~ equal to that of bars in tension & this length shall be measured from the end of inclined portion of bar.

→ inclined bars in compression zone will have the Ld equal to that of bars in tension & this length shall be measured from the middle depth of beam.

→ For stirrups, transverse ties & other secondary reinforcement, ~~compre~~ complete Ld & anchorage are considered to be satisfied if prepared as

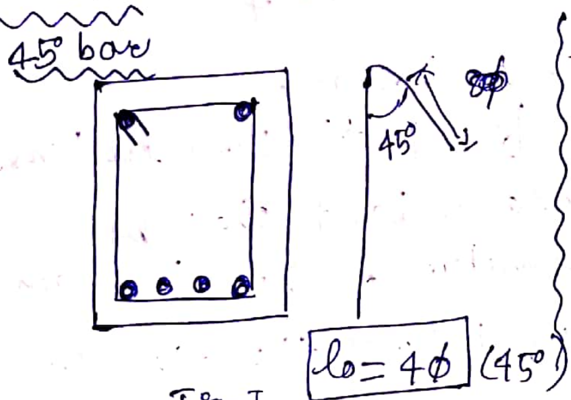


Fig-I

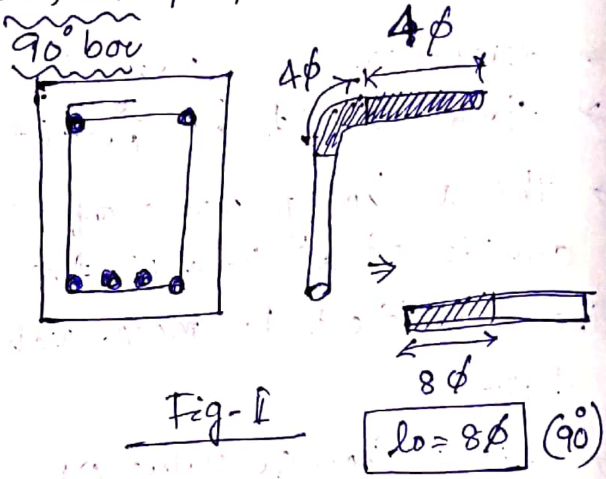


Fig-II

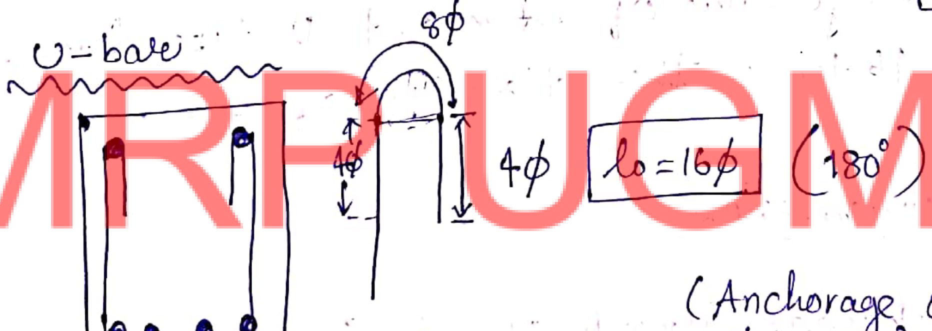


Fig-III

(Anchorage of Stirrups)

Check for development length (Ld)

$$\boxed{\frac{M_1}{V} + l_0 \geq L_d}$$

if not then
- → reduce φ
- → increase l_0
- → reduce no. of bent-up bars

M_1 = moment of resistance of section

V = shear force at section

L_0 = sum of anchorage beyond the centre of the support & equivalent anchorage value of any hook.

anchorage length

At point of inflexion, L_0 = max. { effective depth of member or 12ϕ

→ $\frac{M_1}{V}$ can be increased by 30% when end of the reinforcement are confined by a compressive reaction.

Problem

Q/ An RCC beam 250 mm × 500 mm has a clear span of 5.5m. The beam has 2-20φ bar going into the support. Vu = 140 KN. Check for Ld if Fe415 & M20 is used.

Soln

Given b = 250 mm, D = 500 mm

$\therefore d = 500 - 20 - 10 = 470$ mm

$l = 5.5$m

Ast at support $= 2 \times \frac{\pi}{4} \times 20^2 = 628 \text{ mm}^2$

$V_u = 140$ KN

$$M_1 = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{b d f_{ck}}\right)$$

$$M_1 = \cancel{0.87 f_y}\ 0.87 \times 415 \times 628 \times 470 \left(1 - \frac{628 \times 415}{250 \times 470 \times 20}\right)$$

$$= 94.748 \times 10^6 \text{ N mm}$$

Check for Ld

$$\frac{M_1}{V} + l_o \geq L_d$$

$$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}}$$

$$\tau_{bd} = 1.6 \times 1.2 = 1.92 \text{ N/mm}^2$$

$$\therefore L_d = \frac{0.87 \times 415 \times 20}{4 \times 1.92} = 940.2 \text{ mm}$$

* Provided a 90° bend at centre of Support

anchorage length $l_o = 8\phi = 160$ mm

$$\frac{M_1}{V} + l_o = \frac{94.748 \times 10^6}{140 \times 10^3} + 160$$

$$= 836.77 < L_d \quad (\text{Not satisfied})$$

* Provide a U-bend at the end of bar

$$\frac{M_1}{V} + l_o = \frac{94.748 \times 10^6}{140 \times 10^3} + 320$$

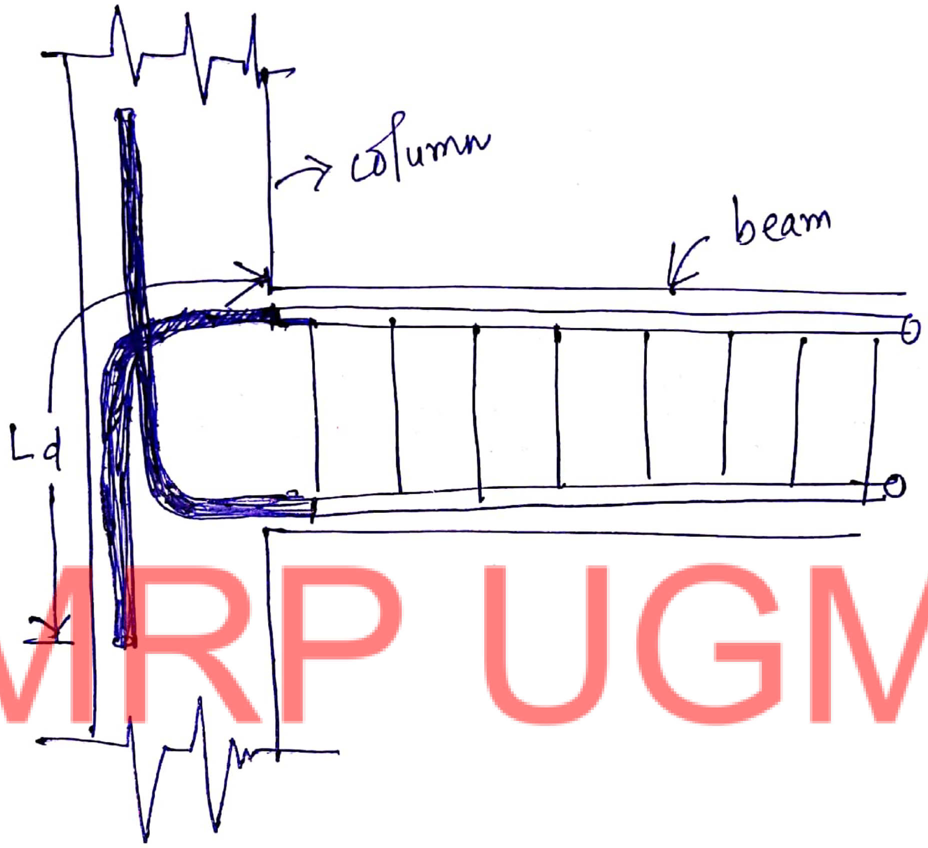
$$= 997 \text{ mm} > L_d \text{ (OK)}$$

(Soln)

—x—

~~Position 0:~~

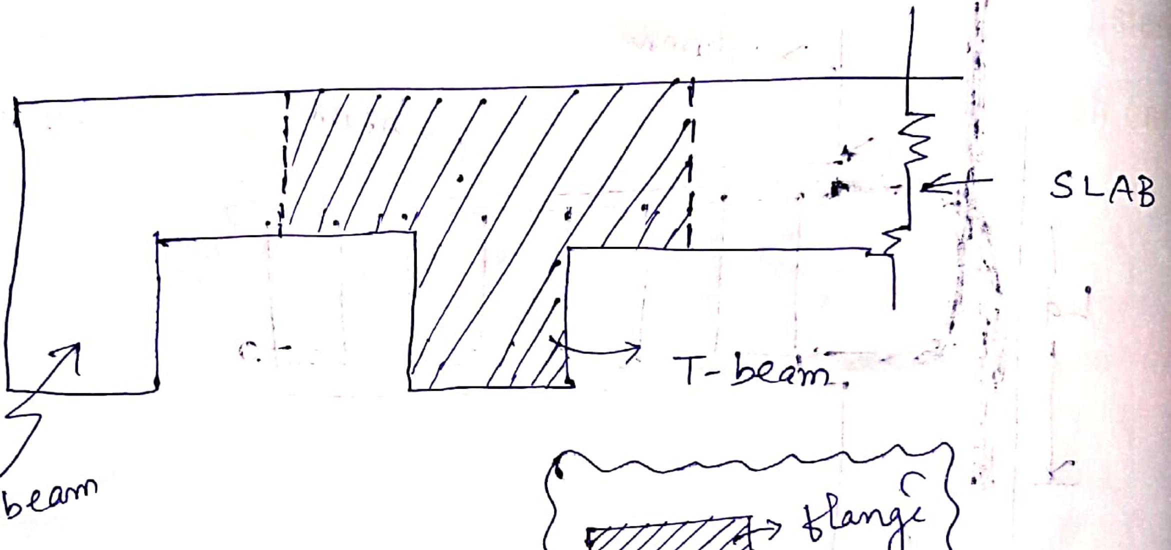
Diagram regarding Ld

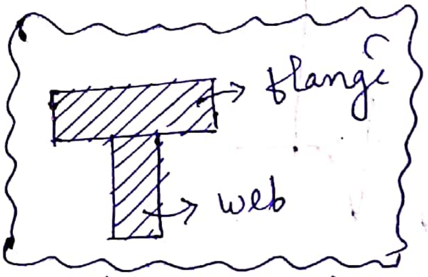


Chapter-05

Analysis & Design of T-beam

→ In RCC structure slabs & beams are cast monolithically.





⇒ The intermediate beam supporting the slab is called T-beam & the end-beam is called L-beam.

Advantages of T-beam

1/ Since the beam is cast monolithically with the slab, the flange take up the compressive stress, so it will be more effective in resisting sagging moment acting on the beam.

2/ Better headroom

3/ Less deflection

4/ lighweight, stable, provide 3 point contact and assembly points, increases structural rigidity

Disadvantages

(Stress distribution)

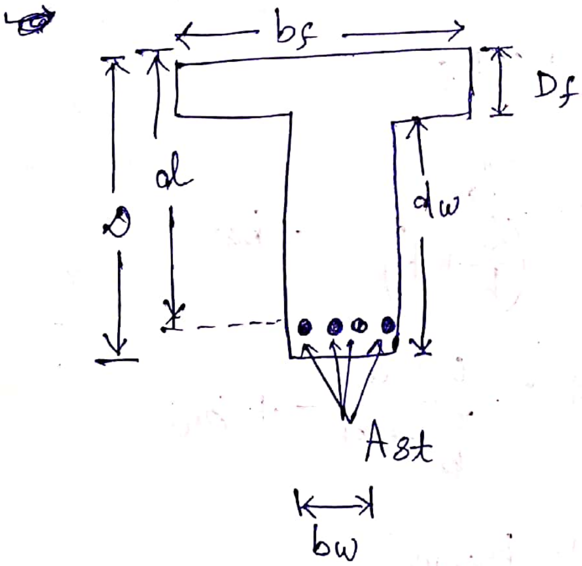
→ The shear stress increases at the flange & web junction due to change in c/s (cross section). So casting should be ~~done very~~

done very carefully & ensure proper bonding..

→ In earthquake prone zones T- beam is used with mechanical stiffners in the junction as T-beam is very weak in resisting lateral shear force.

→ There will be small savings in steel too.

Effective width of flange as per IS: 456-2000 codal provision



b_f = width of flange

D_f = depth of flange

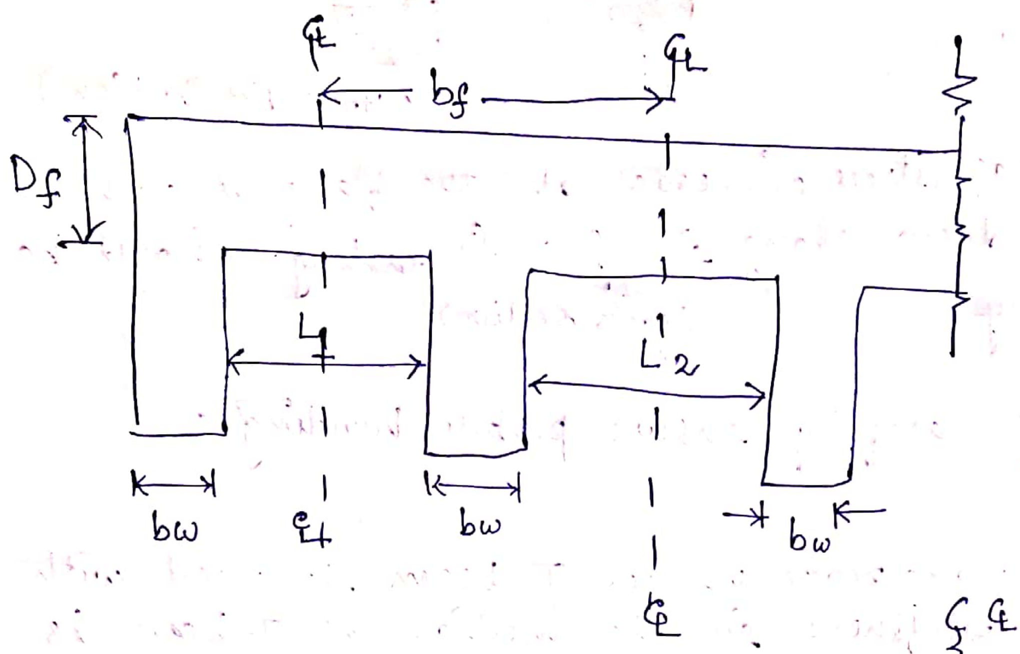
D = overall depth of T- beam

d = effective depth of T- beam

d_w = depth of web

b_w = width of web

A_{st} = area of steel



{ ℄ = centre line }

effective width (b_f)

It is the portion of the slab which acts integrally with the beam & extends on either side of the beam forming the compression zone.

→ For T-beams $\quad b_f = \frac{l_o}{6} + b_w + 6D_f$

L-beam $\quad b_f = \frac{l_o}{12} + b_w + 3D_f$

→ for isolated beam

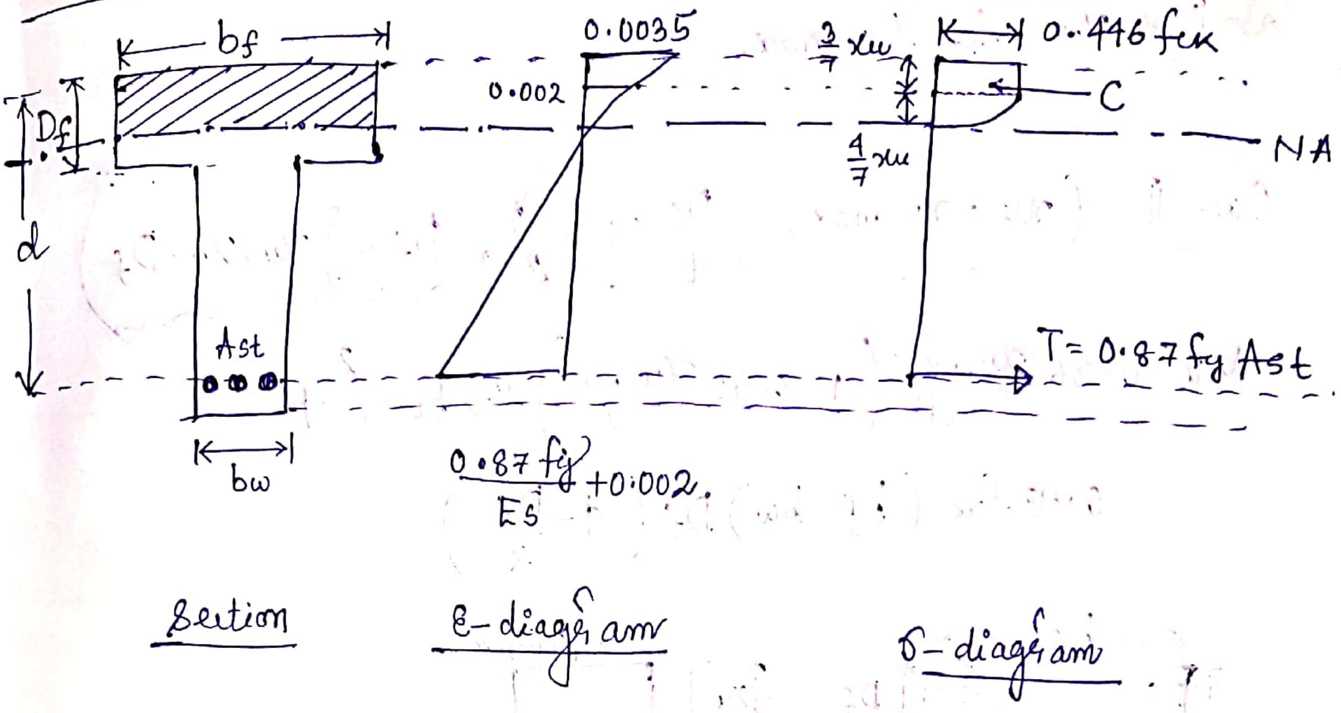
T-beam $\quad b_f = \frac{l_o}{\left(\frac{l_o}{b} + 4\right)} + b_w$

L-beam $\quad b_f = \frac{0.5\,l_o}{\frac{l_o}{b} + 4} + b_w$

Analysis of Single Reinforced T-beam

Consider a T-beam having flange width b_f, web width b_w, flange thickness D_f, the T-beam is reinforced with area of steel A_{st} in tension zone (single reinforced).

Case-I ($x_u < D_f$: NA lies within the flange)



→ when the depth NA (x_u) falls within the flange (D_f). the T- beam formulation for calulation will be same as of rectangular beam c/s.

$$\therefore x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f}$$

→ **Moment of resistance (MOR)**

* $M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$ if $x_u < x_{u,max}$ (under reinforced)

* $M_u = 0.36 f_{ck} b_f x_{u,max} (d - 0.42 x_{u,max})$

or

$M_u = 0.87 f_y A_{st} (d - 0.42 x_{u,max})$ if $x_u \approx x_{u,max}$ (balanced)

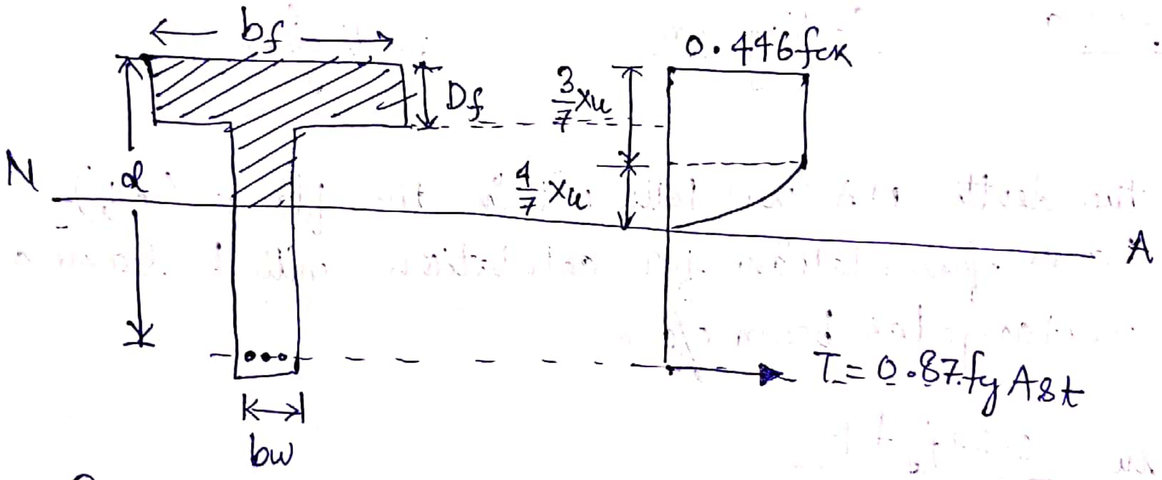
* M_u. $x_u > x_{u,max}$ then section should be redesigned.

~~Case-II (x_u = D_f, $x_{u,max}$)~~

Case-II ($x_u = x_{u,max}$, $\frac{D_f}{d} < 0.2$), $D_f < \frac{3}{7} x_u$, $x_u > D_f$)

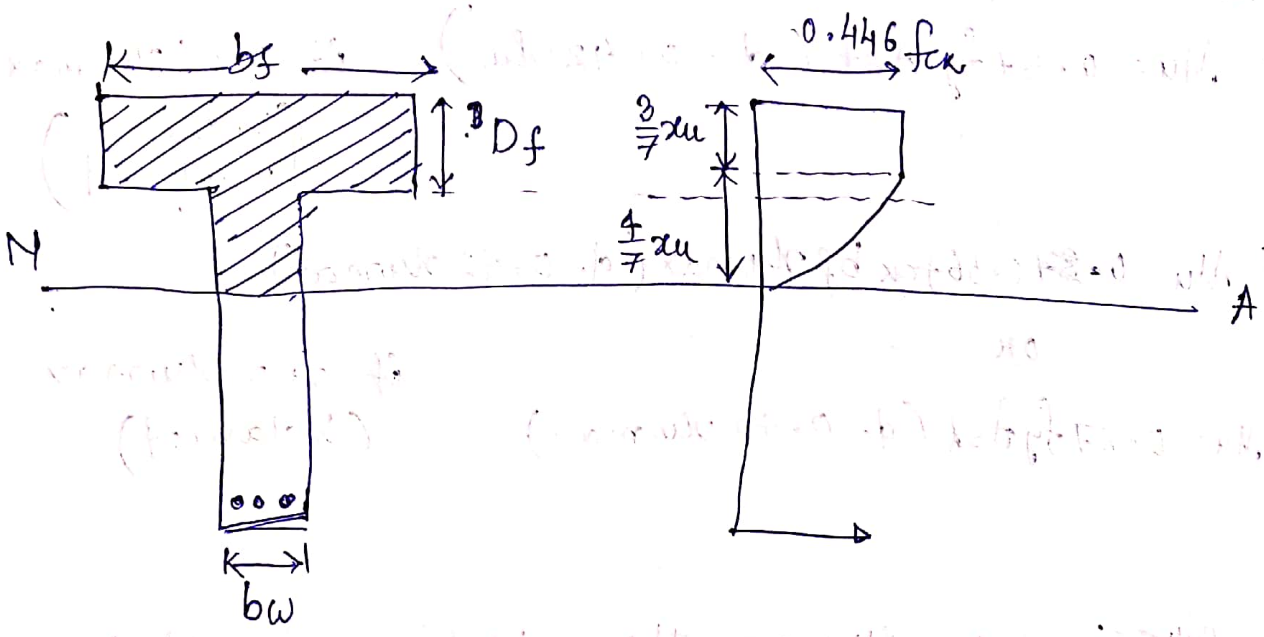
$$M_u = 0.36 \frac{x_{u,max}}{d}\left(1 - 0.42 \frac{x_{u,max}}{d}\right) f_{ck} b_w d^2 +$$

$$0.45 f_{ck} (b_f - b_w) D_f \left(d - \frac{D_f}{2}\right)$$



Case-III

($x_u \approx x_{u,max}$, $\frac{D_f}{d} > 0.2$, $D_f > \frac{3}{7} x_u$, $x_u > D_f$)



~~$M_u = 0.36 f_{ck}$~~

$$M_u = 0.36 \frac{x_{u,max}}{d}\left(1 - 0.42 \times \frac{x_{u,max}}{d}\right) f_{ck} b_w d^2 + 0.45 f_{ck} (b_f - b_w) y_f \left(d - \frac{y_f}{2}\right)$$

where $\boxed{y_f = 0.15 x_u + 0.65 D_f < D_f}$

Case-IV $\left(x_{u,max} > x_u > D_f,\ \frac{D_f}{d} < 0.2,\ D_f < \frac{3}{7} x_u\right)$

$$M_u = 0.36 \frac{x_u}{d}\left(1 - 0.42 \frac{x_u}{d}\right) f_{ck} b_w d^2 + 0.45 f_{ck} (b_f - b_w) D_f \left(d - \frac{D_f}{2}\right)$$

Case-V $\left(x_{u,max} > x_u > D_f,\ \frac{D_f}{d} > 0.2,\ D_f > \frac{3}{7} x_u\right)$

$$M_u = 0.36 \frac{x_u}{d}\left(1 - 0.42 \frac{x_u}{d}\right) f_{ck} b_w d^2 + 0.45 f_{ck} (b_f - b_w) \times y_f \left(d - \frac{y_f}{2}\right)$$

where $\boxed{y_f = 0.15 x_u + 0.65 D_f < D_f}$

* for all the cases $\boxed{M_u = 0.87 f_y A_{st} (d - 0.42 x_u)}$

x_u changes as per UR, OR & Balanced section with $x_{u,max}$.

Problem 1

Q/ find the Mu (→ Moment of resistance) of a T beam having a web width of 240 mm, effective depth of 400mm, flange width of 740 mm & flange thickness is 100mm. The beam is reinforced with 5-16 ϕ Fe 415, M20.

Soln

$b_f = 740$ mm; $b_w = 240$ mm, $d = 400$ mm, $D_f = 100$ mm

$A_{st} = 5 \times \frac{\pi}{4} \times 16^2 = 1005.3$ mm^2

$f_{ck} = 20$ N/mm^2

$f_y = 415$ N/mm^2

Assume N.A lies within the flange.

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = 68.1 \text{ mm} < D_f \ (100\text{mm})$$

⇒ N.A. lies within the flange.

$x_{u,max} = 0.48 \times d = 192$ mm

$x_u < x_{u,max}$ ⇒ ~~over~~ under-reinforced section.

∴ Moment of resistance (Mu)

$$M_u = 0.87 f_y A_{st}\, d \left(1 - \frac{A_{st} f_y}{b_f\, d\, f_{ck}}\right)$$

$= 134.95$ kNm (Ans)

Problem - 2

Q/ An isolated s/s T-beam has ~~bf = 2'~~ flange width of 2400mm & flange thickness 120mm; $l_{eff} = 3.6\,m$. $d = \cancel{35}\ 580mm$ & $b_w = 300mm$. It is reinforced with 8 - 20 ϕ, Fe415, M20. Find out M_u ?

Soln. Given $b = 2400$, $D_f = 120$ mm, $b_w = 300$ mm, $d = 580$ mm

$A_{st} = 5 \times \frac{\pi}{4} \times 20^2 = 3041\ mm^2$

$f_{ck} = 20\ N/mm^2$

$f_y = 415\ N/mm^2$

$l = 3.6m = 3600\ mm$

$\therefore$ effective width $b_f = \dfrac{l_o}{\frac{l_o}{b} + 4} + b_w$

here $l = l_o = 3600$

$\therefore b_f = \dfrac{3600}{\frac{3600}{2400} + 4} + 300 = 954.54\ mm$

* Assume N. A. lies within the flange

$x_u = \dfrac{0.87 f_y A_{st}}{0.36 f_{ck} b_f} = 159.75\ mm > D_f$

$\Rightarrow$ Our assumption is wrong.

$\Rightarrow \boxed{x_u > D_f}$

$\therefore \dfrac{D_f}{d} = \dfrac{120}{580} = 0.206$

$\Rightarrow \boxed{\dfrac{D_f}{d} > 0.206}$

* Assume $D_f > \frac{3}{7} x_u$

Our Neutral Axis formulation is $C = T$

$$\therefore C = 0.36 f_{ck} b_w x_u + 0.45 f_{ck} y_f (b_f - b_w)$$

$$T = 0.87 f_y A_{st}$$

$$\Rightarrow x_u = 209.76 \text{ mm}$$

$$\Rightarrow \frac{3}{7} x_u = 90.2 \text{ mm} < D_f$$

$\Rightarrow$ Our assumption is CORRECT.

$$x_{u,max} = 0.48 \times d = 278.4 \text{ mm} > x_u$$

$\Rightarrow$ Our section is under-reinforced.

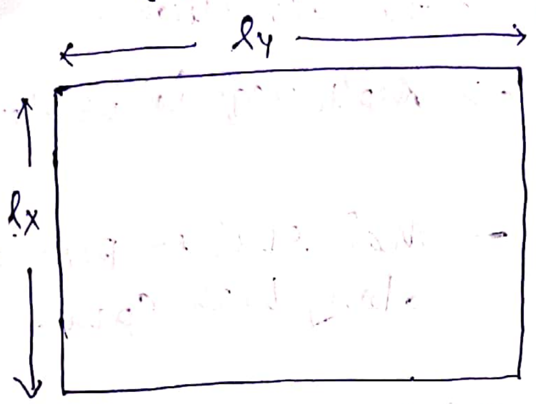
$$M_u = 0.36 \frac{x_u}{d}\left(1 - 0.42 \frac{x_u}{d}\right) f_{ck} b_w d^2 + 0.45 f_{ck} (b_f - b_w) y_f \left(d - \frac{y_f}{2}\right)$$

$$\Rightarrow M_u = 562.5 \text{ KNm}$$ (Ans)

Chapter - 6

Analysis & Design of slab & staircase

Slab – It is a 2-dimensional/planar element, used in all type of structure such as floors & roof coverings.



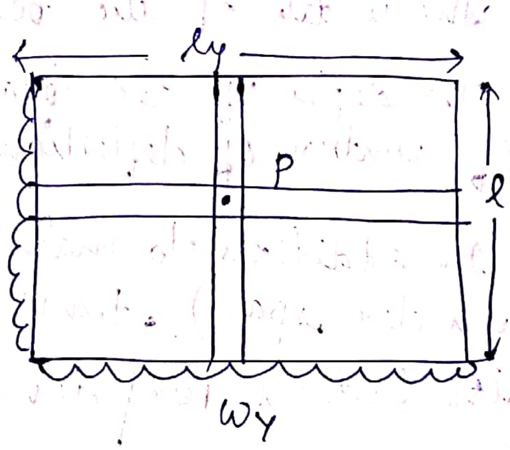
where l_y = length of longer span

l_x = length of shorter span

$r = \frac{l_y}{l_x} > 2 \Rightarrow$ One way slab $\Rightarrow$ bending in shorter direction.

$r = \frac{l_y}{l_x} < 2 \Rightarrow$ Two way slab
$\Rightarrow$ bending in both direction.

load distribution in slab



Δ_P = deflection at P

$$(\Delta P)_x = \frac{5}{384} \frac{w_x l_x^4}{EI}$$

$$(\Delta P)_y = \frac{5}{384} \frac{w_y l_y^4}{EI}$$

$$\therefore (\Delta P)_x = (\Delta P)_y \Rightarrow \frac{w_x}{w_y} = \left(\frac{l_y}{l_x}\right)^4 = r^4$$

$$\therefore w = w_x + w_y = w_y r^4 + w_y = w_y(1+r^4)$$

$$\Rightarrow w_y = \frac{w}{1+r^4} \quad \therefore w_x = w - \frac{w}{1+r^4} = \frac{w + wr^4 - w}{1+r^4} = w\left(\frac{r^4}{1+r^4}\right)$$

Difference between 1-way & 2-way slab :-

One way slab	Two-way slab
→ $\frac{l_y}{l_x} > 2$	→ $\frac{l_y}{l_x} \leq 2$
→ bending occurs in shorter span.	→ bending occurs in both direction.
→ Depth req. is more.	→ Depth require is less.
→ Main steel is ~~proper~~ provided along shorter span.	→ Main steel is provided along both span.
→ less economical as 't' is more.	→ more economical as 't' is less.

Design of simply supported ~~slab~~ one way slabs for flexure check for deflection control & shear

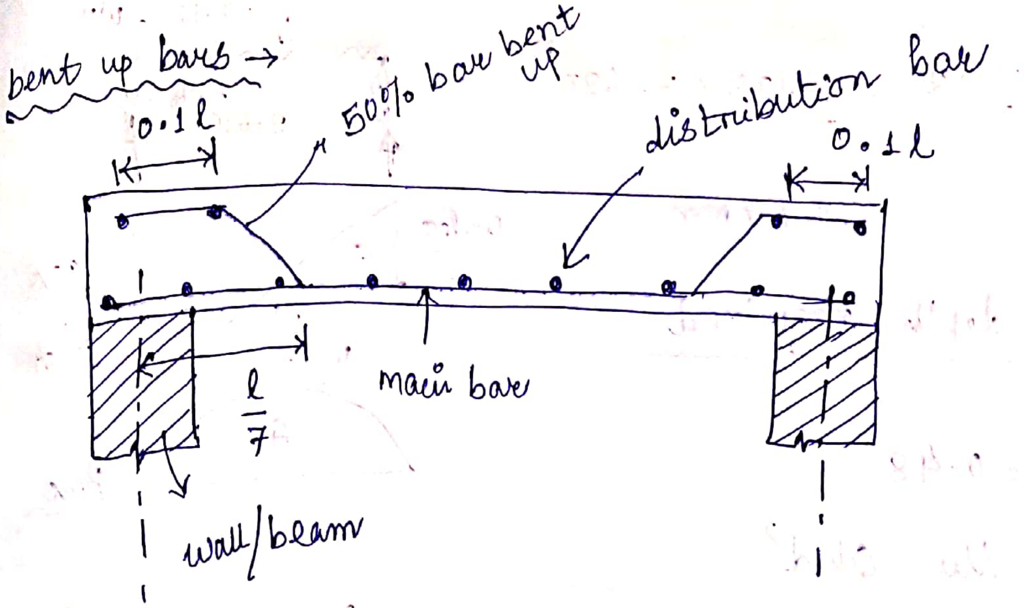
→ The width of the beam is assumed as 1m.

→ The depth of the beam is assumed on basis of control of deflection. ($\frac{l}{d} = 25-30$ (SS); 10 (cantilever))

→ In addition to main reinforcement (along shorter span), transverse reinforcement / distribution reinforcement is provided.

→ Some main bars in the slab are bent up near the support ($\frac{l}{7}$ from centre of support)

→ shear is to be checked only.



Problem

Q/ Design a simply supported roof slab for a room 7.5m X 3.5m clear in size. The slab is carrying an imposed load of 5 kN/m². Use M20, Fe415.

Sol^n

$$\frac{l_y}{l_x} = \frac{7.5}{3.5} > 2 \Rightarrow \text{One-way slab.}$$

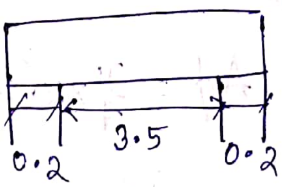
Assume depth (D) = 150 mm $\left[d = \frac{l}{25} = \frac{3500}{25} = 140\text{ mm} \right]$

$\therefore d = 150-20-5 = 125$ mm (where cc = 20mm, ϕ = 10mm)

↳ assumed effective depth

$\therefore l_{eff}$

minimum



$= 3.5 + \frac{0.2}{2} + \frac{0.2}{2} = 3.7$ m

OR

clear span + d $= 3.5 + 0.125 = 3.625$ m

$\therefore$ $l_{eff} = 3.625$ m

* **Design load (w_u) & factored moment (M_u)**

$\therefore$ Self weight (~~w~~) $= 0.15 \times 1 \times 25 = 3.75$ kN/m

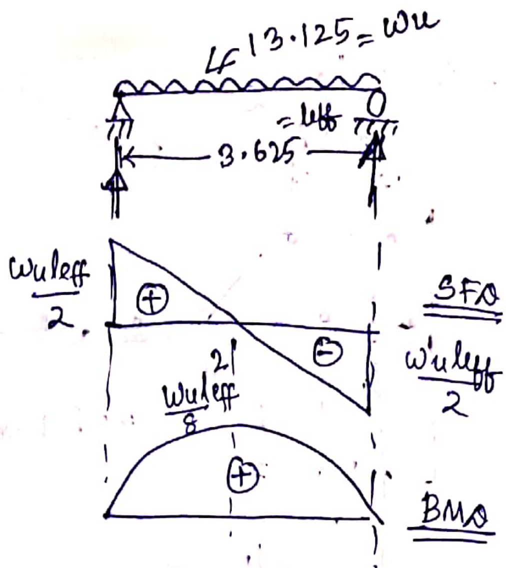
Imposed load $=$ 5 kN/m

$w =$ 8.75 kN/m

$\Rightarrow w_u = 1.5\,w = 13.125$ kN/m

$$\therefore M_u = \frac{w_u l_{eff}^2}{8} = \frac{13.125 \times 3.625^2}{8}$$

$$= 21.6 \text{ kNm}$$



* Effective depth required

$$\frac{x_{u,max}}{d} = 0.48$$

We know $M_u = Qbd^2$

$$\therefore Q = 0.36 f_{ck} \frac{x_{u,max}}{d}\left(1 - 0.42 \frac{x_{u,max}}{d}\right)$$

$$= 2.76$$

$$\therefore d_{req} = \sqrt{\frac{M_u}{Qb}} = 88 \text{ mm} < 125 \text{ mm}$$ {under-reinforced section}

* Ast calculation

$$M_u = 0.87 f_y A_{st} \times d \left(1 - \frac{A_{st} \times f_y}{f_{ck} \times bd}\right)$$ (★ - refer other way to calculate Ast)

$\Rightarrow (A_{st})_{provided} = 523.4 \text{ mm}^2$

using 10 mm ϕ bar $A\phi = \frac{\pi}{4} \times 10^2 = 78.5 \text{ mm}^2$

Spacing of 10ϕ bar $= \frac{A\phi}{A_{st}} \times 1000 = \frac{78.5}{523.4} \times 1000$

$$= 150.17 \text{ mm}$$

$\therefore$ Provide 10 mm ϕ @ 150 mm c/c

check for spacing
{ $3 \times d = 3 \times 125 = 375$ mm > 150 mm
300 mm > 150 mm }

(OK)

$(A_{st})_{required} = \frac{78.5}{150} \times 1000 = 523 \text{ mm}^2$ {Spacing $= \frac{A\phi}{A_{st}} \times 1000$}

∴ bending alternate bar at $\frac{l_{eff}}{7}$

$$= \frac{3625}{7} = 517 \text{ mm}$$

$\simeq$ 510 mm from centre of Support.

Distribution steel

↳ distribution steel bar is provided in longer direction.

$= 0.15\% \times$ gross sectional area

$= 0.15\% \times 150 \times 1000 = 225 \text{ mm}^2$

Use 6 mm ϕ bar $A_\phi = \frac{\pi}{4} \times 6^2 = 28.3 \text{ mm}^2$

Spacing of 6 mm ϕ bar $= \frac{A_\phi}{A_{st}} \times 1000 = \frac{28.3}{225} \times 1000 = 125.7 \text{ mm}$

∴ Provide 6 mm ϕ @ 125 mm c/c in longer direction.

Check for shear

$$V_u = \frac{w_u L}{2} = \frac{13.125 \times 3.5}{2} = 22.97 \text{ KN}$$

Nominal shear stress $(\tau_v) = \frac{V_u}{bd} = \frac{22.97 \times 10^3}{1000 \times 125} = 0.18 \text{ N/mm}^2$

Design shear strength (τ_c) ∈ Pt

$$p_t = \frac{A_{st}}{bd} \times 100 = \frac{(523/2)}{1000 \times 125} \times 100$$

$$= 0.21\% \quad \left\{ \text{each Support } A_{st} = \frac{523}{2} \right\}$$

from table $p_t = 0.21\%$ & M20.

$$\tau_c = 0.28 + \frac{0.36 - 0.28}{0.25 - 0.15} (0.21 - 0.15)$$

$$= 0.328 \text{ N/mm}^2$$

∴ $\boxed{\tau_v < \tau_c}$ hence OK.

check for deflection

$$p_t = \frac{100 \times A_{st}}{bd} = \frac{100 \times 523}{1000 \times 125} = 0.40\%$$

$$\therefore f_s = 0.58 f_y \times \frac{(A_{st})_{req.}}{(A_{st})_{provided}}$$

$$= 0.58 \times 415 \times \frac{523}{523} = 240 \text{ N/mm}^2$$

for $p_t = 0.40\%$, $f_s = 240 \text{ N/mm}^2$, $k_t = 1.55$

$$\left(\frac{l}{d}\right)_{max} = 20 \times k_t = 20 \times 1.55 = 31$$

$$\left(\frac{l}{d}\right)_{provided} = \frac{3625}{125} = 29$$

$$\therefore \left(\frac{l}{d}\right)_{max} > \left(\frac{l}{d}\right)_{provided} \quad \text{(OK)}$$

deflection

$$\frac{l}{d} \begin{cases} = 7 & \text{cantilever} \\ = 20 & \text{s/s} \\ = 26 & \text{continuous beam} \end{cases}$$

check for development length

Moment of resistance at support by 10 mm ϕ @ 300mm c/c = M_1

$$A_{st} = \text{at support} = \frac{523}{2} = 266 \text{ mm}^2$$

$$M_1 = 0.87 f_y A_{st} \times d \left(1 - \frac{f_y A_{st}}{f_{ck}\, b\, d}\right)$$

$$= 0.87 \times 415 \times 266 \times 125 \left(1 - \frac{415 \times 266}{20 \times 1000 \times 125}\right)$$

$$= 11.47 \times 10^6 \text{ N mm}$$

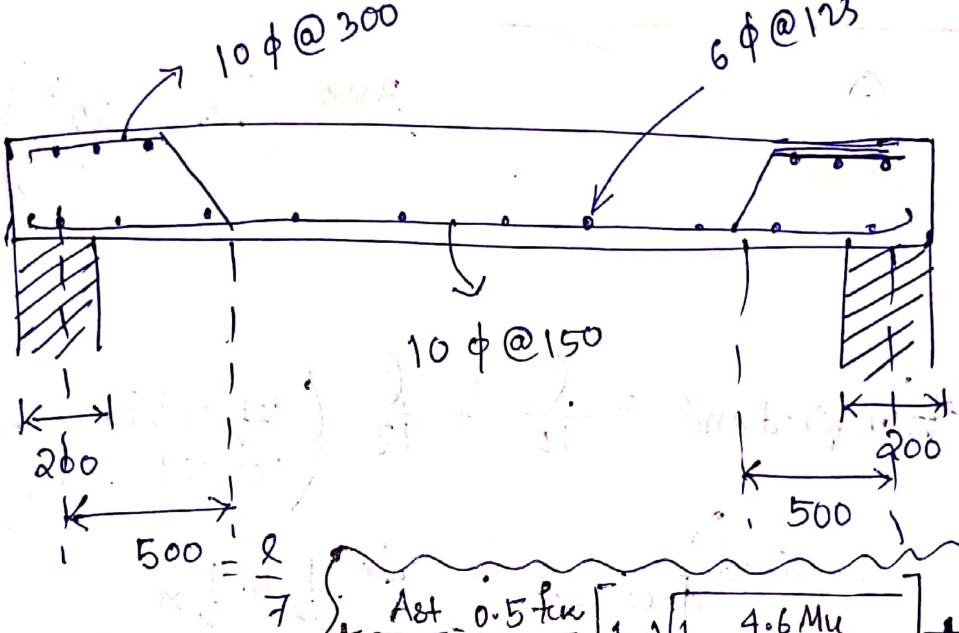
$$V_u = 22970 \text{ N}$$

provide no hookes $l_o = 0$

$$\therefore \frac{M_1}{V} + l_o = \frac{11.47 \times 10^6}{22970} = 500 \text{ mm}$$

$$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}} = \frac{0.87 \times 415 \times 10}{4 \times 1.2 \times 1.6} = 470 \text{ mm}$$

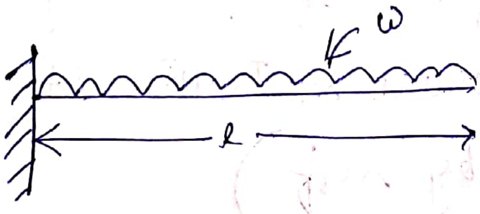
$$\therefore \frac{M_1}{V} + l_o > L_d \text{ (OK)}$$

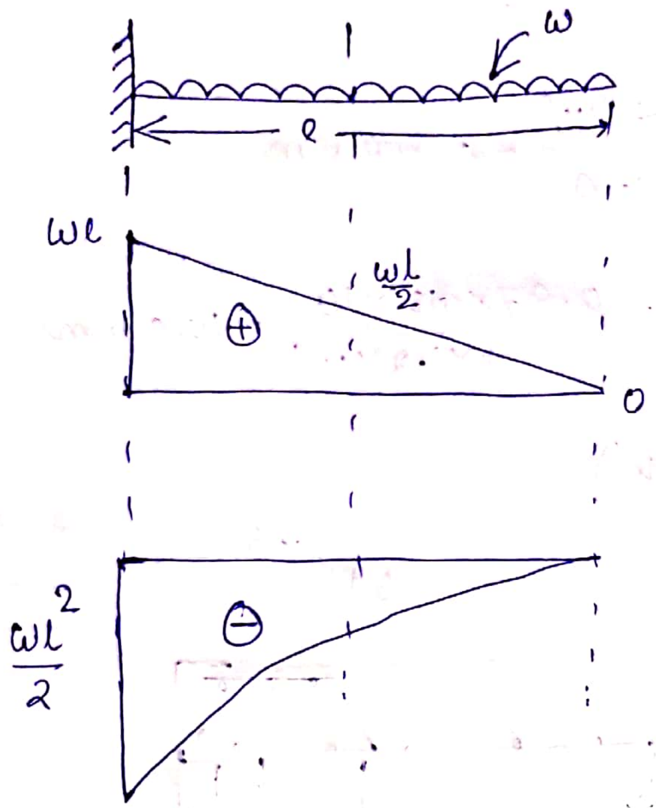


$$\bigstar \quad \frac{A_{st}}{bd} = \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} b d^2}}\right] \quad \bigstar \quad \text{(Ans)}$$

Design of one-way cantilever slabs & cantilever chajjas for flexure check ~~foor~~ for deflection control & check for development length & shear

→ In this case all steps are same as simply supported one but only difference is the calculation of bending moment & shear force.





SFD $\left(S_x = +wx\right)$

BMD $\left(M_x = -\dfrac{wx^2}{2}\right)$

↳ here (~~deff~~ at fixed end $= \dfrac{l}{10}$ to $\dfrac{l}{12}$ (deflection consideration)

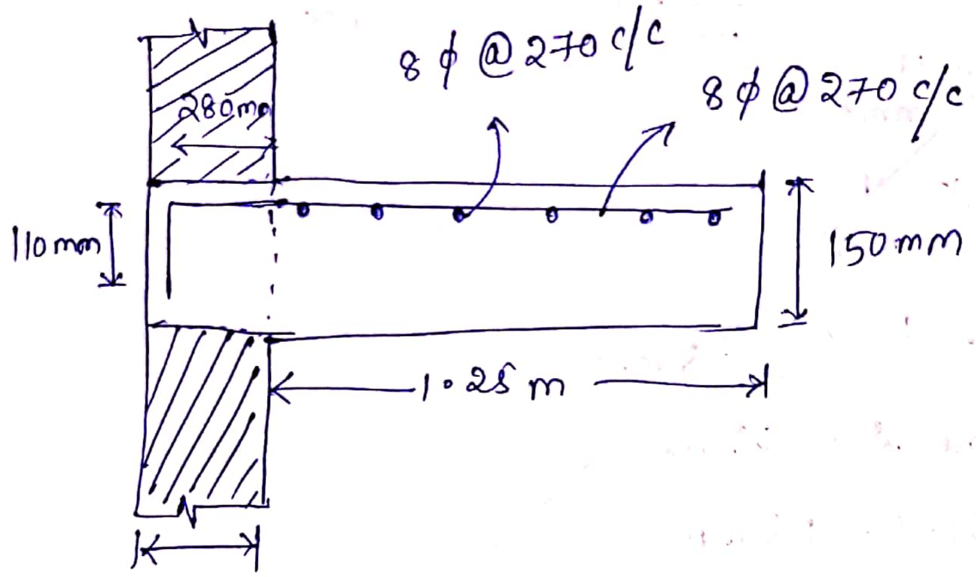
→ (depth required) free end $=$ $\dfrac{1}{2}$ to $\dfrac{1}{3}$ × (depth at fixed end)

→ Main reinforcement is provided at top & to be curtailed.

Q/ Design a cantilever slab for an overhang of 1.25m. The imposed load on slab consists of $1\ \dfrac{kN}{m^2}$ of live load & wt. of finishing 800 N/m^2. Use M20 concrete & Fe415 steel.

(Do the problem by self)

* final bar distribution for cantilever slab case →



here bend is 90°.

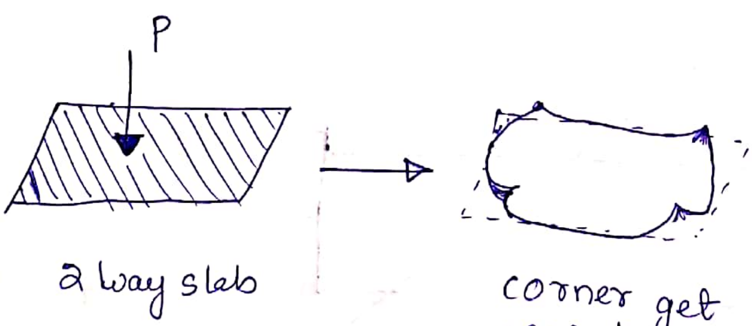
$l_0 = 8\phi$ { ϕ = dia of bar }

$L_d = 390$ mm (280 mm + 110 mm)

~~x~~

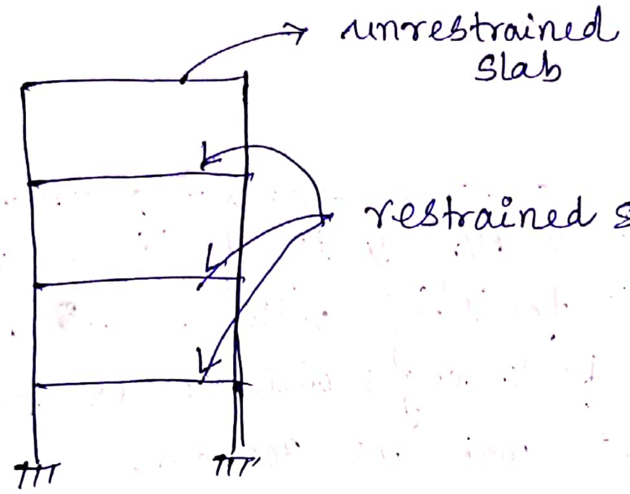
Design of 2-way S/S slab for flexure with corner free to lift (unrestrained).

$\frac{l_y}{l_x} < 2 \Rightarrow$ Two-way slab



Restrained slab → corners are prevented from ~~sit~~ lifting.

Unrestrained slab → corners are not prevented from lifting.
(free to lift)



building frame

Design step

1/ → max. moment $M_x = \alpha_x w l_x^2$

$M_y = \alpha_y w l_x^2$

α_x, α_y can be ~~for~~ found from IS-456 code.

2/ At least 50% of tension reinforcement provided at mid span should extend to support & remaining 50% should extend 0.1 l_x or 0.1 l_y of support.

3/ shear check $V_x = w l_x \cdot \frac{r}{2+r}$

$V_y = \frac{w l_x}{3}$ $\left\{ r = \frac{l_y}{l_x} \right\}$

* for S/S beam for 2-way slab (upto 3.5m)

$$\frac{l}{d} = 35 \quad (Fe250)$$

$$\frac{l}{d} = 35 \times 0.8 = 28 \quad (Fe415)$$

Problem

Q/ Design a RCC slab for a room measuring 4m × 5m. The slab carries LL = 2 kN/m^2 & is finished with 20mm thick granolithic finishing with $\gamma = 25$ kN/m^3. Use M20, Fe415. The four edges with corner free to lift. Width of supporting wall is 300 mm.

Solution

Given data slab panel = 4m × 5m
$(l_x) \times (l_y)$

$f_{ck} = 20$ N/mm^2, $f_y = 415$ N/mm^2, $\gamma = 24$ kN/m^3, $t = 200$ thick (mm)

slab is SS, corner free to lift, bearing = 300 mm

step-I. $\frac{l_y}{l_x} = 1.25 < 2 \Rightarrow$ Two way slab.

step-II (fixing depth of slab)

with respect to control of deflection

$$\frac{l}{d} = 20 \qquad \text{Pg - 37 (IS- 456)}$$

$$\Rightarrow \frac{l_x}{d} = 20 \qquad (\text{Pg 39, IS 456})$$

$\Rightarrow \frac{4000}{20} = d$

$\Rightarrow \boxed{d = 200\ mm}$

* But depth is reduced to $\boxed{120\ mm}$

assume effective c = ~~200m~~ 30 mm

$\therefore D = d + d' = 120 + 30 = 150\ mm$

$\boxed{D = 150\ mm}$

step-III Calculation of effective span (pg 34, IS 456) ~~for span~~ short span

$$l_{eff} = \begin{cases} l\ c/c = 4 + \frac{0.3}{2} + \frac{0.3}{2} = 4.3\ m \\ \text{less}\ \ l_x + d = 4 + 0.12 = 4.12 m \end{cases}$$

$\Rightarrow \boxed{(l_{eff})_x = 4.12 m}$

$$(l_{eff})_y = \begin{cases} l\ c/c = 5 + \frac{0.3}{2} + \frac{0.3}{2} = 5.3 m \\ l_y + d = 5 + 0.12 = 5.12 m \end{cases}$$

$\boxed{(l_{eff})_y = 5.12 m}$

step-IV load calculation

1- Dead load = $2\cancel{4}5 \times 0.\cancel{12}15 \times 1 = 3.\cancel{75} kN/m^2$
$(\gamma \times D \times b)$

2- live load = $2\ kN/m^2$

3- granolithic finish = $0.02 \times 1 \times 24 = 0.48\ kN/m^2$

$w = 6.23\ kN/m^2$

$\Rightarrow w_u = 1.5 w = 1.5 \times 6.23 = 9.345 \frac{kN}{m^2}$

$$\boxed{w_u = 9.345 \frac{kN}{m^2}}$$

Stg-V

Calculation of bending moments

Let's calculate B.M. along short span (M_x) & BM. along long span (M_y) using IS-456.

$$\left.\begin{aligned} M_x &= \alpha_x w_u l_x^2 \\ M_y &= \alpha_y w_u l_y^2 \end{aligned}\right\} \text{Pg - 91 (IS - 456)}$$

$\frac{l_y}{l_x} = 1.2 \quad \alpha_x = 0.084 \quad \alpha_y = 0.059$

$= 1.3 \quad \alpha_x = \cancel{0.059}$ 0.093 $\quad \alpha_y = 0.055$

by interpolation

$$\alpha_x = 0.084 + \frac{\cancel{0.059}\ 0.093 - 0.084}{1.3 - 1.2}(1.25 - 1.2)$$

$$\boxed{\alpha_x = 0.0885}$$

$$\alpha_y = 0.055 - \frac{\dfrac{0.055 - 0.059}{\cancel{1.3 - 1.2}}}{1.3 - 1.2}(1.3 - 1.25)$$

$$\boxed{\alpha_y = 0.057}$$

$\therefore M_x = \alpha_x w_u l_x^2$

$= \cancel{0.057}$ 0.0885 $\times 9.345 \times 4.12^2$

$= 14.038$ kN m

$M_y = \alpha_y w_u l_x^2 = 0.057 \times 9.345 \times 4.12^2$

$$\boxed{M_y = 9.041 \text{ kNm}}$$

Calculation of effective depth required

$$d_{req} = \sqrt{\frac{M_u}{0.138 f_{ck} b}}$$

$$\left\{ \begin{array}{l} Q = 0.148 f_{ck} \ (Fe250) \\ \quad = 0.138 f_{ck} \ (Fe415) \\ \quad = 0.133 f_{ck} \ (Fe500) \end{array} \right.$$

$$\Rightarrow d_{req} = \sqrt{\frac{14.038 \times 10^6}{0.138 \times 20 \times 1000}}$$

$= 71$ mm

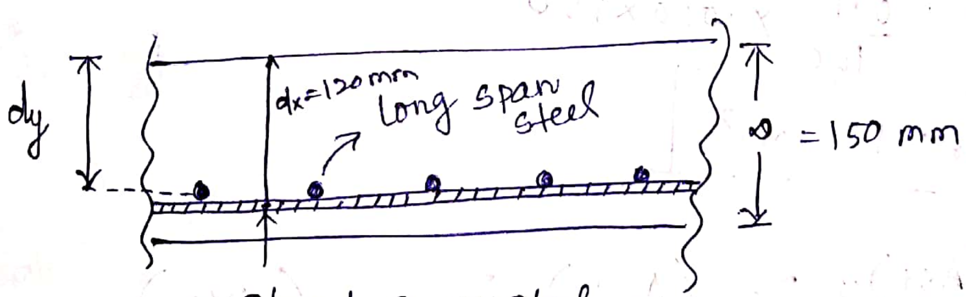
but $d_{provided} = 120$ mm $> d_{req}$

$\Rightarrow$ ok & UR reinforced.

Step VI

Calculation of Ast

Let's assume 8 mm ϕ bar along both the spans. Now shorter span steel will be kept below longer span steel ($M_x > M_y$)



Short span steel

$$d_y = 120 \text{ mm} - \frac{8}{2} - \frac{8}{2} = 112 \text{ mm}$$

Now Ast along short span $= (A_{st})_x$

$$(A_{st})_x = \frac{0.5 f_{ck}}{f_y} \left[1 - \sqrt{1 - \frac{4.6 (M_u)_x}{f_{ck} b (d_x)^2}} \right] b (d)_x$$

$$= \frac{0.5 \times 20}{415} \left[1 - \sqrt{1 - \frac{4.6 \times 14.038 \times 10^6}{20 \times 1000 \times 120^2}} \right] 1000 \times 120$$

$(Ast)_x = 344.71\ mm^2$

Ast along long span $= (Ast)_y$

$$(Ast)_y = 0.5\frac{f_{ck}}{f_y}\left[1-\sqrt{1-\frac{4.6(M_u)_y}{f_{ck}\, b (d_y)^2}}\right] b\, d_y$$

$$= 0.5 \times \frac{20}{415}\left[1-\sqrt{1-\frac{4.6 \times 9.041}{20 \times 1000 \times 112^2}}\right] 1000 \times 112$$

$$\boxed{(Ast)_y = 233.81\ mm^2}$$

Check for min. Ast

$(Ast)_{min} = 0.12\%\ bD$ (Fe 415 & Fe 500)

$0.15\%\ bD$ (Fe 250)

$$(Ast)_{min} = \frac{0.12}{100} \times 1000 \times 150$$

$$\boxed{(Ast)_{min} = 180\ mm^2}$$

$(Ast)_x$ & $(Ast)_y > (Ast)_{min}$ (OK)

Now for 8 mm ϕ bar

$ast = \frac{\pi}{4} \times 8^2 = 50.26\ mm^2$

Spacing S_x = shorter span

$$s_x = \frac{a_{st}}{(A_{st})_x} \times 1000$$

$$= \frac{50.26}{344.71} \times 1000$$

$$= 145.8 \text{ mm c/c}$$

$\therefore$ provide $s_x = 140$ mm c/c along short span.

* (Spacing is reduced otherwise Ast will reduced)

$$s_y = \frac{a_{st}}{(A_{st})_y} \times 1000$$

$$= \frac{50.26}{233.81} \times 1000 = 214 \text{ mm c/c}$$

$\therefore$ provide $s_y = 200$ mm c/c along long span.

$$(A_{st})_x \text{ provided} = \frac{a_{st}}{s_x} \times 1000 = \frac{50.26}{140} \times 1000$$

$$\boxed{(A_{st})_x \text{ provided} = 359 \text{ mm}^2}$$

$$\boxed{(A_{st})_{yp} = \frac{a_{st}}{s_y} \times 1000 = \frac{50.26}{200} \times 1000 = 251.3 \text{ mm}^2}$$

* alternate bars of short span can be bent $0.15 l_x \left(\frac{l_x}{7}\right)$ from centre of support.

$0.15 l_x = 0.15 \times 4.12 = 0.62$m from centre of support.

* alternate bars of long span to be bent $0.15 l_y \left(\frac{l_y}{7}\right)$ from centre of support.

$0.15\,l_y = 0.15 \times 5.12 = 0.768\,m$ from centre of support & remaing alternate bar will continue to the support.

~~Check for SF~~

Step-VII / Check for shear

$$V_{ux} = \frac{W_u l_x}{2} = \frac{9.345 \times 4.12}{2} = 19.25\ KN$$

$$V_{uy} = \frac{W_u l_y}{2} = \frac{9.345 \times 5.12}{2} = 23.92\ KN$$

$$\therefore (\tau_v)_x = \frac{V_{ux}}{b\,d_x} = \frac{19.25 \times 1000}{1000 \times 120} = 0.16\ N/mm^2$$

$$(\tau_v)_y = \frac{V_{uy}}{b\,d_y} = \frac{23.92 \times 1000}{1000 \times 112} = 0.21\ N/mm^2$$

Now $p = \frac{(A_{st})_{xp}}{b\,d_x} \times 100$

W all highest values are taken

$$= \frac{359}{1000 \times 120} \times 100$$

$$= 0.3\%$$

from IS table $\tau_c = 0.3\ N/mm^2$

Compare $\tau_c > (\tau_v)_x$

$\tau_c > (\tau_v)_y$

$\therefore$ Slab is safe in shear.

$\Rightarrow$ No design required for shear.

Step-VIII / check for deflection

$$f_s = 0.58\, f_y \times \frac{(A_{st})_r}{(A_{st})_p}$$

$$= 0.58 \times 415 \times \frac{344.71}{359}$$ (all highest values have been taken)

$$= 231.11\ N/mm^2 \quad , p = 0.30\%$$

modification factor (k) = 1.45

$$\frac{l}{d} = 20k$$

$$\Rightarrow \frac{(l)_x}{d_x} = 20 \times 1.45 \Rightarrow d_x = \frac{4120}{20 \times 1.45} = 142\ mm$$

$$\boxed{d_x = 142\ mm}$$

- In earlier step we have assumed $(d_x)_p = 120$ mm

$\Rightarrow$ slab fails in deflection.

$\Rightarrow$ ~~slab~~ $(d_x)_p < (d_x)_{req}$

$\Rightarrow$ slab has to be re-design assuming $d = 200$ mm

Step-IX / Check for development length

Step-X / steel distribution diagram.

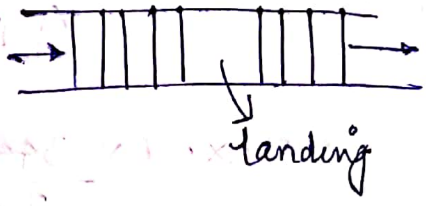
(soln)

STAIR

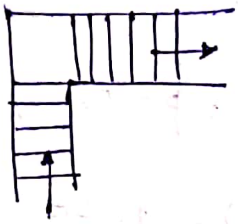
Stairs → A staircase is a means of giving access to different floors/levels of a building.

Types –

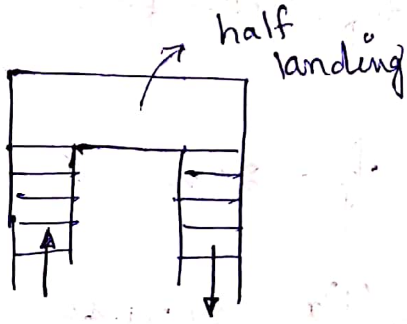
① straight stair



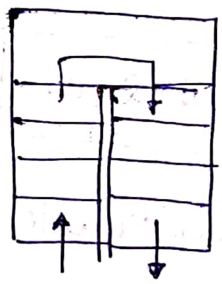
② quarter turn ~~Stra~~ stair



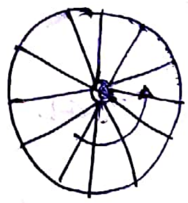
③ half turn stair



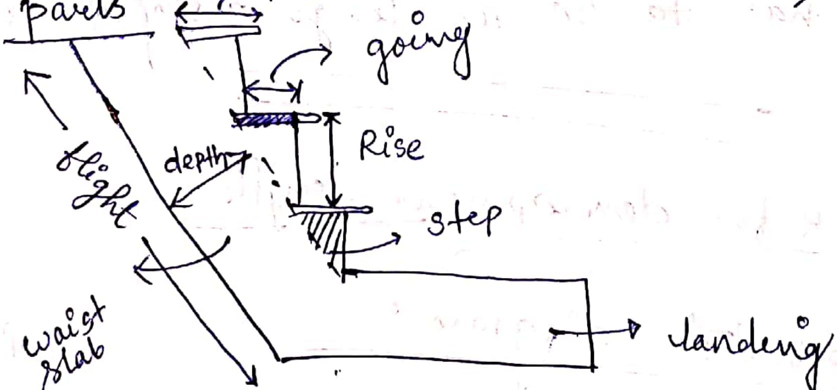
④ Dog legged stair



⑤ Circular / spiral stair

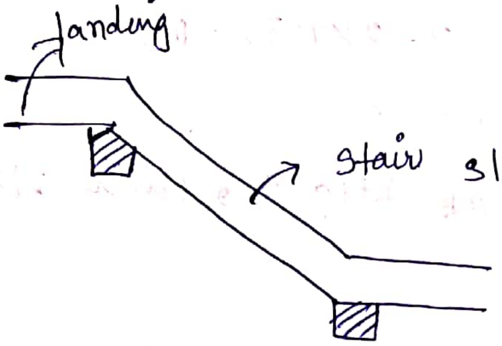


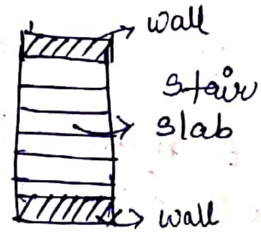
parts



Stair slab spanning longitudinally

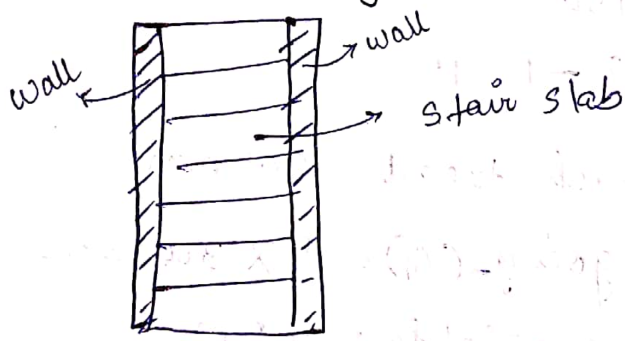
The stair slab supported at the bottom & top of flight is called stair slab spanning longitudinally.





Stair slab spaning horizontally

The stair slab supported both sides over the wall is called stair slab spanning horizontally.



Problem

Q//

Design a dog legged stair-case for an office building in a room measuring 3.0m × 6.0m. Floor to floor height is 3.5m. The building is a public building liable to over-crowding. Stairs are supported on brickwalls 230mm thick at the end of landing. Use M20, Fe415.

Soln width of stair case = 3 m

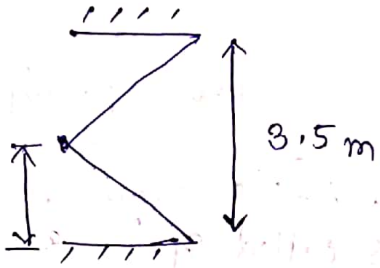
considering 2 - flights of dog legged stair case, let's assume width of each flight = 1.35 m

Space between flights = 3 − 2 × 1.35 = 0.3 m

floor to floor height = 3.5 m

As there will be 02 no. of flight, each flight will have a height of

$$\frac{3.5}{2} = 1.75\ m$$



Assume height of riser = 150 mm

No. of risers $= \frac{1750}{150} = 11.66 \simeq 12$

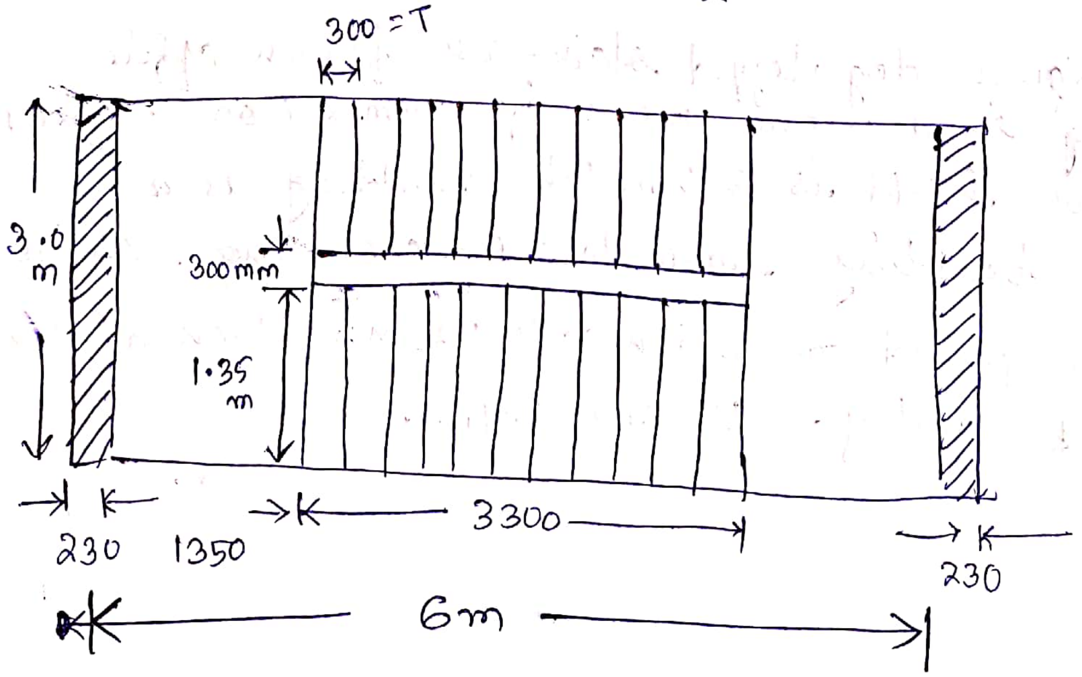
No. of tread = 12 − 1 = 11

Let width of each tread = 300 mm

Total no. of going (G) = 11 × 300 = 3300 mm

Total length available = 6.0 m

width of each landing $= \frac{6 - 3.3}{2} = 1350$ mm



Design of dog-legged stair case

→ effective span of flight = c/c dist. of wall

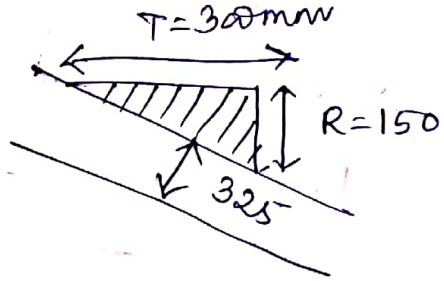
$$= 6 + \frac{0.23}{2} + \frac{0.23}{2} = 6.23\,m$$

→ Thickness of waist slab = $\frac{1}{20}$ of span.

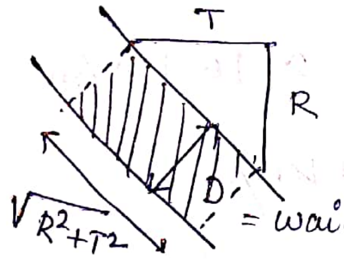
$$= \frac{1}{20} \times 6230$$

$$= 311.5\,mm$$

Let's take $d = 300\,mm$ & $D = 325\,mm$



NOTE



D' = waist slab

Dead load of 1 step (1 m width)

w_1 = area of each step × 1 × γ

$$= \frac{R \cdot T}{2} \times 1 \times 25 = \frac{25\,RT}{2}$$

wt. of steps per m. length in plan $= \frac{25}{2} \cdot \frac{RT}{T} = \frac{25}{2} R$

Dead load of waist slab

$$w_2 = \sqrt{R^2+T^2} \cdot D \times 1 \times 25 = 25\sqrt{R^2+T^2} \cdot D$$

wt. of waist slab per m. length in plan

$$= \frac{25 \cdot D \sqrt{R^2+T^2}}{T}$$

Load calculation

→ ~~Wat~~ weight of wl. slab per m. width of flight

$$= D\sqrt{1+\frac{R^2}{T^2}} \cdot 25 = 0.325\sqrt{1+\frac{0.15^2}{0.3^2}} \times 25$$

$$= 9.1 \text{ kN/m}$$

→ weight of steps per m. width of flight

$$= \frac{25RT}{2T} = \frac{1}{2}25R = 1.875 \text{ kN/m}$$

∴ Total load (w) $= 9.1 + 1.875 = 10.975$ kN/m

LL $= 5 \text{ kN/m}^2 = 5$ kN/m per m. width

$$w = 15.975 \simeq 16 \text{ kN/m}$$

$\Rightarrow w_u = 1.5w = 1.5 \times 16 = 24$ kN/m

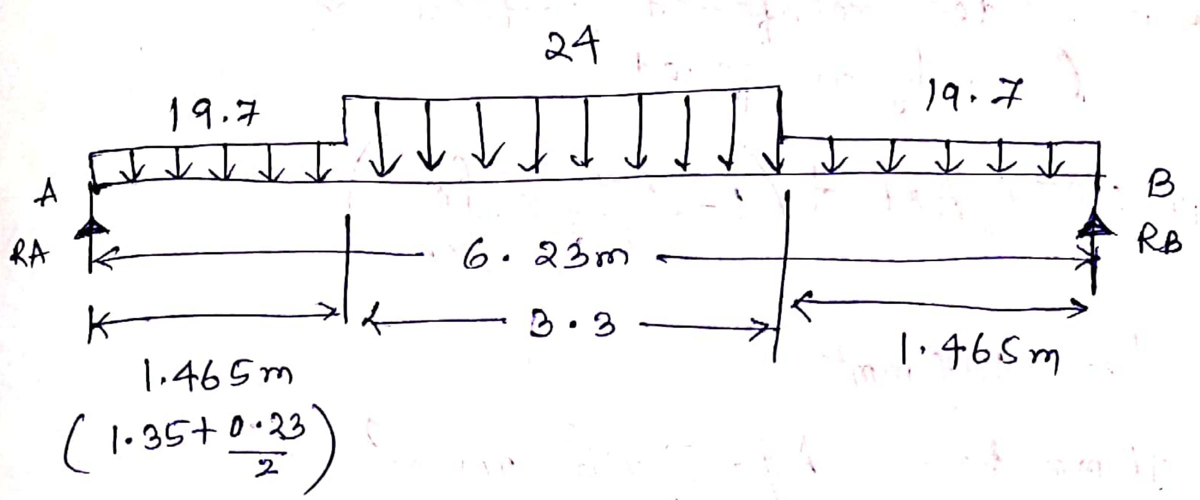
~~Flo~~ for landing $DL = 0.325 \times 25 \times 1 = 8.125 \frac{kN}{m}$

$LL = 1 \times 5 = 5 \frac{kN}{m}$

$$w = 13.125 \text{ kN/m}$$

(w_u) landing $= 1.5w = 19.7$ kN/m

The load diagram is



Design Moment : $R_A = R_B = \dfrac{(2\times 19.7\times 1.465)+(24\times 3.3)}{2}$

$= 68.5$ kN

Bending moment at mid span

$$M_u = 68.5\times\frac{6.23}{2} - \left(19.7\times 1.465\times \frac{1.465+3.3}{2}\right)$$

$$- \left(24\times\frac{3.3}{2}\times\frac{3.3}{4}\right)$$

$= 112$ kNm

$M_{u,lim} = 0.138 f_{ck} b d^2 = 0.138\times 20\times 1000\times 300^2$

$= 2484$ kNm > 112 kNm

(hence section is UR)

Ast calulation

for under-reinfored section &
~~a~~ singly reinfored

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st}}{bd} \times \frac{f_y}{f_{ck}}\right)$$

$\Rightarrow A_{st} = 1117 \ mm^2$

Use 16 mm bars, $A\phi = \frac{\pi}{4} \times 16^2 = 201 \ mm^2$

$$\text{Spacing} = \frac{A\phi}{A_{st}} \times 1000 = \frac{201}{1117} \times 1000 = 179 \ mm$$

provide 16 mm ϕ @ 170 mm c/c

Distribution steel = 0.12 % of area
(use 10 ϕ bar)

$$= \frac{0.12}{100} \times 1000 \times 325$$

$$= 390 \ mm^2$$

$$\therefore \text{spacing} = \frac{78.5}{390} \times 1000 = 201 \ mm$$

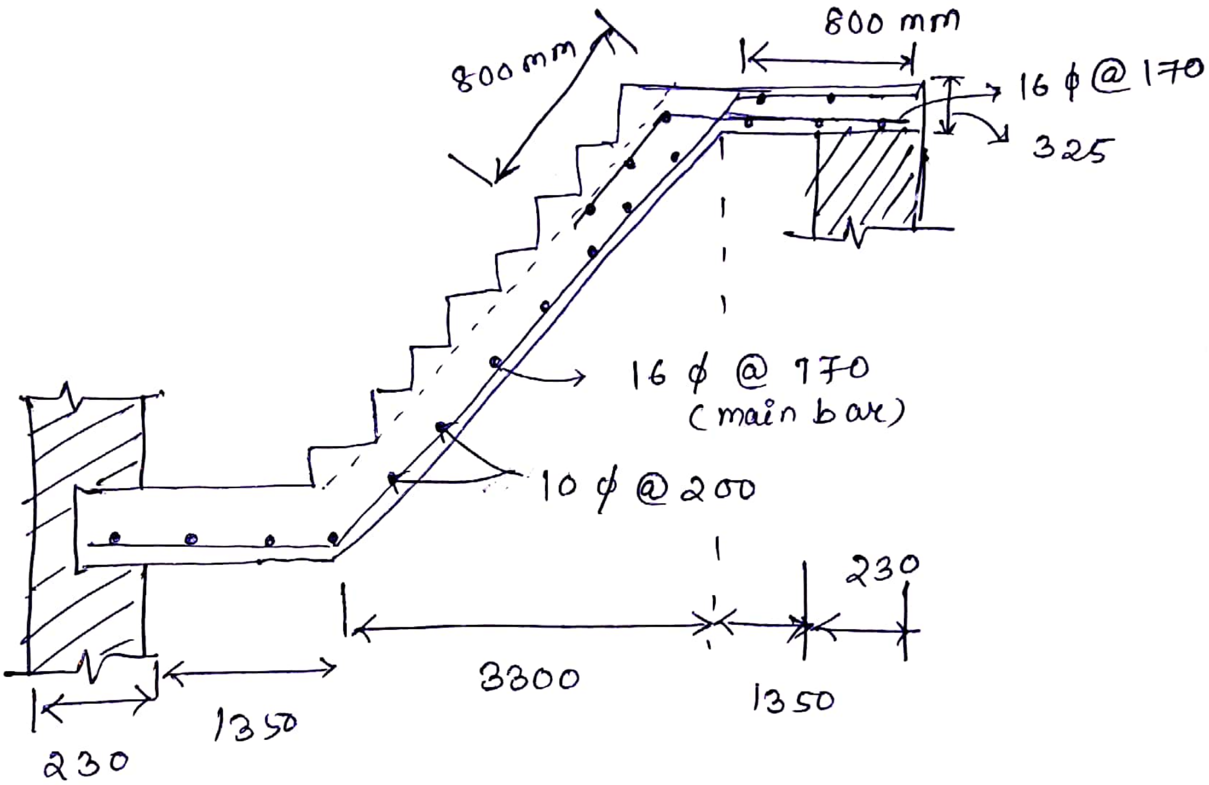
provide 10 mm ~~@~~ ϕ @ 200 mm c/c

Development length

$$L_d = \frac{\phi (0.87 f_y)}{4 \tau_{bd}}$$

$$\therefore \quad L_d = \frac{16 \times 0.87 \times 415}{4 \times 1.6 \times 1.2} = 752 \text{ mm}$$

$\Rightarrow$ provide 800mm length of bars where L_d is required.



Chapter-07

Design of axially loaded columns & footings

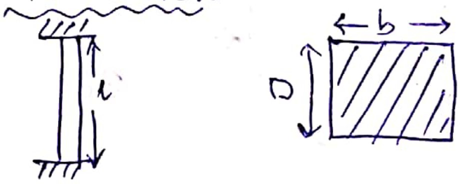
Assumptions in Limit State of Collapse (Compression)

↳ The max. compressive strain in concrete in axial compression is taken as 0.002.

↳ The max. compressive strain at the highly compressed extreme fibre in concrete subjected to axial compression & bending & when there is no tension on the section shall be 0.0035 minus 0.75 times the strain at the least compressed extreme fibre.

Definition of colums

FRONT VIEW



TOP VIEW

↳ It is a vertical compression member which is mainly subjected to axial loads & the effective length of which exceeds three times the least lateral dimension.

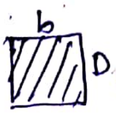
→ Always STRAIGHT.

P = axial compressive load

original length = l

l' = effective length

P



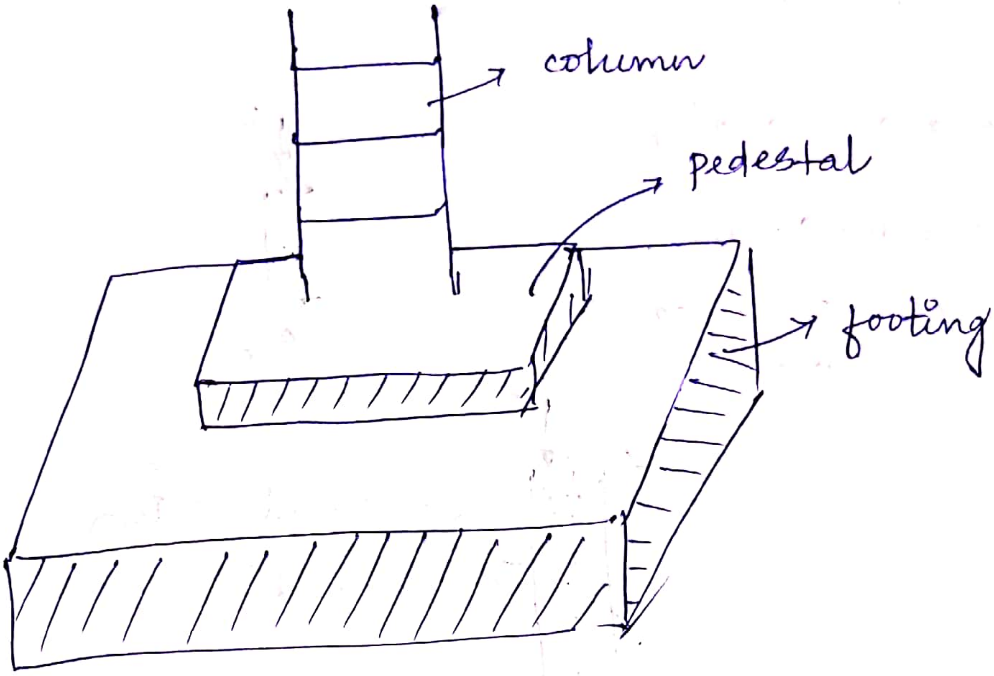
b = least lateral dimension

$$\boxed{l' > 3b}$$

Pedestal → The compression member whose effective length is less than three times its lateral dimension is called pedestal.

$$\boxed{l' < 3 \times \text{lateral dimension}}$$

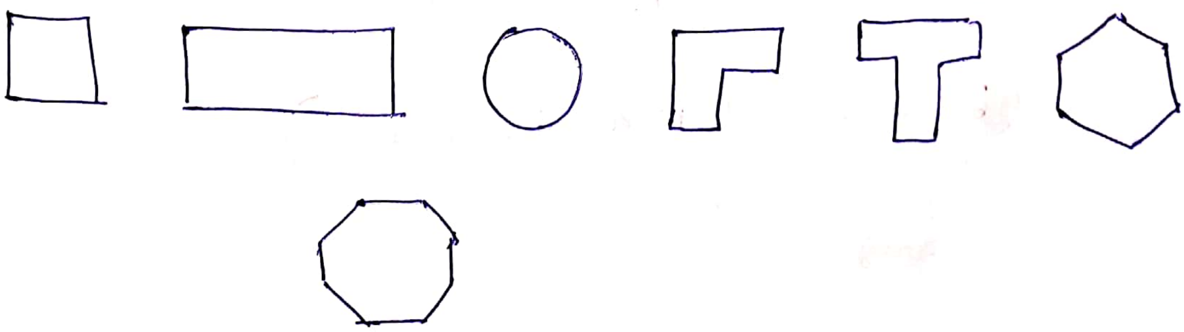
~~Strut~~ →



Strut → the compression member which is either inclined/straight & the effective length is less than three(3) times the least lateral dimension.

Classification of column

1. Shape of c/s



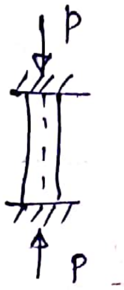
MRP,Civil,SDTE(O)

2. Material of construction

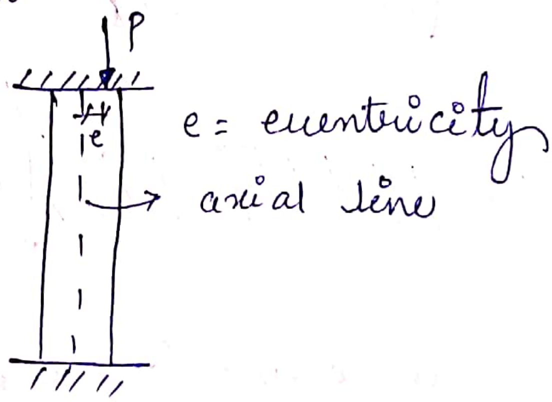
- → Timber column
- → masonary "
- → RCC "
- → Steel "
- → composite "

3. based on loading

- → axially loaded column

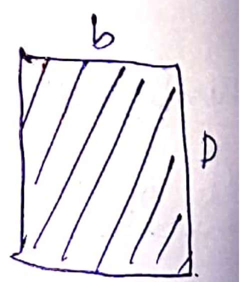


- → eccentrically loaded ~~to~~ column



4. based on slenderness ratio

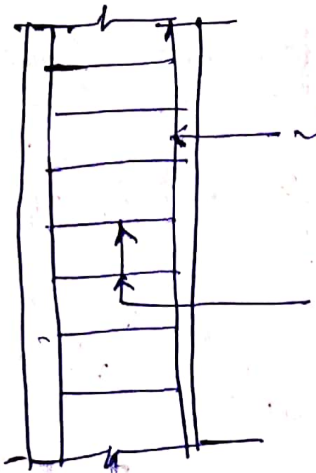
- → Short column $\frac{l_{eff}}{b} \leq 12$
- → long column $\frac{l_{eff}}{b} > 12$



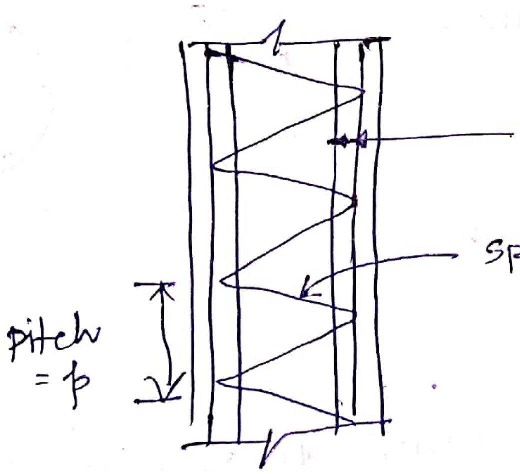
(b = least lateral dimension)

5. Type of lateral reinforcement

→ column with longitudinal steel & lateral ties

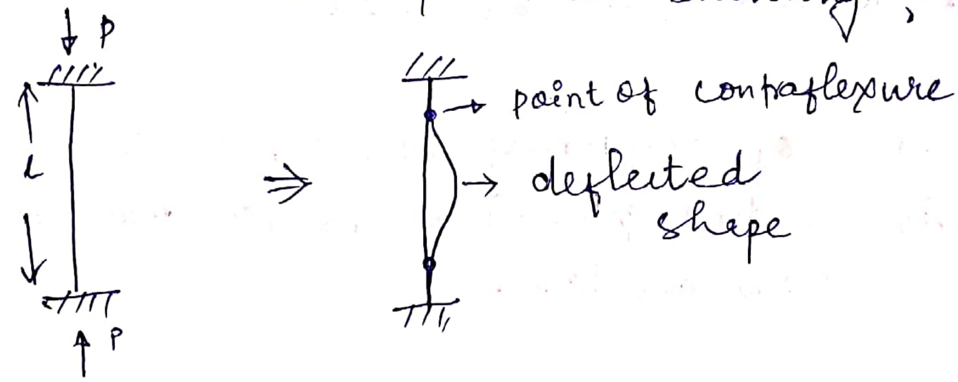


→ column with helical (spiral) ties



Effective length of column

It is defined as that ~~part of column~~ length of column which takes part in buckling.



→ Note

Effective length of various end conditions can be checked from IS-456.

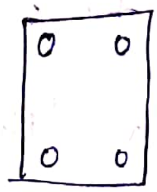
Specification for minimum reinforcement & max. reinf

↳ longitudinal reinforcement ≮ 0.8% of gross c/s area (minimum)

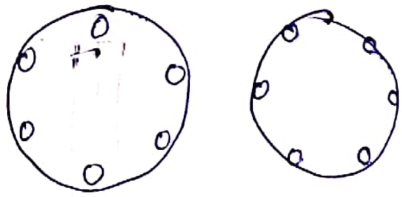
≯ 6% of gross c/s area (maximum)

↳ minimum no of longitudinal bars

= 4 in rectangular column



= 6 in circular column



⟼ min. dia of bar in column = 12mm

↳ for pedestal in where longitudinal reinforcement is not taken in account in strength calculation, nominal longitudinal reinforcement not less than 0.15% of the cross sectional area shall be provided.

↳ max. of reinforcing bar (rebar) in column is 6% but preferred as 4% max due to 50% overlap.

MRP,Civil,SDTE(O)

→ In beam max. rebar is 4% of c/s in compression & tension.

↳ minimum rebar area in tension is $\frac{0.85bd}{fy}$.

→ in slabs minimum rebar is 0.120% in LSM & 0.15% in WSM.

Specification for maximum reinforcement

cover

The nominal cover
↳ for a longitudinal reinf. bar in column $\nless \begin{cases} 40\ mm \\ \phi \text{ (dia of bar)} \end{cases}$

↳ In case of small sized column (where $\phi \leq 12mm$) nominal cover 25 mm is used.

slenderness limit

the unsupported length length between the end supports shall not exceed 60 times the least lateral dimension.

minimum eccentricity

$$e_{min} = \frac{\text{unsupported length}}{500} + \frac{\text{lat. dimension}}{30}$$

$e_{min} \geq 20\ mm$

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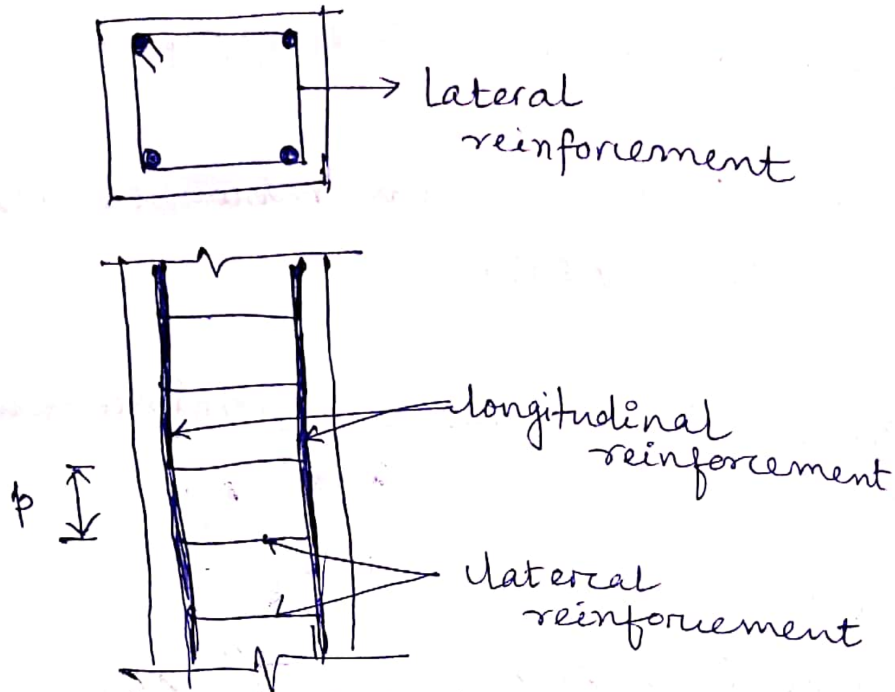
Lateral/traverse reinforcement



↳ dia of lateral tie $= \max \begin{cases} \phi/4 \ (\phi_{large} = \text{dia of long. bar}) \\ 6\ mm \end{cases}$

↳ pitch (p) $\ngtr \begin{cases} \text{least lateral dimension of column} \\ 16\phi_{small} \\ 300\ mm \end{cases}$

↳ dia. of spiral reinforcement $> \begin{cases} \dfrac{\phi_{large}}{4} \\ 6\ mm \end{cases}$

↳ pitch of helical reinf. $\ngtr \begin{cases} 75\ mm \\ \dfrac{\phi_{core\ of\ concrete}}{6} \end{cases}$

$\nless \begin{cases} 25\ mm \\ 3 \times \text{dia of steel bar forming the helix} \end{cases}$

MRP,Civil,SDTE(O)

Analysis & Design of axially loaded short Square column

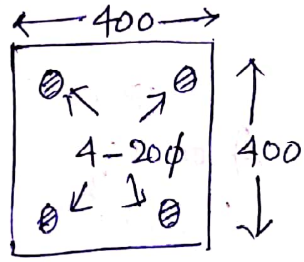
Q// A reinforced concrete short column is 400 mm X 400 mm and has 4 bars of 20 mm ϕ. Determine the ultimate load carrying capacity of column is M20 concrete & Fe415 Steel is used. Assume $e_{min} < 0.05D$

Soln $b = 400\,mm$, $d = 400\,mm$

A_{sc} = area of steel $= 4 \times \frac{\pi}{4} \times 20^2 = 1256.6\ mm^2$

A_g = gross area of ~~concrete~~ column $= 400 \times 400$
$= 160000\ mm^2$

$f_{ck} = 20\ N/mm^2$, $f_y = 415\ N/mm^2$



A_c = area of concrete $= A_g - A_{sc}$
$= 158743.4\ mm^2$

As per IS-456 & $e_{min} < 0.05\,D$

$$\boxed{P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}}$$

$= 1619.3\ KN$

Analysis & Design of rectangular column :-

Q// An RCC short column of size 400mm x 500mm is carrying a factored load of 3000 KN. Design the column assuming $e_{min} < 0.05D$. Use M25 concrete & Fe415 steel.

MRP,Civil,SDTE(O)

Soln

given $b = 400\ mm$

$D = 500\ mm$

$P_u = 3000\ kN \times 3 \times 10^6\ N$

$e_{min} \leq 0.05D$

$f_{ck} = 25\ N/mm^2$

$f_y = 415\ N/mm^2$

Area of steel (A_{sc})

as $e_{min} \leq 0.05D$

$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$

$A_c = A_g - A_{sc}$

$= 400 \times 500 - A_{sc}$

$\Rightarrow A_c = 200000 - A_{sc}$

From eqn

$\therefore 3 \times 10^6 = 0.4 \times 20 \times (200000 - A_{sc}) + 0.67 f_y A_{sc}$

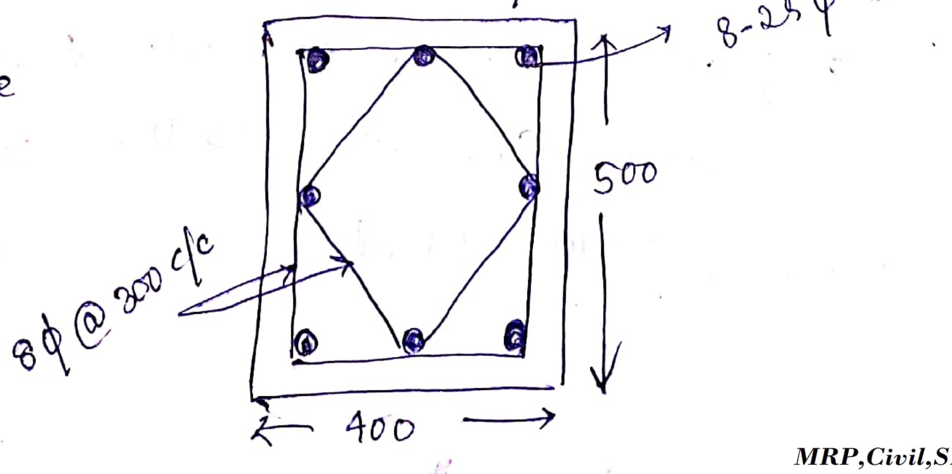
$\Rightarrow A_{sc} = 3730.6\ mm^2$

Use 25ϕ bar $\quad A_\phi = \frac{\pi}{4} \times 25^2 = 490\ mm^2$

No. of bars $= \frac{3730.6}{490} \simeq 7.6 \simeq 8$

$\therefore$ provide ~~8 no~~ $8 - 25\phi$ bars.

~~tate~~



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lateral ties

dia of lateral tie $= \begin{cases} 25/4 = 6.25\,mm \\ 6\,mm \end{cases}$

$\Rightarrow$ use 8 mm ϕ ties.

spacing of ties

The pitch $\ngtr \begin{cases} 400\,mm \\ 16 \times 25 = 400\,mm \\ 300\,mm \end{cases}$

$\therefore$ provide 8 mm ϕ @ 300 mm c/c as double ties.

(soln)

Analysis & Design of circular column

Q// Design a circular column of diameter 400mm subjected to a load of 1200 kN. The column is having spiral ties. The column is 3m long & is effectively held in position at both ends but not restrained against rotation. Use M20, Fe415 steel.

soln

$l = 3m$

$P = 1200\,kN$

$D = 400mm$

$f_{ck} = \cancel{2}\ 25\,\frac{N}{mm^2}$

$f_y = 415\,\frac{N}{mm^2}$

$\therefore$ effective length (l_{eff}) = $1.0\,l$ (IS-456)

= 3 m.

slenderness ratio = $\frac{l}{d} = \frac{3000}{400} = 7.5 < 12$

$\Rightarrow$ Hence it is a short column.

Minimum eccentricity

$$e_{min} = \frac{l_{eff}}{500} + \frac{D}{30}$$

$$= \frac{3000}{500} + \frac{400}{30} = 19.33 \leq 20$$

$\therefore e_{min} = 20$ mm

$$\frac{e_{min}}{D} = \frac{20}{400} = 0.05$$

$\Rightarrow$ It is designed as axially loaded column.

Area of steel (A_{sc})

$P_u = 1.5\,P = 1.5 \times 1200 = 1800$ kN

for a circular column with helical ties

$$P_u = 1.05\,(0.4 f_{ck} A_c + 0.67 f_y A_{sc})$$

$$A_g = \frac{\pi}{4} \times 400^2 = 125663.7\,mm^2$$

$$A_c = A_g - A_{sc} = 125663.7 - A_{sc}$$

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from eq^n

$$1800\times10^3 = 1.05\left[0.4\times25\times(125663.7-A_{sc})+0.67\times415\times A_{sc}\right]$$

$\Rightarrow$ $268.05\,A_{sc} = 480531.15$

$\Rightarrow A_{sc} = 1793\ mm^2$

% of steel $= \dfrac{1793}{125663.7}\times100 = 1.43\%$

$\boxed{0.8\% < 1.43\% < 8\%}$ (ok)

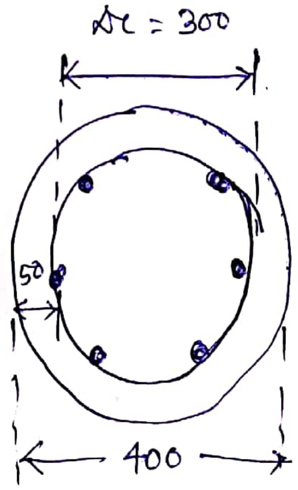
use 20 ϕ bars $A\phi = 314\ mm^2$

No. of bars $= \dfrac{1793}{314} \simeq 6$

$\therefore$ Provide 6 - 20 ϕ bars $\left((A_{sc})_p = 6\times\frac{\pi}{4}\times20^2 = 1884\ mm^2\right)$

Helical Reinforcement

core diameter $D_c = 400 - 2\times50 = 300\ mm$



Area of core

$= \frac{\pi}{4}\times300^2 - 1884$

$= 68880.8\ mm^2$

assume pitch $= p$

volume of core per pitch

$= 68880.8\times p$

MRP,Civil,SDTE(O)

using 8 mm ϕ spiral

Volume of one spiral per pitch $= \frac{\pi}{4} \times 8^2 \times \pi (300-8)$

$= 46110.8 \, mm^2$

$$\frac{\text{Vol. of helical reinf.}}{\text{Vol. of core}} = \frac{46110.8}{68801.8p}$$

As per IS-456

$$\frac{46110.8}{68801.8p} \leq 0.36\left(\frac{A_g}{A_c} - 1\right)\frac{f_{ck}}{f_y}$$

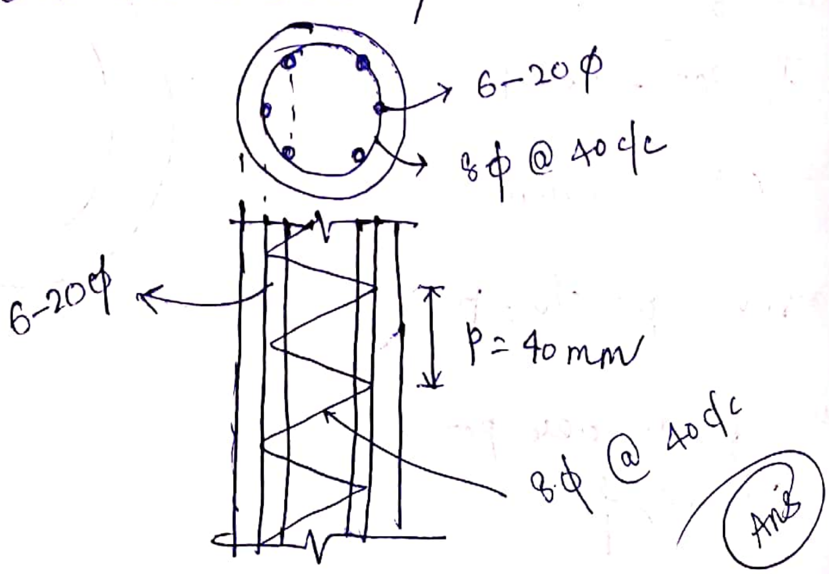
$$\leq 0.36\left(\frac{125663.7}{68801.8} - 1\right)\frac{25}{415}$$

$\Rightarrow p \geq 37 mm$

max. pitch $\begin{cases} \not> 75 mm \\ \frac{\phi \text{ core}}{6} = \frac{300}{6} = 50 mm \end{cases}$

min. pitch $\begin{cases} 25 mm \\ 3 \times \phi \text{ helical reinf} = 3 \times 8 = 24 mm \end{cases}$

$\therefore$ provide 8 ϕ. spirals @ 40 mm c/c.



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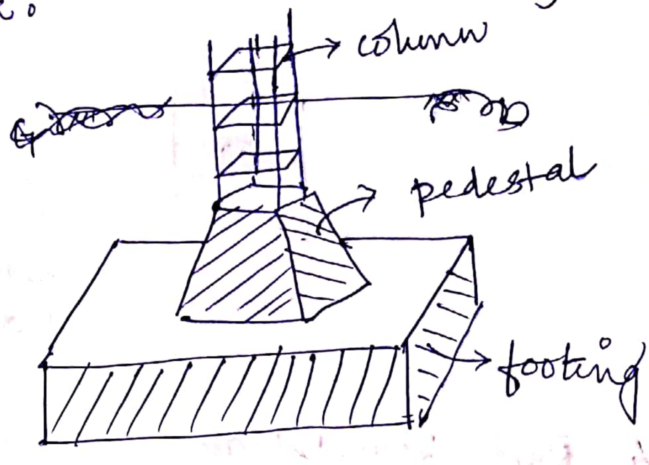
Footing

Definition.

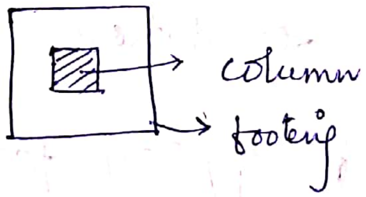
It is the bottom most part of a vertical structure (column) which ultimately transfers the weight from walls & columns to the soil / bedrock.



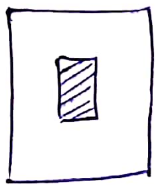
Types.

1. Isolated footing

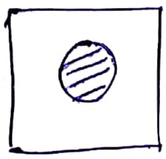
↳ Square footing



↳ Rectangular footing

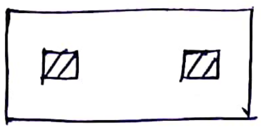


↳ circular footing

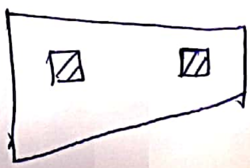


2. Combined footing.

↳ Rectangular



↳ Trapezoidal



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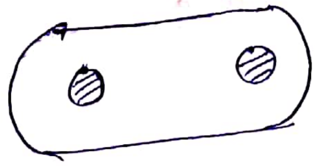
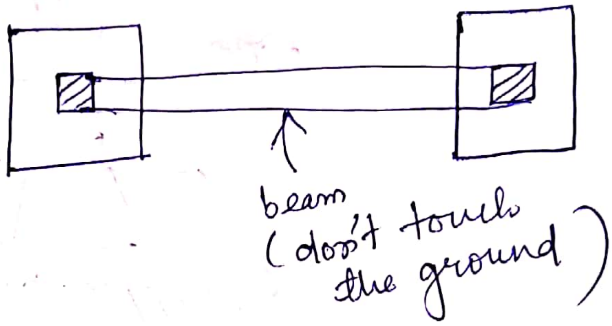
↳ elliptical



3. Strap footing



4. Raft foundation

5. Pile foundation

6. Well foundation

7. Wall footing / strip footing

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Design of isolated square column footing of uniform thickness for flexure & shear :-

Q/ Design a square footing of uniform thickness for an axially loaded column of 450 mm X 450 mm size. The safe bearing capacity of soil is 190 kN/m². Load on column is ~~45~~ 850 kN. Use M20 concrete & Fe415 st.

Soln

Given $P = 850\, kN$

(q_0) bearing capacity $= 190\, kN/m^2$

$f_{ck} = 20 \frac{N}{mm^2}$, $f_y = 415 \frac{N}{mm^2}$

Load calculation

$P = 850\, kN$.

self wt. of footing $= 10\% \times P = 85\, kN$

$\therefore P' = 850 + 85 = 935\, kN$

Area of footing

$$A = \frac{P'}{q_0} = \frac{935}{190} = 4.92\, m^2$$

side of square footing $= \sqrt{4.92} = 2.22 \simeq 2.25\, m$

$\therefore$ Factored ~~shear~~ soil pressure due to column load only $= \dfrac{1.5 \times 850}{2.25 \times 2.25} = 251.85 \frac{kN}{m^2}$

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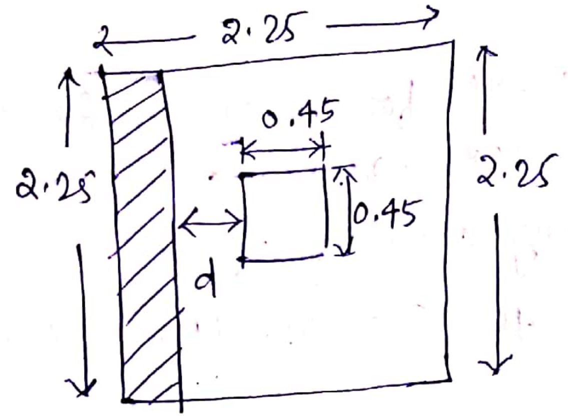
Depth of footing by one-way shear



$$SF = \text{shear force} = 2.25 \times \left(\frac{2.25-0.45}{2} - d\right) \times 251.85$$

$$= 566.66(0.9-d) \quad \{d = \text{eff. depth of footing}\}$$

Assume 0.20% steel, $\tau_c = 0.32\ N/mm^2$

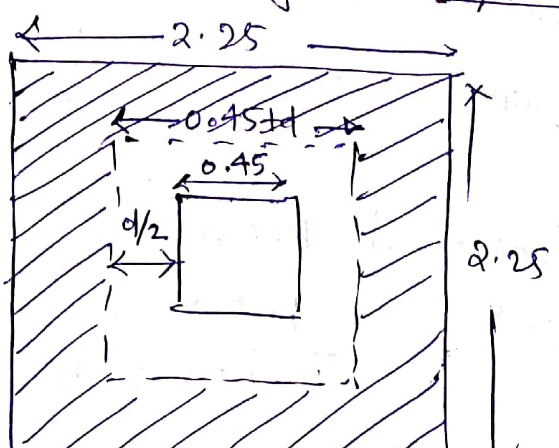
S.F. resisted by section $= \tau_c \times 2.25 \times d$

$= 720\,d$

$\therefore\ 720d = 566.66(0.9-d)$

$\Rightarrow d = 0.396m$

Depth of footing by two-way shear/punching shear



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Consider critical section at a distance of $\frac{d}{2}$ from face of column.

Perimeter of critical section $= 4(0.45+d)$

$= 1.8 + 4d$

S.F. at critical section $= 251.85 \left\{2.25^2 - (0.45+d)^2\right\}$

$= 1274.99 - 251.85(0.2025 + d^2 + 0.9d)$ — (1)

Max. allowable shear stress $= 0.25\sqrt{f_{ck}} = 1118\,kN/m^2$

Shear force resisted $= 1118(1.8+4d) \times d$

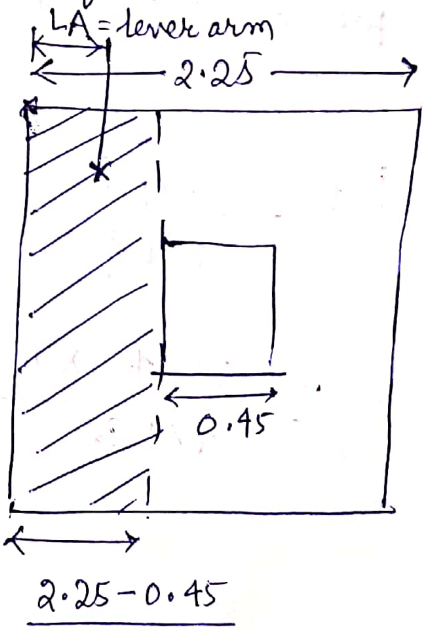
$= 2012.4d + 4472d^2$ — (II)

equating (1) & (II) $\quad d = 0.367m$

Depth of footing by BM criteria

↳ critical section is at face of column.

$\therefore M_u = 251.85 \times 2.25 \times \left(\frac{2.25-0.45}{2}\right) \times \left(\frac{2.25-0.45}{4}\right)$



$= 229.498 \times 10^6\, Nmm$

$M_{u,lim} = Qbd^2 = 2.76 \times 2250 \times d^2$

$\therefore 2.76 \times 2250 \times d^2 = 229.498 \times 10^6$

$\Rightarrow d = 0.192m$

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∴ So the highest value of d = 0.396 m

$\Rightarrow d = 400\,mm$

Overall depth (D) = 400 + 8 + 50 = 458 ≃ 460 mm

Ast

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{b d f_{ck}}\right)$$

$\Rightarrow A_{st} = 1652.03\,mm^2$

Use 16 ϕ bar . $A_\phi = \frac{\pi}{4} \times 16^2 = 201\,mm^2$

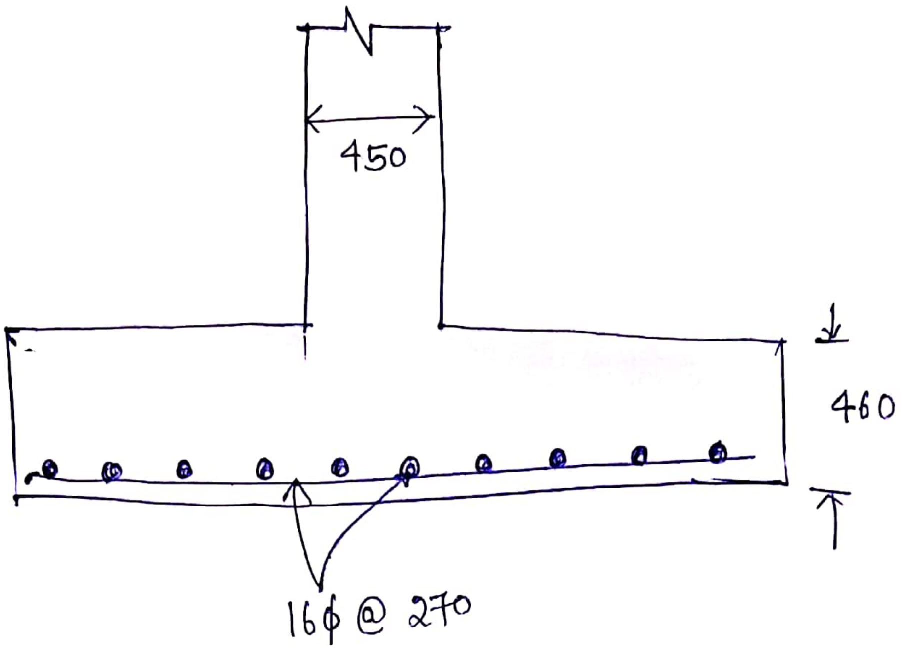
$$\text{Spacing} = \frac{201}{1652.03} \times 2250 = 273\,mm$$

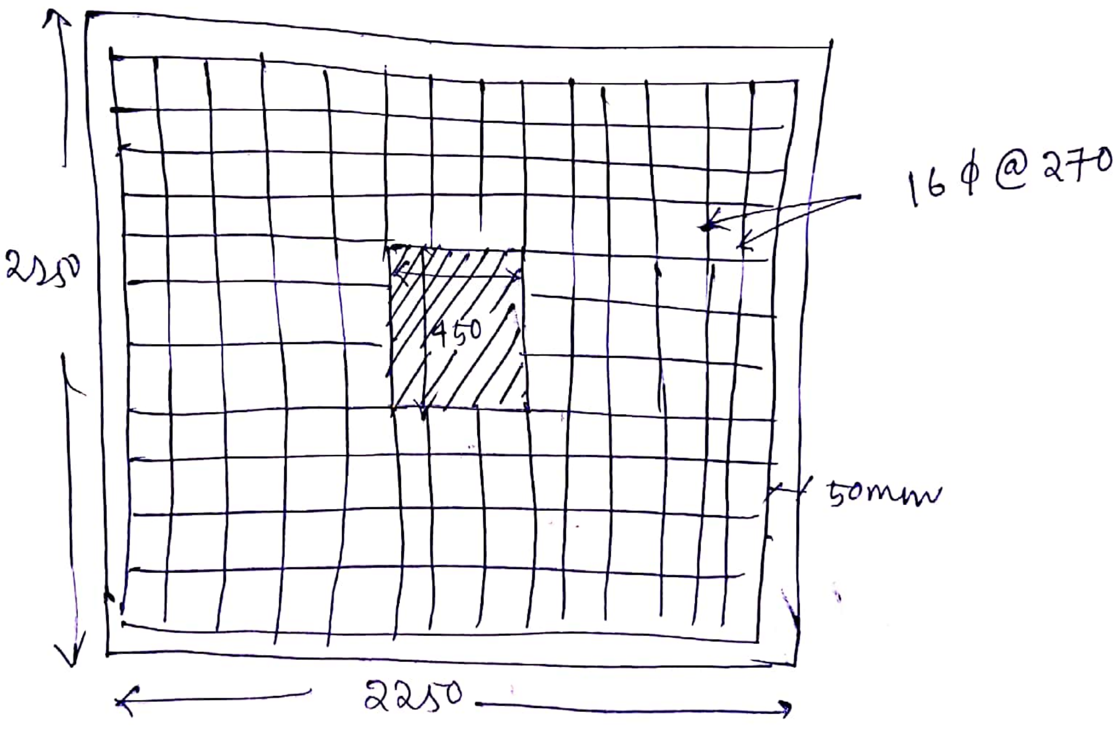
∴ Provide 16 ϕ bar @ 270 mm c/c in each dir^n^.

Check for development length (L_d)

$$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}}$$

$$= \frac{0.87 \times 415 \times 16}{4 \times 1.92} = 752.2\,mm$$





Lect. Note - SD-I
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