Unit -4

Rotational Motion

Translational and Rotational Motion

1. Translational Motion :-

→ The motion of a body is purely translational if the velocity of all the particles of the body is the same at any instant of motion.

→ During such motion all particles have same displacement velocity and acceleration at an ~~inst~~ instant.

→ There are three types of translation motion.

(a) Linear or one dimensional motion:

→ when a body is moving along a straight line ~~then~~ then its motion is called linear motion.

→ examples :- Bus running on a straigh road.

(b) Two Dimensional motion:

→ when a body is moving in a plane then two coordinates changes this types of motion is called two dimensional motion. [(x,y) or (y,z) or (x,z)]

→ examples:- A car moving on a zig-zag road, Projectile motion.

(C) Three dimensional motion:

→ When a body is moving in the sky (space) then all the three coordinates changes this is called three dimensional motion.

→ Example:- A bird flying in the sky.

Let a system have n-particles of mass $m_1, m_2, \ldots m_n$

→ If the body is moving in translational motion then,

$$V_1 = V_2 = \ldots = V_n = a$$

and $a_1 = a_2 = a_3 = \dots\dots = a_n = a$

→ By newton's laws of motion we know that

$F = ma$

then, $\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \dots\dots + \vec{F_n}$

$\vec{F} = m_1\vec{a_1} + m_2\vec{a_2} + m_3\vec{a_3} + \dots\dots + m_n\vec{a_n}$

$\Rightarrow \vec{F} = m_1\vec{a} + m_2\vec{a} + m_3\vec{a} + \dots\dots + m_n\vec{a}$

$\Rightarrow \vec{F} = (m_1 + m_2 + m_3 + \dots + m_n)\vec{a}$

$\Rightarrow \vec{F} = M\vec{a}$

For momentum,

we know $\vec{P} = m\vec{v}$

$\vec{P} = m_1\vec{v_1} + m_2\vec{v_2} + \dots\dots + m_n\vec{v_n}$

$\Rightarrow \vec{P} = m_1\vec{v} + m_2\vec{v} + \dots\dots + m_n\vec{v}$

$\Rightarrow \vec{P} = (m_1 + m_2 + \dots\dots + m_n)\vec{V}$

$\Rightarrow \vec{P} = M\vec{V}$

For kinetic energy

we know, $K.E. = \frac{1}{2}mv^2$

$K.E. = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2 + \dots\dots + \frac{1}{2}m_nv_n^2$

$\Rightarrow K.E. = \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}m_3v^2 + \dots\dots + \frac{1}{2}m_nv^2$

$\Rightarrow K.E. = \frac{1}{2}(m_1 + m_2 + m_3 + \dots\dots + m_n)v^2$

$\Rightarrow K.E. = \frac{1}{2}Mv^2$

2. <u>Pure</u> <u>Rotational</u> <u>Motion</u>:-

→ A rotation is said to be purely rotational motion if each point of the body moves in a circle whose center lies on the axis of rotation and each point traces the same angle in a particular time interval.

→ when a rigid body rotate about a ~~fixed~~ fixed line then that line is called axis of rotation.

→ In pure rotation each particle is rotated with same angular velocity (ω)

	Linear	Rotational/ Angular	Relation between linear and angular quantity
a) Displacement →	s →	θ →	$\Delta\theta = r\Delta s$
b) Velocity →	v →	ω →	$v = r\omega$
c) Acceleration →	a →	α →	$a = r\alpha$
d) Mass →	m →	I (moment of Inertia) →	$I = mr^2$
e) Force →	$\vec{F}$ →	$\vec{\tau}$ (Tau) →	$\vec{\tau} = \vec{r} \times \vec{F}$
f) Momentum →	$\vec{P}$ →	$\vec{L}$ →	$\vec{L} = \vec{r} \times \vec{P}$

Angular Displacement (θ):

→ The angle traced by a particle is called angular displacement.

$$\theta = \frac{arc}{radius} = \frac{s}{r}$$

Angular Velocity (ω):

→ Change in angular position with time is called angular velocity.

$$\omega = \frac{d\theta}{dt}$$

Angular acceleration (α):

→ The rate of change of angular velocity is called as angular acceleration.

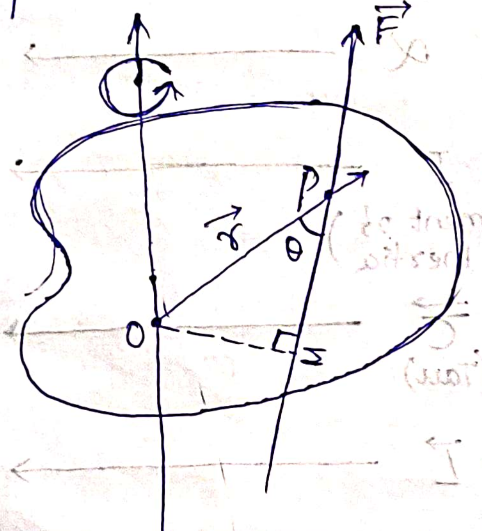
$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{d\theta}{dt}\right) = \frac{d^2\theta}{dt^2}$$

Torque(τ)

→ It is given as the product of force and perpendicular distance of force from the axis of rotation.

→ Let a force $\vec{F}$ acts on a point p of rigid body the axis of rotation passes throw origin.

$\vec{r}$ is the position vector of point p.



→ The rotation of the body due to the torque exerted by a force $\vec{F}$ is given by

Torque = force × (~~Perpend~~ Perpendicular distance of the line of action of a force from axis of rotation)

Draw a perpendicular OM on the line of action of force from the origin.

In triangle OMP

$$\sin\theta = \frac{OM}{OP}$$

$$\Rightarrow \sin\theta = \frac{OM}{r}$$

$$\Rightarrow OM = r\sin\theta$$

Now, Torque $\tau = F \cdot (OM)$

$\Rightarrow \tau = F \cdot r\sin\theta$

$\Rightarrow \tau = r\sin\theta$

$\Rightarrow \vec{\tau} = \vec{r} \times \vec{F}$ → In vector form

→ SI unit of torque is Nm

→ Dimension of torque is

→ Torque is also called as moment of force

∴ We know

$\tau = rF\sin\theta$

Case I, $\theta = 0°$ or $180°$

$\sin\theta = 0$

$\tau = rF(0)$

$\Rightarrow \tau = 0$

Case II, $\theta = 90°$

$\sin\theta = 1$

$\tau = rF(1)$

$\Rightarrow \tau = rF$

Angular Momentum:

→ It is defined as the product of linear momentum and the perpendicular distance of linear momentum from the axis of momentum.

Angular momentum = linear momentum × (Perpendicular distance of linear momentum from the axis of rotation).

→ Let the linear momentum $\vec{p}$ is applied at point P of a rigid body.

Now, Torque $\tau = F \cdot (OM)$

$\Rightarrow \tau = F \cdot r \sin\theta$

$\Rightarrow \tau = r \sin\theta$

$\Rightarrow \vec{\tau} = \vec{r} \times \vec{F}$ → In vector form

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Angular Momentum :

→ It is defined as the product of linear momentum and the perpendicular distance of linear momentum from the axis of momentum.

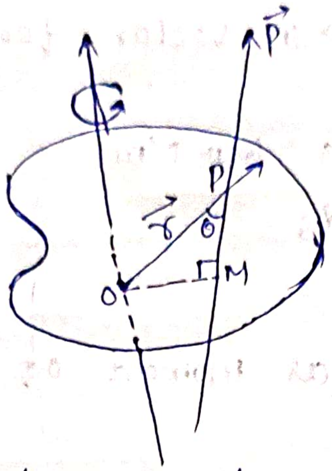
Angular momentum = linear momentum × (Perpendicular distance of linear momentum from the axis of rotation.)

→ Let the linear momentum $\vec{p}$ is applied at point P of a rigid body.

→ The position vector of point P is $\vec{r}$ from the origin.

→ Angular momentum of the particle about O is given by

$L = p.(OM)$



→ Here, OM is perpendicular drawn from origion.

→ In triangle OMP

$$\sin\theta = \frac{OM}{OP}$$

$$\Rightarrow \sin\theta = \frac{OM}{r}$$

$$\Rightarrow OM = r\sin\theta$$

Then, $L = rp\sin\theta$

$\Rightarrow \vec{L} = \vec{r} \times \vec{P}$ ⟶ in vector form

→ SI unit of angular momentum is kgm^2s^{-1} or. Js

∴ We ~~have~~ have

$L = rp\sin\theta$

<u>Case-I</u>, $\theta = 0° \text{ or } 180°$

$\sin\theta = 0$

$\Rightarrow L = rp(0)$

$\Rightarrow L = 0$

<u>Case-II</u>, $\theta = 90°$

$\sin\theta = 1$

$L = rp(1)$

$\Rightarrow L = rp$

$\Rightarrow L = r m v$

$\Rightarrow L = m v r$

Moment of Inertia:-

a) Moment of inertia:

→ Moment of inertia of a particle is defined as the product of mass of the particle and the square of the distance of the particle from the axis of rotation.

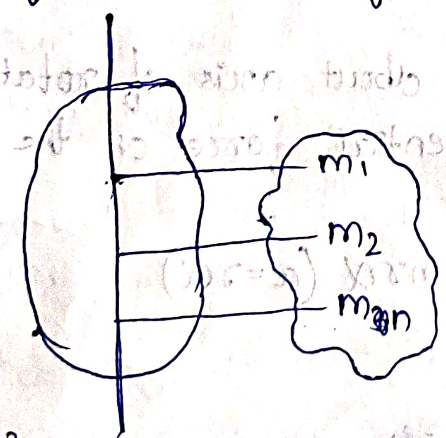
formula, $I = Mr^2$

→ If a body contains n number of particle then the moment of inertia of the body is given as

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots + m_n r_n^2$$

here, $m_1, m_2, m_3, \dots m_n$ are masses and $r_1, r_2, r_3, \dots r_n$ are distance of the particle from axis of rotation

SI unit of moment of inertia is kgm^2



$I = m_1 r_1^2$

$I = m_2 r_2^2$

$I = m_3 r_3^2$

$\vdots$

$I = m_n r_n^2$

$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots m_n r_n^2$

b) Physical Significant of (M.I.):

→ A moment of inertia is a property of a body due to which it opposes the change in it rotational motion.

→ It ~~placed the same~~ plays the same role as the mass in translational motion.

→ Moment of inertia depends on:- (i) mass of the body.

(ii) Distribution of mass (or distance of the particle from the axis of rotation)

c) Relation of torque and moment of inertia:

We know that acting on a body given as

$$\vec{\tau} = \vec{r} \times \vec{F}$$

If position vector r of a body is perpendicular to applied force then $\theta = 90°$, $\sin\theta = 1$

$$\tau = rF\sin\theta$$

$$\Rightarrow \tau = rf$$

→ A particle of mass m rotated about axis of rotation in circle of radius r then tangential force on the particle is

$$F = ma$$

$$\Rightarrow F = mr\alpha \quad (a = r\alpha)$$

Therefore, The torque is given as

$$\tau = rf$$

$$\Rightarrow \tau = r(ma)$$

$$\Rightarrow \tau = rmr\alpha$$

$$\Rightarrow \tau = mr^2\alpha$$

→ If the body n number of particle then the total torque on the body can be ~~obt~~ ~~obtaned~~ obtained by summing ~~over~~ over all torque exerted due to all the forces acting on the particle.

~~Torque~~

$$\tau_{net} = \sum_{i=1}^{n} m_i r_i^2 \alpha = I\alpha$$

$$\tau_{net} = I\alpha$$

here, I is the moment of inertia of a body about axis of rotation.

Net torque of the body,

$$\vec{\tau}_{net} = \sum_{n=1}^{n} \vec{r}_i \times \vec{F}_i$$

(d) <u>Relation of angular momentum and moment of inertia</u>:

We know that, Angular momentum of ~~ab~~ a body is given as $L = \vec{r} \times \vec{P}$

→ If position vector $\vec{r}$ of the body is perpendicular to P then $\theta = 90°$, $\sin\theta = 1$

Therefore, $L = rp\sin\theta$

$\Rightarrow L = rp$

→ A particle of mass m rotated about the axis of rotation in a circle of radius r then angular momentum,

$L = rp$

$\Rightarrow L = r\,mv \quad (v = r\omega)$

$\Rightarrow L = r\,mr\omega$

$\Rightarrow L = mr^2\omega$

→ If the body have n number of particles then the total angular momentum of the body can be obtained by summing the angular momentum of all the particle due to the linear momentum of all the particles of the body.

$$L_{net} = \left(\sum_{i=1}^{n} m_i r_i^2\right)\omega$$

$$L_{net} = I\omega$$

here, I is moment of inertia of the body about the axis of rotation.

Total angular momentum is given as = $\vec{L} = \vec{r} \times \vec{P}$

Conservation of angular momentum and its applications.

Conservation of angular momentum:

we know that, the angular momentum of a body is given as

$$\vec{L} = \sum_{i=1}^{n} \vec{r}_i \times \vec{P}_i$$

Differentiating both side with respect to time (t)

formula for derivating
1) $\frac{d}{dx}(x^n) = n x^{n-1}$
2) $\frac{d}{dx}(\text{constant}) = 0$
3) $\frac{d}{dx}(u \times v) = \frac{du}{dx} \times v + u \times \frac{dv}{dx}$

$$\frac{d}{dt}(\vec{L}) = \sum_{i=1}^{n} \frac{d}{dt}(\vec{r}_i \times \vec{P}_i)$$

$$\frac{d\vec{L}}{dt} = \sum_{i=1}^{n} \frac{d\vec{r}_i}{dt} \times \vec{P}_i + \vec{r}_i \times \frac{d\vec{P}_i}{dt}$$

$$\frac{d\vec{L}}{dt} = \sum_{i=1}^{n} (\vec{v}_i + \vec{P}_i + \vec{r}_i \times \vec{F}_i)$$

$$\frac{d\vec{L}}{dt} = \sum_{i=1}^{n} (\vec{v}_i \times m_i\vec{v}_i + \vec{r}_i \times \vec{F}_i) \quad [\because \vec{v}_i \times \vec{v}_i = 0]$$

$$\frac{d\vec{L}}{dt} = \sum_{i=1}^{n} (\vec{r_i} \times \vec{F_i})$$

$$\frac{d\vec{L}}{dt} = \vec{\tau}_{net}$$

here, $\vec{\tau}_{net}$ total torque on the ~~bad~~ body due to external forces.

→ If $\tau_{net} = 0$, then $\frac{d\vec{L}}{dt} = 0$

$\Rightarrow L =$ ~~contant~~ constant

$\Rightarrow I\omega =$ constant

or, $I_1\omega_1 = I_2\omega_2$

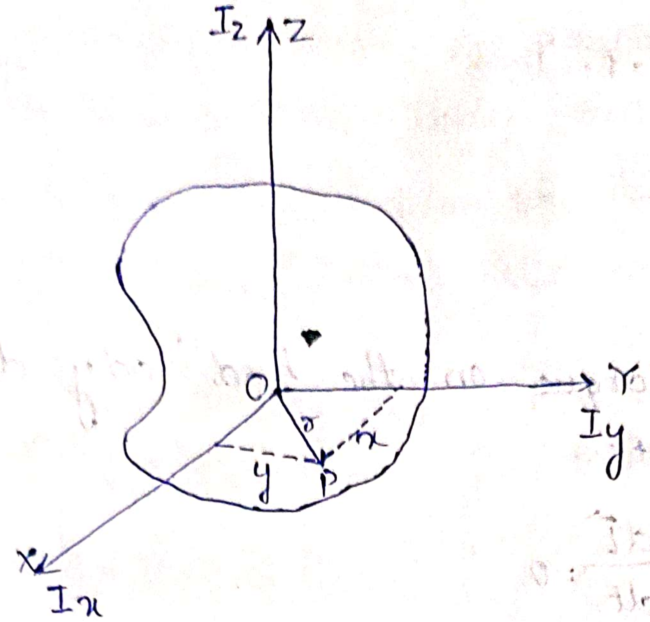
→ If the net external torque on a body is 0, then total angular momentum of the body remains constant. This concept is called principle of conservation of angular momentum.

Theorum of Parallel and Perpendicular Axis :

A. Perpendicular axis Theorum :

→ This theorum states that the ~~momentum~~ moment of Inertia of a lamina about an axis perpendicular to its plane (I_z) is equal to the sum of the moment of inertia of the lamina about two mutually perpendicular axis (I_x and I_y) lie in its plain and intersecting at a point where the perpendicular axis passes.

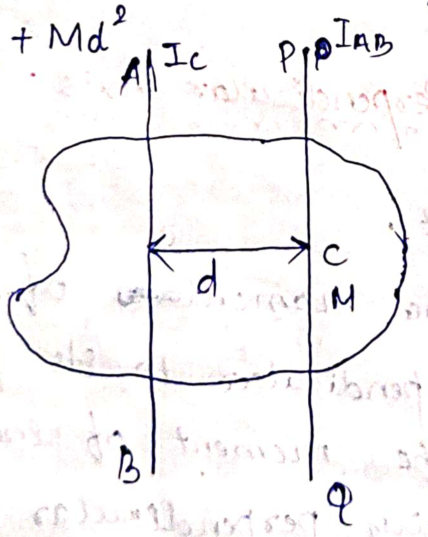
$$I_z = I_x + I_y$$



B. Parallel axis theorem of M.I.

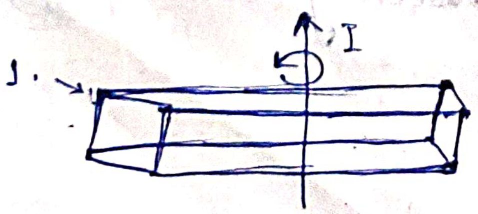
→ This Theorem states that the moment of inertia I_{AB} of a body about any given axis (AB) is equal to the sum of inertia about a parallel axis passes through the centre of mass (C) of the body and the product of the mass M of the body and the square of the distance 'd' between the two parallel axis.

$$I_{AB} = I_c + Md^2$$



Moment of Inertia of the Following Bodies:

(a) Rod:



1. Moment of Inertia about an axis of rotation passes through the centre & perpendicular to the length.

$$I = M\left(\frac{l^2}{12} + \frac{r^2}{4}\right)$$

If $r <<< l$

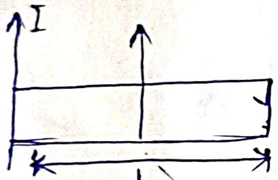
then, $I = \frac{Ml^2}{12}$

M = mass

l = Length

r = Radius

2. Moment of Inertia ~~abou~~ about an axias of rotation passes through 1 end & perpendicular to the length of the rod.



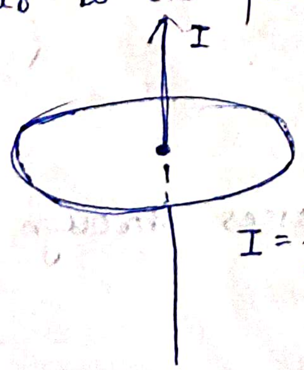
$$I = \frac{Ml^2}{3}$$

3. Moment of Inertia about ~~an~~ the axis ~~passes~~ of rod.

$$I = \frac{1}{2}mr^2$$

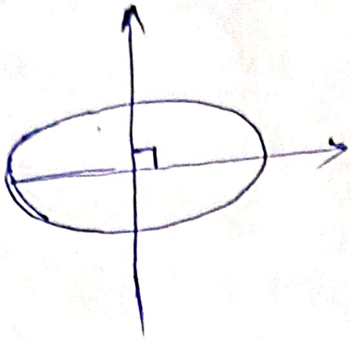
(b) Disc :

1. Moment of Inertia about an axis passes through centre & perpendicular to its ~~plane~~ plain



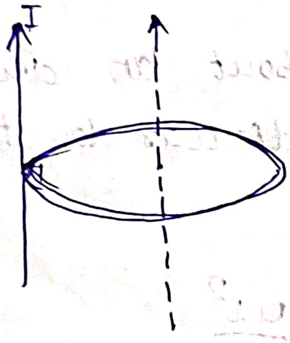
$$I = \frac{1}{2}Mr^2$$

2. Moment of Inertia about an axis passes through a centre & parallel to its ~~plane~~ plain



$I = \frac{1}{4} Mr^2$

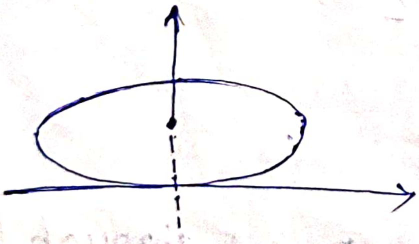
3. Moment of Inertia about an axis is tangent & perpendicular to ~~plane~~ plain



$I = \frac{1}{2} Mr^2$ & Mr^2

$I = \frac{Mr^2 + 2Mr^2}{2} = \frac{3Mr^2}{2} = \frac{3}{2} Mr^2$

4. Moment of Inertia about the tangent lies in ~~plane~~ plain

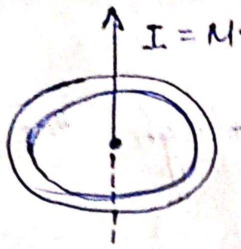


$I = \frac{1}{4} Mr^2 + Mr^2$

$\Rightarrow I = \frac{5}{4} Mr^2$

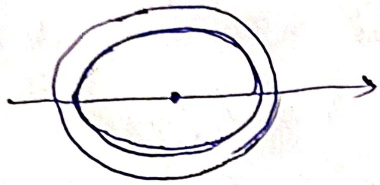
(c) <u>Ring:</u> 

1. Moment of Inertia about the axis passes through centre & Perpendicular to plane.



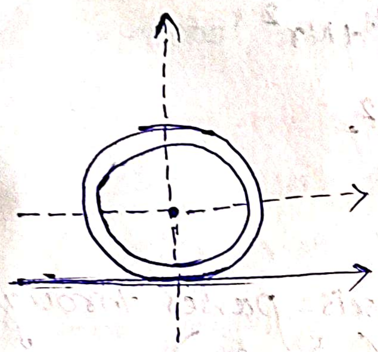
$I = Mr^2$

2. Moment of Inertia about the axis passes through a centre lies in its plain



$I = \frac{1}{2} Mr^2$

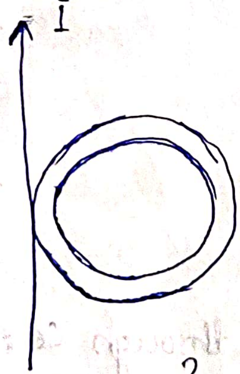
3. Moment of Inertia about an axis as tangent & lies in its plane.



$I = \frac{1}{2} Mr^2 + Mr^2$

$I = \frac{3}{2} Mr^2$

4. Moment of Inertia about tangent & perpendicular to plane.



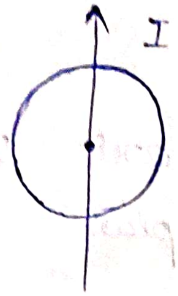
$I = Mr^2 + Mr^2$

$I = 2Mr^2$

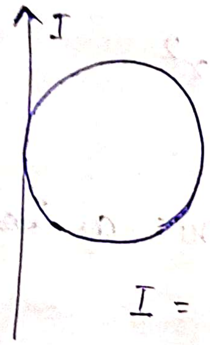
(d) Sphere (Solid):

1. Moment of Inertia about an axis passes through the centre

$$I = \frac{2}{5} M r^2$$



2. Moment of Inertia about tangent



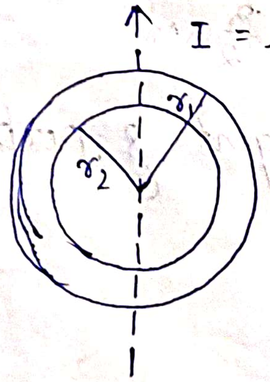
$$I = \frac{2}{5} M r^2 + M r^2$$

$$I = \frac{7}{5} M r^2.$$

(e) Hollow sphere:

1. Moment of Inertia about an axis passes through centre.

$$I = \frac{2}{5} M \left(\frac{r_2^5 \quad r_1^5}{r_2^3 - r_1^3} \right)$$



(f) ~~Spherical~~ Spherical Shell:

1. Moment of Inertia about axis passes through centre or diameter

$$I = \frac{2}{3} M r^2$$

Important Questions.

1. Define translational motion and wright down ity types.

Ans: The motion of a body is purely translational if the velocity of all the particles of the body is the same at any instant of motion.

- During such motion all particles have same displacement, velocity and acceleration at an ~~ato~~ instant.

Types of translational motion

(a) Linear or one dimensional ~~exact~~ motion

(b) Two dimensional motion

(c) Three dimensional motion.

2. Define Pure Rotational motion with examples

Ans: A rotation is said to be purely rotational motion if each ~~point~~ point of the body moves in a circle whose ~~centre~~ centre lies on the axis of rotation and each point traces the same angle in a particular time interval.

- example
- Rotation of the earth about its axis.
- The motion of the wheels in the vehicles.

3. Define Angular Displacement.

Ans: The angle traced by a particle is called angular displacement

$$\theta = \frac{arc}{radius} = \frac{s}{r}$$

4. Define Angular Velocity.

Ans: change in anglular position with time is called angular velocity

$$\omega = \frac{d\theta}{dt}$$

5. Define angular acceleration.

Ans:- The rate of change of angular velocity is called as angular acceleration.

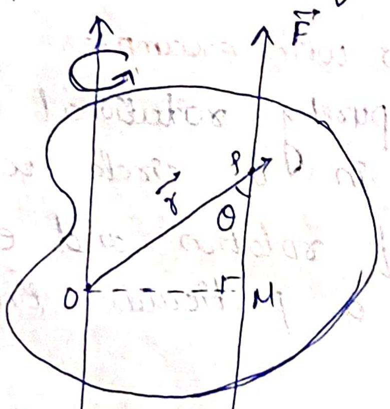
$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\frac{d\theta}{dt} = \frac{d^2\theta}{dt^2}$$

6. What do you mean by torque. Derive its expression.

Ans →It is given as the product of force and perpendicular distance of force from ~~the~~ the axis of rotation.

Let a force $\vec{F}$ acts on a point P of rigid body the axis of rotation ~~process~~ passes through origin.

$\vec{r}$ is the position vector of point p



→ The rotation of the body due to the torque exerted by a force $\vec{F}$ is given by

Torque = force + (perpendicular distance of the line of action of a force from axis of rotation)

• Draw a perpendicular OM on the line of action of force from the origin.

In triangle OMP

$$\sin\theta = \frac{OM}{OP} \Rightarrow \sin\theta = \frac{OM}{r} \Rightarrow OM = r\sin\theta$$

Now, torque. $\tau = F \cdot OM$

$\Rightarrow \tau = F \cdot r\sin\theta$

$\Rightarrow \tau = ~~\ldots~~ \, r s \sin\theta$

$\Rightarrow \vec{\tau} = \vec{r} \times \vec{F}$

→ Torque is also called as moment of ~~...~~ force.

∴ We know
$\vec{\tau} = rF\sin\theta$

Case-I, $\theta = 0°$ or $180°$

$\sin\theta = 0$

$\tau = r\,F(0)$

$\Rightarrow \tau = 0$

Case-II, $\theta = 90°$

$\sin\theta = 1$

$\tau = rF(1)$

$\Rightarrow \tau = rF$

7. Define angular momentum and its examples.

Ans:- It is defined as the product of linear momentum and the perpendicular distance of linear momentum from the axis of momentum.

Angular momentum = linear momentum × (perpendicular distance of linear momentum from the axis of rotation)

→ SI unit of angular momentum is ~~Kgm~~ Kgm^2s^{-1} or Js.

Example

- Orbital angular momentum of the earth due to its revolution about the sun.
- Spin angular momentum of a rotation chair about its axis.

8. What do you mean by moment of Inertia and derive its expression.

Ans:- → Moment of Inertia of a particle is defined as the product of mass of the particle and the ~~...~~ square of the distance of the particle from the axis of rotation

Formula, $I = Mr^2$

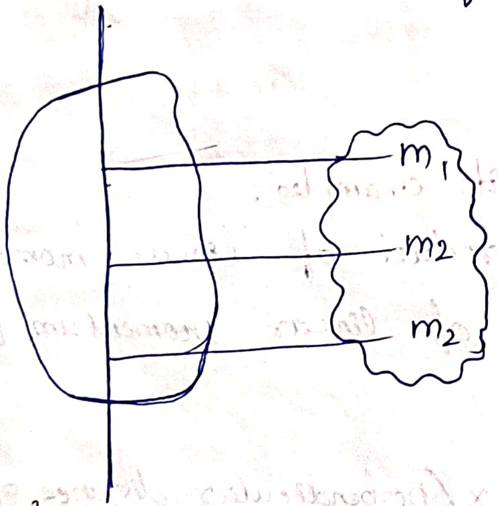
→ If a body contains n number of particles then the moment of inertia of the body is given as

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2 + \text{-------} + m_n r_n^2$$

here, $m_1, m_2, m_3, \text{-----} m_n$ are masses and

$r_1, r_2, r_3, \text{--------} r_n$ are distance of the particle from axis of rotation.

→ SI unit of moment of inertia is kg m^2



$I = m_1 r_1^2$

$I = m_2 r_2^2$

$I = m_3 r_3^2$

$\vdots$

$I = m_n r_n^2$

$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \text{------} + m_n r_n^2$

Q. What is the physical significant of moment of inertia.

Ans - A moment of inertia is a property of a ~~do~~ body due to which it opposes the change in it rotational motion.

→ Moment of inertia depends on :- (i) mass of the body
(ii) Distribution of mass (or distance of the particle from the axis of rotation)

10. Derive a relation between torque and moment of inertia.

Ans: We know that acting on a body given as

$$\vec{\tau} = \vec{r} \times \vec{F}$$

→ If position vector r of a body is perpendicular to applied force then $\theta = 90°$, $\sin\theta = 1$

$$\tau = rF\sin\theta$$
$$\Rightarrow \tau = rF$$

→ A particle of mass m rotated about axis of rotation in circle of radius r then tangential force on the particle is

$$F = ma$$
$$\Rightarrow F = mr\alpha \ (a = r\alpha)$$

Therefore, the torque is given as

$$\tau = rF$$
$$\Rightarrow \tau = r(ma)$$
$$\Rightarrow \tau = r\, m r\alpha$$
$$\Rightarrow \tau = mr^2\alpha$$

→ If the body no. of particles then the total torque on the body can be obtained by summing over all torque exerted due to all the forces acting on the particles.

$$\tau_{net} = \sum_{i=1}^{n} m_i r_i^2 \alpha = I\alpha$$

$$\tau_{net} = I\alpha$$

here, I is the moment of inertia of a body about axis of rotation.

Net torque of the body,

$$\vec{\tau}_{net} = \sum_{i=1}^{n} \vec{r_i} \times \vec{F_i}$$

11. Derive a relation between angular momentum and moment of Inertia.

Ans: We know that angular momentum of a body is given as

$L = \vec{r} \times \vec{p}$

→ If position vector $\vec{r}$ of the body is perpendicular to p

then $\theta = 90°$, $\sin 90° = 1$

Therefore, $L = rP\sin\theta$

$\Rightarrow L = rP$

→ A particle of mass m rotated about the axis of rotation in a circle of radius r then angular momentum.

$L = rp$

$\Rightarrow L = rmv$ ($\because v = r\omega$)

$\Rightarrow L = rmr\omega$

$\Rightarrow L = mr^2\omega$

→ If the body have n no. of particles then the total angular momentum of the body can be obtained by summing the angular momentum of all the particle due to the linear momentum of all the particles of the body.

$$L_{net} = \left(\sum_{i=1}^{n} m_i r_i^2\right)\omega$$

$$L_{net} = I\omega$$

here, I is moment of Inertia of the body about the axis of rotation.

→ Total angular momentum is given as $\vec{L} = \vec{r} \times \vec{p}$

12. Derive an expression for conservation of angular momentum.

Ans: We know that the angular momentum of a body is given as

$$\vec{L} = \sum_{i=1}^{n} \vec{r_i} \times \vec{P_i}$$

Differentiating both side with the respect to time (t)

$$\frac{d}{dt}(\vec{L}) = \sum_{i=1}^{n} \frac{d}{dt}(\vec{r_i} \times \vec{P_i})$$

$$\Rightarrow \frac{d\vec{L}}{dt} = \sum_{i=1}^{n} \frac{d\vec{r_i}}{dt} \times \vec{P_i} + \vec{r_i} \times \frac{d\vec{P_i}}{dt}$$

$$\Rightarrow \frac{d\vec{L}}{dt} = \sum_{i=1}^{n} (\vec{V_i} + \vec{P_i} + \vec{r_i} \times \vec{F_i})$$

$$\Rightarrow \frac{d\vec{L}}{dt} = \sum_{i=1}^{n} (\vec{V_i} \times m_i\vec{v_i} + \vec{r_i} + \vec{F_i}) \left[\because \vec{V_i} \times \vec{V_i} = 0\right.$$

$$\Rightarrow \frac{d\vec{L}}{dt} = \sum_{i=1}^{n} (\vec{r_i} \times \vec{F_i})$$

$$\Rightarrow \frac{d\vec{L}}{dt} = \vec{\tau}_{net}$$

here, $\vec{\tau}_{net}$ total torque on the body due to external forces.

→ If $\tau_{net} = 0$, then $\frac{d\vec{L}}{dt} = 0$

$$\Rightarrow L = \text{constant}$$

$$\Rightarrow I\omega = \text{constant}$$

$$\text{or } I_1\omega_1 = I_2\omega_2$$

→ If the net external torque on a body is 0, then total angular momentum of the body remains constant.

This concept is called principle of conservation of angular momentum.

13. Define radius of Gyration.

Ans: Moment of inertia of a body is given as

$$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 \cdots\cdots m_nr_n^2 = \sum_{i=1}^{n} m_ir_i^2$$

Moment of inertia of the body can be written as

$$I = M \times K^2$$

Here, M is the total mass of the body and K is the radius of gyration.

$MK^2 = \sum_{i=1}^{n} m_i r_i^2$ thus, the radius of gyration $K = \sqrt{\frac{\sum_{i=1}^{n} m_i r_i^2}{M}}$

→ The radius of gyration K of a rotating body is equal to the radial distance from the axis of ~~axi~~ rotation, the square of which multiplied by the total mass of the body gives the moment of inertia of that body.

14. State theorum of parallel and perpendicular axis.

Ans:- Perpendicular axis:- This theorum states that the moment of inertia of a lamina about an axis perpendicular to its plain (I_z) is equal to the sum of the moment of inertia of the lamina about two mutually perpendicular axis (I_x and I_y) lie in its plain and intersecting at a point where the perpendicular axis ~~pare~~ passes.

$$I_z = I_x + I_y$$

Parallel axis theorum:-

→ ~~The~~ This theorum states that the moment of inertia I_{AB} of a body about ~~any~~ any given axis (AB) is equal to the sum of moment of inertia about a parallel axis passes through the centre of mass (c) of the body and the product of the mass M of the body and the square of the distance 'd' between the two parallel axis.

$$I_{AB} = I_c + Md^2$$

15. Moment of Inertia of a rigid body.

MI:- (a) Rod:

1. Moment of inertia about an axis of rotation passes through the centre & perpendicular to the length.

$$I = M\left(\frac{L^2}{12} + \frac{r^2}{4}\right)$$

mass = m
length = l
radius = r

If $r << L$

then, $I = \frac{ML^2}{12}$

2. MI about an axis of rotation passes through one end & perpendicular to the length of the rod.

$$I = \frac{ML^2}{3}$$

3. MI about an axis of rod.

$$I = \frac{1}{2}mr^2$$

(b) Disc:

1. MI about an axis passes through centre & perpendicular to its plain.

$$I = \frac{1}{2}Mr^2$$

2. MI about an axis passes through a centre & parallel to its plain.

$$I = \frac{1}{4}Mr^2$$

3. MI about an axis i.e. tangent & perpendicular to plain.

$$I = \frac{1}{2}Mr^2 + Mr^2$$

$$\Rightarrow I = \frac{3}{2}Mr^2$$

4. MI about the tangent lies in plain

$$I = \frac{1}{4}Mr^2 + Mr^2$$

$$\Rightarrow I = \frac{5}{4}Mr^2$$

(c) Ring:

1. M.I about the axis passes through centre & perpendicular to plain.

$$I = Mr^2$$

~~2. M.I about the axis passes through a centre lies in its plain.~~

$$\cancel{I = \frac{1}{4}Mr^2}$$

~~3. M.I about an axis~~

2. M.I about the axis passes through a centre lies in its plain.

$$I = \frac{1}{2}Mr^2$$

3. MI about an axis as tangent & lies in its plain.

$$I = \frac{1}{2}Mr^2 + Mr^2$$
$$= \frac{3}{2}Mr^2$$

4. MI about tangent & perpendicular to plain

$$I = Mr^2 + Mr^2$$
$$= 2Mr^2$$

(d) Sphere (Solid):

1. MI about an axis passes through the centre.

$$I = \frac{2}{5}Mr^2$$

2. MI about tangent

$$I = \frac{2}{5}Mr^2 + Mr^2 = \frac{7}{5}Mr^2$$

(e) Hollow sphere:

1. MI about an axis passes through centre

(c) Ring:

1. M.I about the axis passes through centre & perpendicular to plain.

$$I = Mr^2$$

~~2. M.I about the axis passes through a centre lies in its plain.~~

$$\cancel{I = \frac{1}{4}Mr^2}$$

~~3. M.I about an axis.~~

2. M.I about the axis passes through a centre lies in its plain.

$$I = \frac{1}{2}Mr^2$$

3. MI about an axis as tangent & lies in its plain.

$$I = \frac{1}{2}Mr^2 + Mr^2$$
$$= \frac{3}{2}Mr^2$$

4. MI about tangent & perpendicular to plain

$$I = Mr^2 + Mr^2$$
$$= 2Mr^2$$

(d) Sphere (Solid):

1. MI about an axis passes through the centre.

$$I = \frac{2}{5}Mr^2$$

2. MI about tangent

$$I = \frac{2}{5}Mr^2 + Mr^2 = \frac{7}{5}Mr^2$$

(e) Hollow sphere:

1. MI about an axis passes through centre

$$I = \frac{2}{5}M\left(\frac{r_2^5 - r_1^5}{r_2^3 - r_1^3}\right)$$

(f) Spherical shell:

1. MI about an axis passes through centre.

$$I = \frac{2}{3}Mr^2$$